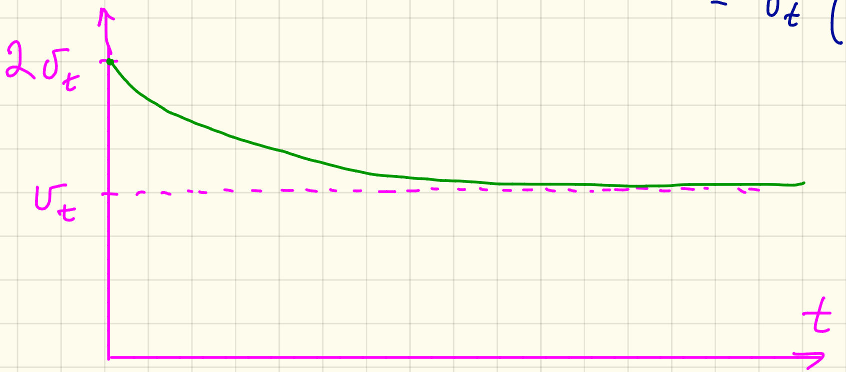
(2.5) The initial drag force in this case is larger than the body's weight. So it initially will accelerate up!

In class, we derived $v(t) = v_t - e^{-t/\tau}(v_t - v_0)$,

where $v_t = \frac{mg}{b}$, $\tau = \frac{m}{b}$

for $v_0 = 2v_t$ we get $v(t) = v_t + v_t e^{-t/\tau} =$

$= v_t \left(1 + e^{-t/\tau}\right)$



2.6 for $t \ll \tau$ $\quad e^{-t/\tau} = 1 - \frac{t}{\tau} + \frac{t^2}{2\tau^2} + \dots$

So $v(t) = v_t - \left(1 - \frac{t}{\tau}\right)\left(v_t - v_0\right) = \tau g - \left(1 - \frac{t}{\tau}\right)\tau g =$

$= \tau g - \tau g + gt = \underline{\underline{gt}}$

one order is enough here

For y:

$y(t) = v_t \cdot t - (v_t - v_0)\cdot\tau\left(1 - e^{-t/\tau}\right) =$

$= \tau g t - \tau g \cdot \tau \cdot \left(1 - \left(1 - \frac{t}{\tau} + \frac{t^2}{2\tau^2}\right)\right) = \tau g t - \tau g t + \frac{g t^2}{2} =$

$= \underline{\underline{\frac{1}{2} g t^2}}$

two orders are needed here.

If you use just one, you get $y(t) = 0$.

That is a sign that you need higher orders.

(2.9)

$$\frac{m\,dv}{v - v_t} = -b\,dt$$

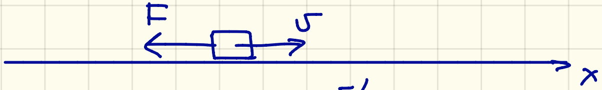
$$\int \frac{dv}{v - v_t} = \ln(v - v_t) + C$$

$$\int_{v(0)}^{v(t)} \frac{m\,dv}{v - v_t} = m \cdot \ln \frac{v(t) - v_t}{v(0) - v_t} = -b \int_0^t dt = -bt$$

$$\ln \frac{v(t) - v_t}{v_0 - v_t} = -t/\tau \Rightarrow v(t) = v_t - (v_t - v_0) \cdot e^{-t/\tau},$$

same as before.

2.14



$$F = -F_0 \cdot e^{v/V}$$

$$m \frac{dv}{dt} = -F_0 e^{v/V}$$

$$\int_{v_0}^{v} e^{-\frac{u}{V}} du = -\int_0^t \frac{F_0}{m} dt$$

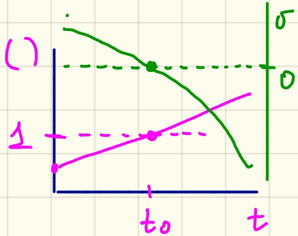
$$\int e^{-\frac{u}{V}} du = -V e^{-\frac{u}{V}}$$

$$-V\left(e^{-v/V} - e^{-v_0/V}\right) = -\frac{F_0}{m} t$$

$$e^{-v/V} = e^{-v_0/V} + \frac{F_0}{mV} t$$

$$v = -\mathbf{V} \cdot \ln\left(e^{-v_0/V} + \frac{F_0}{mV} \cdot t\right)$$

$$t \sim 0: \quad v = -V \cdot \left(-\frac{v_0}{V}\right)$$



$$\tau \frac{F_0}{mV} = 1 - e^{-v_0/V}$$

$$\tau = \frac{mV}{F_0}\left(1 - e^{-\frac{v_0}{V}}\right)$$

2.14

CONT

$$v = -V \cdot \ln\left(e^{-v_0/V} + \frac{F_0}{mV} \cdot t\right)$$

$$x = -V \int_0^t \ln\left(e^{-v_0/V} + \frac{F_0}{mV} t\right) dt =$$

$$\int \ln x \, dx = x \cdot \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - x + C$$

$$\int \ln(ax+b) \, dx = \frac{1}{a} \int \ln u \, du = \frac{1}{a}\left(u \ln u - u + C\right) = \frac{1}{a}(ax+b)\ln(ax+b) - x + C$$

$$\int_0^\tau \ln(\ ) \, dt = \left(\frac{mV}{F_0}\left(\frac{F_0}{mV} t + e^{-v_0/V}\right) \ln\left(\frac{F_0}{mV} t + e^{-v_0/V}\right) - t\right)\Bigg|_0^\tau =$$

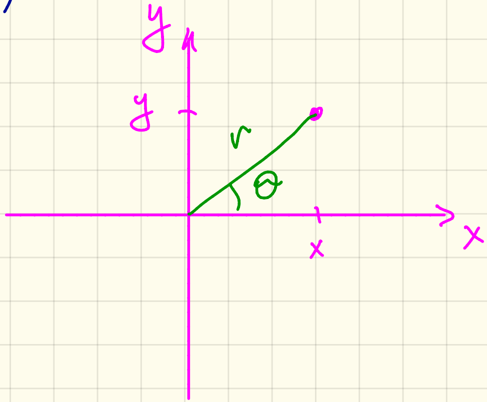
$$= -\tau - \frac{m\cancel{V}}{F_0} \cdot e^{-v_0/V} \cdot \left(-\frac{v_0}{\cancel{V}}\right) = -\frac{mV}{F_0}\left(1 - e^{-\frac{v_0}{V}}\right) + \frac{m v_0}{F_0} e^{-v_0/V}$$

$$x_0 = \frac{mV^2}{F_0}\left[1 - e^{-\frac{v_0}{V}} - \frac{v_0}{V} e^{-\frac{v_0}{V}}\right] = \boxed{\frac{mV^2}{F_0}\left[1 - e^{-\frac{v_0}{V}}\left(1 + \frac{v_0}{V}\right)\right]}$$

2.45 $e^{i\theta} = \cos\theta + i\sin\theta$ ← Euler's formula.

$z = x + iy = re^{i\theta} = r\cos\theta + i r\sin\theta$, for r & θ so

$$x = r\cos\theta, \quad y = r\sin\theta \implies r = \sqrt{x^2+y^2}, \quad \theta = \operatorname{atan} y/x$$

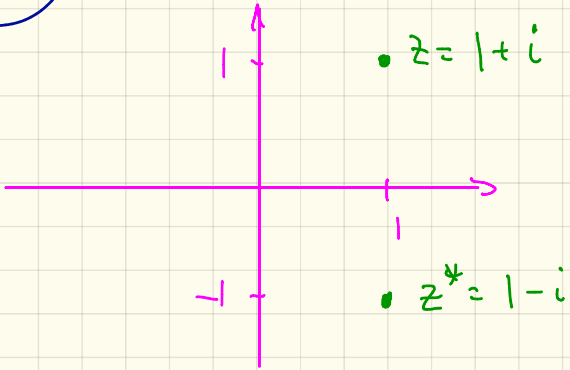


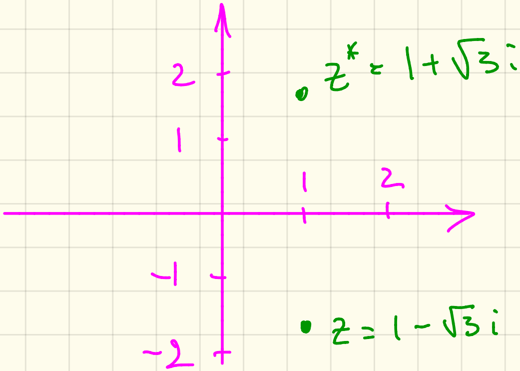
b) $3 + 4i = 5e^{i\operatorname{atan} 4/3}$

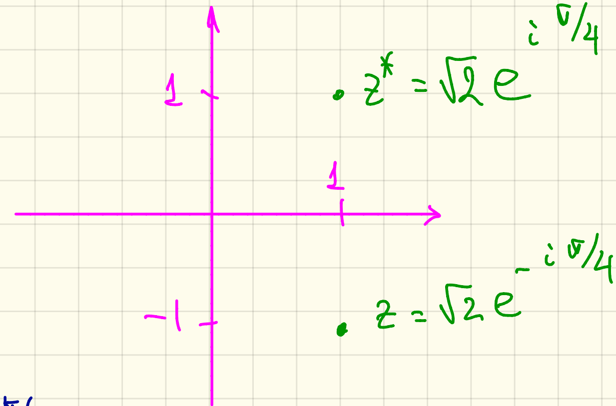
c) $2e^{-i\pi/3} = 2\cdot\cos\frac{\pi}{3} - 2i\sin\frac{\pi}{3} =$

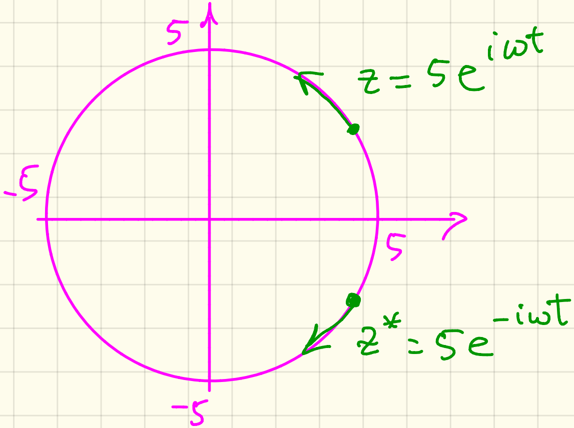
$$= 1 - 2i\frac{\sqrt{3}}{2} = 1 - i\sqrt{3}$$

2,46









$$\sqrt{2}\,e^{-i\pi/4} = \sqrt{2}\cos\frac{\pi}{4} - i\sqrt{2}\sin\frac{\pi}{4} = 1 - i$$

2,47 a) $z = 6+8i \;;\; w = 3-4i$

$z+w = 9+4i$

$z-w = 3+12i$

$zw = 18+32+24i-24i = 50$ (real! Note, $z = 2\cdot w^*$)

$$\frac{z}{w} = \frac{2w^*}{w} = \frac{2w^*\cdot w^*}{ww^*} = \frac{2}{25}\left(9-16+2\cdot 3\cdot 4i\right) = \frac{-14+48i}{25}$$

b) $z = 8e^{i\pi/3} \;;\; w = 4e^{i\pi/6} \Rightarrow z = 8\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right); w = 4\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)$

$z+w = (4+2\sqrt{3}) + (4\sqrt{3}+2)i$

$z-w = (4-2\sqrt{3}) + (4\sqrt{3}-2)i$

$zw = 32e^{i\pi/2} = 32i$

$\frac{z}{w} = 2e^{i\pi/6}$

2,45

$$z = e^{i\theta} = \cos\theta + i\sin\theta$$

a)

$$z^2 = e^{i2\theta} = \cos 2\theta + i\sin 2\theta =$$

$$= (\cos^2\theta - \sin^2\theta) + 2\cos\theta\sin\theta \cdot i$$

$$\Rightarrow \quad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

b)

$$z^3 = e^{i3\theta} = \cos 3\theta + i\sin 3\theta =$$

$$= \left[(\cos^2\theta - \sin^2\theta) + i\,2\cos\theta\sin\theta\right](\cos\theta + i\sin\theta)$$

$$\cos 3\theta = \cos^3\theta - \cos\theta\sin^2\theta - 2\cos\theta\sin^2\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$$

$$\sin 3\theta = 2\cos^2\theta\sin\theta + \sin\theta\cos^2\theta - \sin^3\theta = 3\sin\theta\cos^2\theta - \sin^3\theta$$

2.50

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots =$$

$$a_n = \frac{z^n}{n!}$$

$$\frac{da_n}{dz} = \frac{d}{dz}\left(\frac{z^n}{n!}\right) = \frac{n \cdot z^{n-1}}{n!} = \frac{z^{n-1}}{(n-1)!} = a_{n-1}$$

$$\frac{da_0}{dz} = \frac{d}{dz}(1) = 0.$$

Therefore

$$\frac{de^z}{dz} = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots = e^z$$

2.53

$\vec{E} = E\hat{x}$

$\vec{B} = B\cdot\hat{x}$

$$\vec{F} = q\left(\vec{E} + [\vec{v}\times\vec{B}]\right) =$$

$$= qE\cdot\hat{x} + qB\,[\vec{v}\times\hat{x}]$$

$$[\vec{v}\times\hat{x}] = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ 1 & 0 & 0 \end{vmatrix} =$$

$$= v_z\cdot\hat{y} - v_y\,\hat{z}$$

$$\left.\begin{aligned} m\dot{v}_x &= qE \\ m\dot{v}_y &= v_z\cdot qB \\ m\dot{v}_z &= -v_y\cdot qB \end{aligned}\right\}$$

v_x and E are decoupled from v_y, v_z and B

$$x = x_0 + v_{0x}\,t + \frac{1}{2}\frac{q}{m}E\,t^2$$

$$y + iz = (y_0 + iz_0)\cdot e^{-i\omega t}, \quad \omega = \frac{qB}{m}$$

stretched spiral: