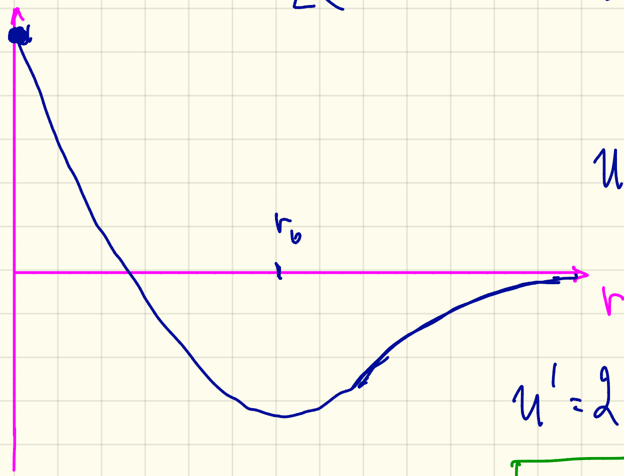
(5.2) $u(r) = A\left[\left(e^{(R-r)/s} - 1\right)^2 - 1\right]$ $s \ll R$



for $r \to \infty$

$u = A\left(\left(1 - e^{-r/s}\right)^2 - 1\right) = -2Ae^{-r/s}$

$r = 0: \quad u \approx e^{2R/s}$

$u' = 2A\left(e^{\frac{R-r_0}{s}} - 1\right)\left(-\frac{r_0}{s}\right) \cdot e^{\frac{R-r_0}{s}} = 0$

$\boxed{r_0 = R}$

$r = r_0 + x = R + x$

$u(x) = A\left[\left(e^{x/s} - 1\right)^2 - 1\right] = A \cdot \left(\frac{x}{s}\right)^2 - A$

$F(x) = -\frac{du}{dx} = -\frac{2A}{s^2} \cdot x$

$\boxed{k = \frac{2A}{s}}$

5.6

$t = 0 \quad x = x_0$

$A = 2x_0$

$$x = A \cdot \cos(\omega t - \delta) = 2x_0 \cos(\omega t - \delta)$$

$$x(0) = x_0 = 2x_0 \cdot \cos\delta \,, \quad \cos\delta = \frac{1}{2} \,, \quad \delta = \pm\frac{\pi}{3}$$

$$v(0) < 0 \,; \quad v = -2x_0\omega \sin(\omega t - \delta)$$

$$v(0) = 2x_0\omega \sin\delta < 0 \Rightarrow \delta = -\frac{\pi}{3}$$

$$\boxed{x = 2x_0 \cdot \cos\left(\omega t + \frac{\pi}{3}\right)}$$

5.23

$$m\ddot{x} + b\dot{x} + kx = 0$$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2$$

$$\frac{dE}{dt} = \frac{1}{2}k \cdot 2x\dot{x} + \frac{1}{2}m 2\dot{x}\ddot{x} = kx\dot{x} + \dot{x}(-b\dot{x} - kx) =$$

$$\frac{dE}{dt} = -b(\dot{x})^2$$

$$F_{damp} = -b\dot{x}$$

$$dW = F_{damp} \cdot dx$$

$$\frac{dW}{dt} = F_{damp} \cdot \dot{x} = -b(\dot{x})^2$$

5.30 **Overdamped:**

$$x(t) = c_1 e^{-(\beta - \sqrt{\beta^2-\omega_0^2})t} + c_2 e^{-(\beta + \sqrt{\beta^2-\omega_0^2})t}$$

$$\dot{x} = -(\beta - \sqrt{\beta^2-\omega_0^2})\, c_1 e^{-(\beta - \sqrt{\beta^2-\omega_0^2})t} - (\beta + \sqrt{\beta^2-\omega_0^2})\, c_2 e^{-(\beta + \sqrt{\beta^2-\omega_0^2})t}$$

a) $x(0) = x_0$, $v(0) = v_0$:

$$x_0 = c_1 + c_2$$

$$v_0 = -(\beta - \sqrt{\beta^2-\omega_0^2})c_1 - (\beta + \sqrt{\beta^2-\omega_0^2})c_2$$

$$c_1 = x_0 - c_2$$

$$v_0 = -\beta x_0 + \sqrt{\ }\, x_0 + \beta c_2 - \sqrt{\ }\cdot c_2 - \beta c_2 - \sqrt{\ }\cdot c_2 = -\beta x_0 + \sqrt{\ }\cdot x_0 - 2c_2\sqrt{\ }$$

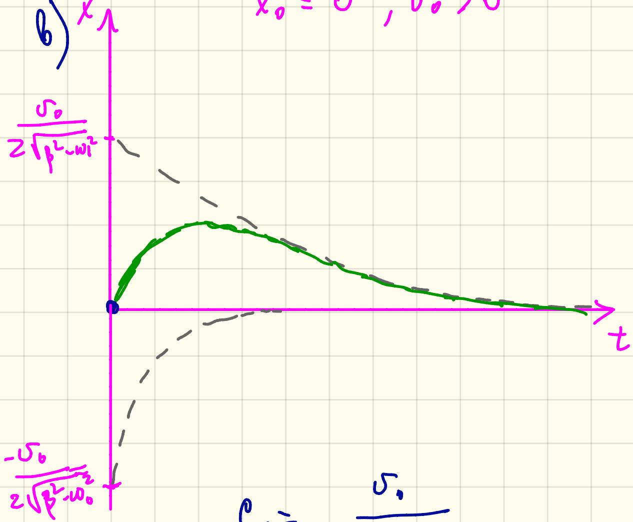
$$2c_2\sqrt{\ } = -\beta x_0 - v_0 + \sqrt{\ }\cdot x_0$$

$$\boxed{c_2 = \frac{x_0}{2} - \frac{\beta x_0 + v_0}{2\sqrt{\beta^2-\omega_0^2}}}$$

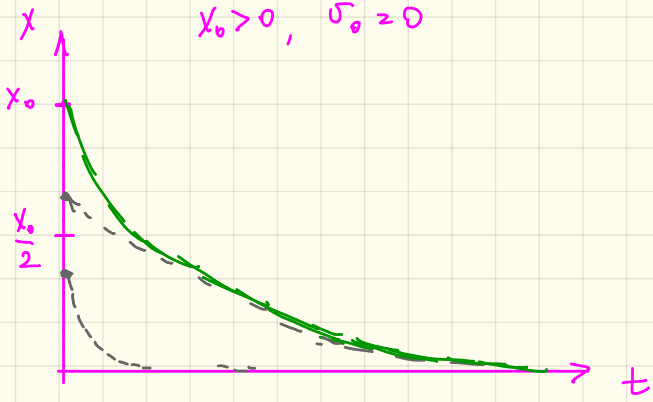
$$\boxed{c_1 = \frac{x_0}{2} + \frac{\beta x_0 + v_0}{2\sqrt{\beta^2-\omega_0^2}}}$$

5.30 cont

b) $x_0 = 0\ ,\ v_0 > 0$



$x_0 > 0\ ,\ v_0 = 0$



$$C_1 = \frac{v_0}{2\sqrt{\beta^2-\omega_0^2}}$$

$$C_2 = -\frac{v_0}{2\sqrt{\beta^2-\omega_0^2}}$$

$$C_1 = \frac{x_0}{2}\left(1 + \frac{\beta}{\sqrt{\beta^2-\omega_0^2}}\right)$$

$$C_2 = \frac{x_0}{2}\left(1 - \frac{\beta}{\sqrt{\beta^2-\omega_0^2}}\right)$$

5.30 cont

$\beta \to 0$:

$$C_1 = \frac{x_0}{2} + \frac{v_0}{2i\omega_0}$$

$$C_2 = \frac{x_0}{2} - \frac{v_0}{2i\omega_0}$$

$$x = \left(\frac{x_0}{2} + \frac{v_0}{2i\omega_0}\right) e^{i\omega_0 t} + \left(\frac{x_0}{2} - \frac{v_0}{2i\omega_0}\right) e^{-i\omega_0 t} =$$

$$= x_0 \cdot \frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2} + \frac{v_0}{\omega_0} \frac{e^{i\omega_0 t} - e^{-i\omega_0 t}}{2i} =$$

$$= x_0 \cdot \cos \omega_0 t + \frac{v_0}{\omega_0} \cdot \sin \omega_0 t$$

↑ correct non-damped solution!