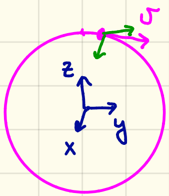
9.8 Coriolis $2m[\dot{\vec{r}} \times \vec{\Omega}]$ Centrif. $m([\vec{\Omega} \times \vec{r}] \times \vec{\Omega})$

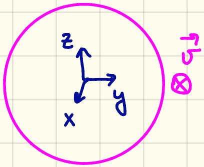
a)



$F_{cent.} \sim$ south

$\vec{F}_{coriolis} \sim [\vec{v} \times \vec{\Omega}] = v \cdot \Omega \cdot [\hat{y} \times \hat{z}] = v \cdot \Omega \cdot \hat{x}$ - west

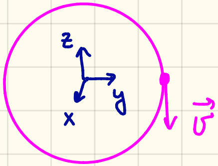
b)



$\vec{F}_{cent} = m\Omega^2 \cdot r \cdot [[\hat{z} \times \hat{y}] \times \hat{z}] = m\Omega^2 r [-\hat{x} \times \hat{z}] = m\Omega^2 r \hat{y}$ - up

$\vec{F}_{coriolis} \sim m r \Omega [-\hat{x} \times \hat{z}] = m r \Omega \hat{y}$ - up

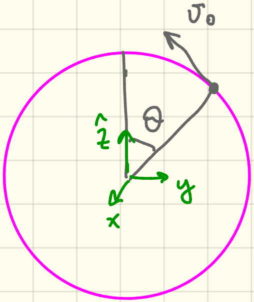
c)



$\vec{F}_{cent} = m\Omega^2 r [[\hat{z} \times \hat{y}] \times \hat{z}] = m\Omega^2 r \hat{y}$ - up

$\vec{F}_{cor} = m r \Omega [-\hat{z} \times \hat{z}] = 0$

9.9



$$\vec{v}_0 = v_0\left(-\cos\theta\,\hat{y} + \sin\theta\,\hat{z}\right)$$

$$\vec{F}_{cor} = 2m\left[\vec{v}_0 \times \vec{\Omega}\right] =$$

$$= 2m v_0 \Omega\left(-\cos\theta\left[\hat{y}\times\hat{z}\right] + \sin\theta\left[\hat{z}\times\hat{z}\right]\right) =$$

$$\boxed{\vec{F}_{cor} = -2m v_0 \Omega \cos\theta\,\hat{x}} \to \underline{\text{east}}$$

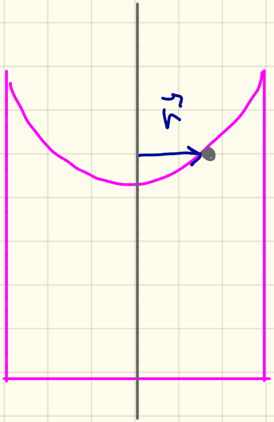
$\theta = 40^0$, $v_0 = 10^3\, m/s$, $\Omega = \dfrac{2\pi}{24\cdot 3600} \simeq 7.3\cdot 10^{-5}$

$\cos\theta \sim 0.77$

$F_{cor} \sim 0.1\cdot m$ $\qquad \dfrac{F_{cor}}{W} \sim 10^{-2}$

$W = mg \simeq 10m$

9:14



Centrifugal force

$$\vec{F}_c = m\Omega^2 \vec{r}.$$ $\Leftarrow$ depends only on coordinate! conservative

$$U_c(r) = -\int_0^r m\Omega^2 r\, dr = -\frac{1}{2} m\Omega^2 r^2$$

$$U_{grav} = mgz$$

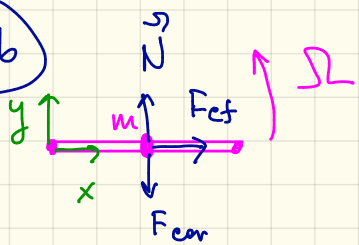
$$U = mgz - \frac{1}{2} m \Omega^2 r^2$$

Water level is equipotential. Therefore

$$z = \frac{\Omega^2}{2g} r^2 + z_0$$

$\hookrightarrow : \dfrac{U_0}{mg}$

9.16



$y = 0$ (rod is solid)

$\vec{F}_{cf} = m\Omega^2 x \cdot \hat{x}$

$\vec{F}_{cor} = 2m\dot{x}\Omega \cdot (-\hat{y}) = -2m\Omega\dot{x}\hat{y}$

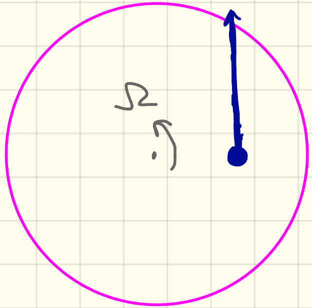
$m\ddot{x} = m\Omega^2 x$

$x = Ae^{\Omega x} + Be^{-\Omega x}$, $A, B \to$ constants

F_{cor} does not change the motion, and is balanced by the normal force $\vec{N}$

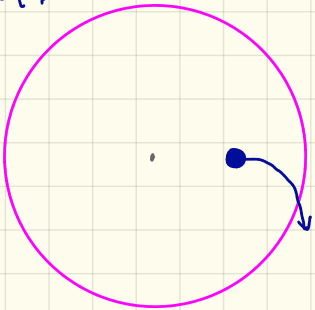
9.19

a) stationary observer:



once released, puck moves with constant velocity

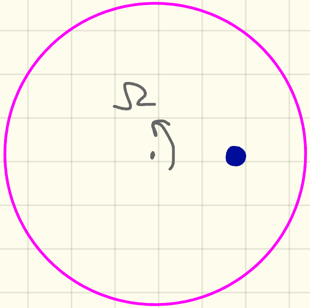
me on the merry-go-round:



centrifugal force is radial, and Coriolis force move it to the right.

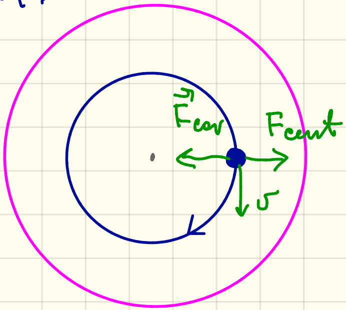
9.19 cont.

b) stationary observer:



← puck remains at rest (no friction!)

me on the merry-go-round:

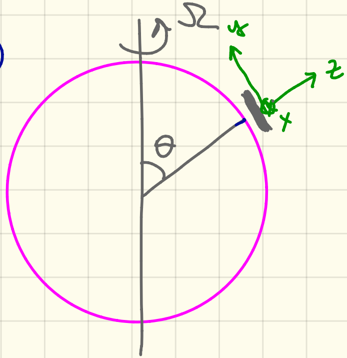


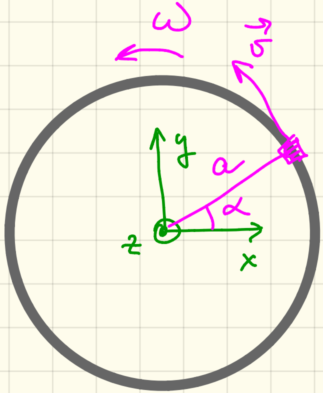
← $\vec{F}_{cor}$ and $\vec{F}_{cent}$ are radial, their sum is mv^2/r → needed centripetal acceleration

$$|\vec{F}_{cent}| = m\Omega^2 r$$

$$|\vec{F}_{cor}| = 2mv\cdot\Omega = 2m\Omega r\cdot\Omega = 2m\Omega^2 r$$

9.30





xyz system rotation:

$$\vec{\Omega} = (0, \Omega\sin\theta, \Omega\cos\theta)$$

$$\vec{v} = (-\omega a\sin\alpha, \omega a\cos\alpha, 0)$$

$$d\vec{F}_{cor} = 2dm[\vec{v}\times\vec{\Omega}] = 2dm\,\Omega\omega a\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ -\sin\alpha & \cos\alpha & 0\\ 0 & \sin\theta & \cos\theta\end{vmatrix} =$$

$$d\vec{F}_{cor} = 2m\Omega\omega a\left(\hat{x}\cos\alpha\cos\theta + \hat{y}\sin\alpha\cos\theta - \hat{z}\sin\alpha\sin\theta\right)$$

$$d\vec{\tau} = \vec{a}\times d\vec{F}_{cor} = (a\cos\alpha, a\sin\alpha, 0)\times d\vec{F}_{cor} =$$

$$d\tau = 2 m\omega\Omega a^2 \begin{array}{ccc} \hat{x} & \hat{y} & \hat{z} \end{array}$$

$$\begin{vmatrix} \cos\alpha\cos\theta & \sin\alpha\cos\theta & -\sin\alpha\sin\theta \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} =$$

$$-\hat{x}\sin^2\alpha\cdot\sin\theta - \hat{y}\sin\alpha\cos\alpha\sin\theta + \hat{z}\left(\cancel{\cos\alpha\sin\alpha\cos\theta} - \cancel{\cos\alpha\sin\alpha\cos\theta}\right)$$

$$-\sin\theta\left(\hat{x}\sin^2\alpha + \hat{y}\sin\alpha\cos\alpha\right)$$

$$d\vec{\tau} = -2m\omega\Omega a^2\sin\theta\left(\hat{x}\sin^2\alpha + \hat{y}\sin\alpha\cos\alpha\right)$$

$$\int_0^{2\pi}\sin^2\alpha\, d\alpha = \frac{1}{2} \qquad \int_0^{2\pi}\sin\alpha\cos\alpha\, d\alpha = \frac{1}{2}\int_0^{2\pi}\sin 2\alpha\, d\alpha = 0$$

$$\boxed{\vec{\tau} = -m\omega\Omega a^2\sin\theta\cdot\hat{x}}$$

$-\hat{x} \equiv$ west in our system