7.1

$$L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + \frac{1}{2}m\dot{z}^2 - mgz$$

$$\frac{\partial L}{\partial x} = 0 \qquad \frac{\partial L}{\partial y} = 0 \qquad \frac{\partial L}{\partial z} = -mg$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \qquad \frac{\partial L}{\partial \dot{y}} = m\dot{y} \qquad \frac{\partial L}{\partial \dot{z}} = m\dot{z}$$

1) $m\ddot{x} = 0$ $\qquad a_x = 0$

2) $m\ddot{y} = 0$ $\qquad a_y = 0$

3) $-mg - m\ddot{z} = 0$ $\qquad a_z = -g$, as expected

7.3

$U = \frac{1}{2} k r^2 \qquad r^2 = x^2 + y^2$

$L = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 - \frac{1}{2} k x^2 - \frac{1}{2} k y^2$

$\frac{\partial L}{\partial x} = kx \qquad \frac{\partial L}{\partial y} = ky \qquad \frac{\partial L}{\partial \dot{x}} = m\dot{x} \qquad \frac{\partial L}{\partial \dot{y}} = m\dot{y}$

$$kx - m\ddot{x} = 0 \qquad ky - m\ddot{y} = 0 \quad \rightarrow \quad \boxed{\begin{array}{l} x = A_1 \cdot \cos(\omega t + \delta_1) \\ y = A_2 \cdot \cos(\omega t + \delta_2) \end{array}} \qquad \omega = \sqrt{\frac{k}{m}}$$

$A_1, A_2, \delta_1, \delta_2 \rightarrow$ constants

generally: an ellipse

7.8

$$U = \frac{1}{2} k (x_1 - x_2 - \ell)^2$$

a) $$L = \frac{1}{2} m \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2 - \frac{1}{2} k (x_1 - x_2 - \ell)^2$$

b) $X = \frac{1}{2}(x_1 + x_2)$ — center of mass

$\varkappa = x_1 - x_2 - \ell$ — extention of the spring.

$x_1 = 2X - x_2$

add

$$\varkappa + \cancel{x_1} = \cancel{x_1} - x_2 - \ell + 2X - x_2$$

$$2x_2 = 2X - \ell - \varkappa$$

$$\begin{cases} x_2 = X - \frac{1}{2}(\ell + \varkappa) \\ x_1 = X + \frac{1}{2}(\ell + \varkappa) \end{cases}$$

$$\begin{cases} \dot{x}_1 = \dot{X} + \frac{1}{2}\dot{\varkappa} \\ \dot{x}_2 = \dot{X} - \frac{1}{2}\dot{\varkappa} \end{cases}$$

$$L = \frac{1}{2} m \left(\dot{X} + \frac{1}{2}\dot{\varkappa}\right)^2 + \frac{m}{2}\left(\dot{X} - \frac{1}{2}\dot{\varkappa}\right)^2 - \frac{1}{2} k \varkappa^2$$

$$L = \frac{1}{2} m \left(\dot{X} + \frac{1}{2}\dot{x}\right)^2 + \frac{1}{2}\left(\dot{X} - \frac{1}{2}\dot{x}\right)^2 - \frac{1}{2} k x^2$$

$$\frac{\partial L}{\partial X} = 0 \qquad \frac{\partial L}{\partial x} = -kx$$

$$\frac{\partial L}{\partial \dot{X}} = m\left(\dot{X} + \frac{1}{2}\dot{x}\right) + m\left(\dot{X} - \frac{1}{2}\dot{x}\right) = 2m\dot{X}$$

$$\frac{\partial L}{\partial \dot{x}} = m\left(\dot{X} + \frac{1}{2}\dot{x}\right)\frac{1}{2} + m\left(\dot{X} - \frac{1}{2}\dot{x}\right)\left(-\frac{1}{2}\right) = \frac{1}{2} m \dot{x}$$

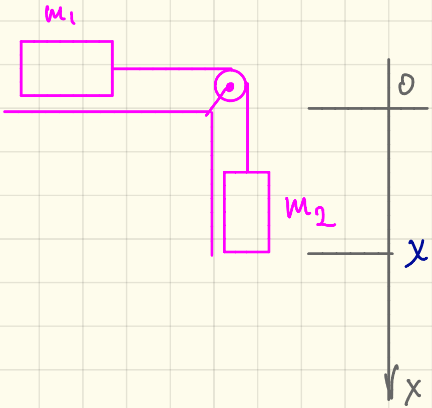
$$\boxed{\begin{aligned} \ddot{X} &= 0 \\ m\ddot{x} &= -2kx \end{aligned}}$$

c) $X = A + Bt$ → motion of center of mass with constant speed

$x = C\cos(\omega t + \delta)$ → oscillations

$$\omega = \sqrt{\frac{2k}{m}}$$

7.15



$$\mathcal{L} = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{x}^2 + m_2 g x$$

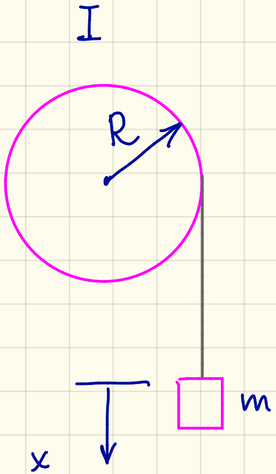
$$\frac{\partial \mathcal{L}}{\partial x} = m_2 g$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = (m_1 + m_2)\dot{x}$$

$$\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} = 0 \implies m_2 g = (m_1 + m_2)\ddot{x}$$

$$\boxed{\ddot{x} = \frac{m_2}{m_1 + m_2} g}$$

7.18



Constraint: $\omega R = \dot{x}$

$$\mathcal{L} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} I \omega^2 + mgx =$$

$$= \frac{1}{2} \left(m + I/R^2 \right) \dot{x}^2 + mgx$$

$$\frac{\partial \mathcal{L}}{\partial x} = mg \qquad \frac{\partial \mathcal{L}}{\partial \dot{x}} = \left(m + \frac{I}{R^2} \right) \dot{x}$$

$$\frac{mR^2 + I}{R^2} \ddot{x} = mg$$

$$\boxed{\ddot{x} = \frac{mR^2}{mR^2 + I} \cdot g}$$