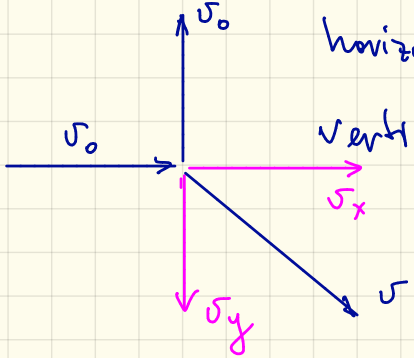
3,2



horizontal: $m v_0 = \frac{1}{2} m v_x \Rightarrow v_x = 2 v_0$

vertical: $0 = \frac{1}{2} m v_0 - \frac{1}{2} m v_y \Rightarrow v_y = v_0$

$$v = v_0 \sqrt{5}$$

3.8 Adding impulse of the weight of the rocket to momentum conservation for the rocket:

$$m\,dv = -u\,dm - mg\,dt$$

$$dv = -u\frac{dm}{m} - g\,dt \qquad \text{Hovers: } v = 0$$

Integrating:

$$0 = -u \ln\frac{m}{m_0} - gt$$

$$\boxed{t = \frac{u}{g}\ln\frac{m_0}{m}}$$

a) $m = m_0(1-\lambda)$, $\underline{t = \frac{u}{g}\ln\frac{1}{1-\lambda}}$

b) $t \approx \frac{3000}{10}\ln\frac{1}{0.9} \approx 300\cdot\ln(1+0.1) \approx$

$\approx 300\cdot(0.1) \simeq 30\,s$

(3.14) $m\dot{v} = -u\dot{m} - bv = -bv + \underbrace{ku}_{\text{constant}}$ $\qquad \frac{dm}{dt} = -k$

The final answer does not explicitely depend on t, only m (which is function of t)

$$\dot{v} = \frac{dv}{dt} = \frac{dv}{dm}\cdot\frac{dm}{dt} = \frac{dv}{dm}\cdot(-k)$$

$-km\frac{dv}{dm} = -bv + ku$ $\rightarrow$ separate variables:

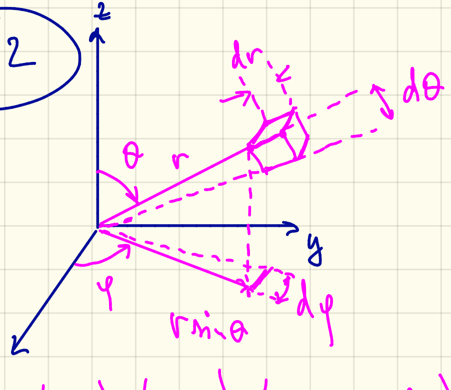
$$-\frac{k}{b}\frac{dv}{\frac{ku}{b} - v} = \frac{dm}{m} \Rightarrow \frac{k}{b}\ln\frac{\frac{ku}{b} - v}{\frac{ku}{b}} = \ln\frac{m}{m_0}$$

$$1 - \frac{b}{ku}v = \left(\frac{m}{m_0}\right)^{b/k} \Rightarrow \boxed{v = \frac{ku}{b}\left(1 - \left(\frac{m}{m_0}\right)^{b/k}\right)}$$

(3,25) The string makes a "central" force $\to \ell = \text{const}$.

$$\ell = m v r = m \omega r^2 \qquad \omega_0 r_0^2 = \omega r^2 \qquad \boxed{\omega = \omega_0 \frac{r_0^2}{r^2}}$$

(3,32)



$$I = \int_V \rho(r) (r \sin\theta)^2 d\tau = \int_0^{\pi}\int_0^{2\pi}\int_0^R \rho \cdot r^2 \cdot \sin^3\theta d\theta \cdot d\varphi \cdot r^2 dr =$$

$$= \rho \int_0^{\pi} \sin^3\theta d\theta \cdot \int_0^{2\pi} d\varphi \int_0^R r^4 dr =$$

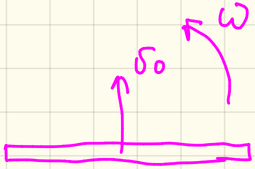
$$dV = (dr)(r d\theta)\cdot(r \cdot \sin\theta \cdot d\varphi)$$

$$\rho \cdot \frac{4}{3}\pi R^3 = M$$

$$= M \cdot \frac{\cancel{3}}{\cancel{4\pi} R^3} \cdot \frac{\cancel{4}}{3} \cdot 2\cancel{\pi} \frac{R^5}{5} = \boxed{\frac{2}{5} M R^2 = I}$$

$$\int_0^{\pi} \sin^3\theta d\theta = \left| \begin{array}{l} t = \cos\theta \\ dt = -\sin\theta d\theta \end{array} \right| = -\int_{-1}^{1} (1-t^2) dt = \int_{-1}^{1} (1-t^2) dt = 2 - 2\frac{1}{3} = \frac{4}{3}$$

3.34



Time it takes for the rod to return:

$$v = v_0 - g t^{top} = 0 \Leftarrow \text{at the top}$$

$$t^{top} = v_0 / g$$

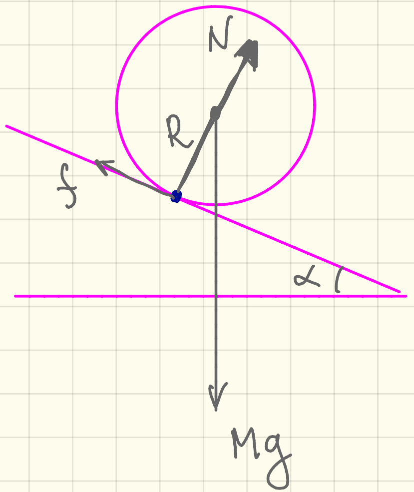
$$t_{return} = 2 t^{top} = \frac{2 v_0}{g}$$

No forces except gravity → no torge w.r.t. CM → $\omega = \text{const}$

time for n rotations $t_n = \frac{2\pi}{\omega} \cdot n$

$$\frac{2\pi}{\omega} \cdot n = \frac{2 v_0}{g} \Rightarrow v_0 = n \frac{\pi g}{\omega}$$

Extra class problem



from the class: $a = \frac{MR^2}{MR^2+I} \cdot g \sin\alpha$

$$\dot{\omega} = a/R = \frac{MR}{MR^2+I} \cdot g \sin\alpha$$

$$I\dot{\omega} = f \cdot R$$

$$f = \frac{I}{R}\dot{\omega} = \frac{MI}{MR^2+I} g \sin\alpha < f_{max}$$

$$f_{max} = \mu N = \mu M g \cos\alpha$$

$$\frac{\cancel{M} I}{MR^2+I} \cancel{g} \sin\alpha < \cancel{M}\cancel{g} \mu \cos\alpha$$

$$\boxed{\mu > \tan\alpha \cdot \frac{I}{I+MR^2}}$$

for $I=0$ wheel never slips!

for max I, $I = MR^2$ (all mass in the rim)

$\mu > \frac{1}{2}\tan\alpha$

extra class problem: (another way, without torque)

$$a = \frac{MR^2}{I + MR^2} g \sin\alpha \Rightarrow \text{Net force is } \frac{MR^2}{I+MR^2} Mg\sin\alpha$$

Weight contributes $Mg\sin\alpha$, the rest must come from friction:

$$\frac{MR^2}{I+MR^2} Mg\sin\alpha = Mg\sin\alpha - f$$

$$f = Mg\sin\alpha \cdot \frac{I}{I+MR^2}$$ ← same as before