1.2

$\vec{c} = (3,2,1)$ $\qquad \vec{b}+\vec{c} = (4,4,4)$

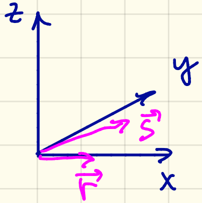
$\vec{b} = (1,2,3)$ $\qquad 5\vec{b}-2\vec{c} = (-1,6,13)$

$$\vec{b}\cdot\vec{c} = 3+4+3 = 10$$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = (2-6,\ 9-1,\ 2-6) = (-4, 8, -4)$$

1.15

Choose coord system so $\hat{x} \parallel \vec{r}$ and $\vec{s}$ is in xy plane:



$\vec{r} = (r, 0, 0)$

$\vec{s} = (s\cdot\cos\alpha,\ s\sin\alpha, 0)$

$$\vec{r}\times\vec{s} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r & 0 & 0 \\ s\cos\alpha & s\sin\alpha & 0 \end{vmatrix} =$$

$\hat{z} \perp \vec{r}, \vec{s}$

$$= \hat{x}\cdot 0 + y\,0 + \hat{z}\cdot r\cdot s\cdot\sin\alpha \Rightarrow |\vec{r}\times\vec{s}| = r\,s\cdot\sin\alpha$$

prove that triple product $\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$ and use it to prove the circular rule: $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$

$$\vec{A}\cdot\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \vec{A}\cdot\left(\hat{x}\begin{vmatrix} B_y & B_z \\ C_y & C_z \end{vmatrix} - \hat{y}\begin{vmatrix} B_x & B_z \\ C_x & C_z \end{vmatrix} + \hat{z}\begin{vmatrix} B_x & B_y \\ C_x & C_y \end{vmatrix}\right) =$$

$$= A_x\begin{vmatrix} B_y & B_z \\ C_y & C_z \end{vmatrix} - A_y\begin{vmatrix} B_x & B_z \\ C_x & C_z \end{vmatrix} + A_z\begin{vmatrix} B_x & B_y \\ C_x & C_y \end{vmatrix} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

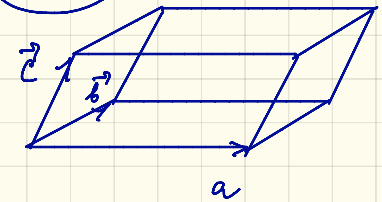
By determinant rules

$$\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} B_x & B_y & B_z \\ C_x & C_y & C_z \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} C_x & C_y & C_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

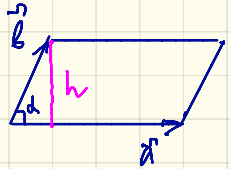
1.19 $\frac{d}{dt}\left(\vec{a}\cdot[\vec{v}\times\vec{r}]\right) = \dot{\vec{a}}\cdot[\vec{v}\times\vec{r}]$!

$$\dot{\vec{a}}\,[\vec{v}\times\vec{r}] + \vec{a}\,\cancel{[\vec{a}\times\vec{r}]} + \vec{a}\,\cancel{[\vec{v}\times\vec{v}]}$$

1.21

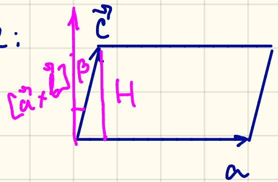


area of base:



$$A = |\vec{a}|\cdot h = |a||b|\cdot\sin\alpha = \left|[\vec{a}\times\vec{b}]\right|$$

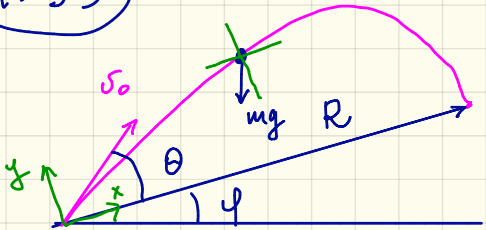
volume:



$$V = A\cdot H = |\vec{a}\times\vec{b}|\cdot|\vec{c}|\cdot\cos\beta = \underline{\left(\vec{c}\cdot[\vec{a}\times\vec{b}]\right)}$$

Note: $= \left(\vec{a}\cdot[\vec{b}\times\vec{c}]\right) = \left(\vec{b}\cdot[\vec{c}\times\vec{a}]\right)$

1.39



$$\begin{cases} m a_x = - m g \sin\varphi \\ m a_y = - m g \cos\varphi \end{cases}$$

$$\begin{cases} y = v_0 \sin\theta \cdot t - \frac{1}{2} g \cos\varphi \cdot t^2 \\ x = v_0 \cos\theta\, t - \frac{1}{2} g \cdot \sin\varphi \cdot t^2 \end{cases}$$

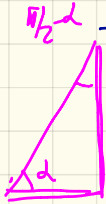
$$y = 0 \Rightarrow \quad v_0 \sin\theta = \frac{1}{2} g \cos\varphi \cdot t \Rightarrow t_L = \frac{2 v_0 \sin\theta}{g \cdot \cos\varphi}$$

$$x(t_L) = \left(v_0 \cdot \cos\theta - \frac{1}{2} \cancel{g} \cdot \sin\varphi \cdot \frac{2 v_0 \sin\theta}{\cancel{g} \cdot \cos\varphi} \right) \frac{2 v_0 \sin\theta}{g \cdot \cos\varphi} =$$

$$= \frac{2 v_0^2}{g} \frac{\cos\theta \cos\varphi - \sin\theta \sin\varphi}{\cos^2\varphi} \cdot \sin\theta = \frac{2 v_0^2}{g} \frac{\cos(\theta + \varphi) \sin\theta}{\cos^2\varphi}$$

$$R = \frac{2v_0^2}{g}\,\frac{\cos(\theta+\varphi)\sin\theta}{\cos^2\varphi}$$

$$\tan\left(\frac{\pi}{2}-\alpha\right) = \frac{1}{\tan\alpha}$$



$$\frac{d}{d\theta}R = \frac{2v_0^2}{g\cos^2\varphi}\Big[-\sin(\theta+\varphi)\sin\theta + \cos(\theta+\varphi)\cos\theta\Big] = 0$$

$$\sin(\theta+\varphi)\sin\theta = \cos(\theta+\varphi)\cos\theta$$

$$\tan(\theta+\varphi)\cdot\tan\theta = 1$$

$$\theta+\varphi = \frac{\pi}{2}-\theta$$

$$\theta = \frac{\pi}{4}-\frac{\varphi}{2}$$

$$2\theta = \frac{\pi}{2}-\varphi$$

$$\cos(\theta+\varphi)\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)\sin\theta = \sin^2\theta = \ldots$$

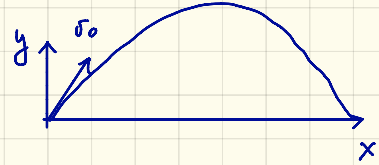
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1-2\sin^2\theta$$

$$\ldots = \frac{1}{2}\left(1-\cos 2\theta\right) = \frac{1}{2}\left(1-\cos\left(\frac{\pi}{2}-\varphi\right)\right) =$$

$$= \frac{1}{2}\left(1-\sin\varphi\right)$$

$$\boxed{R = \frac{v_0^2}{g}\,\frac{1-\sin\varphi}{\cos^2\varphi}} = \frac{v_0^2}{g}\,\frac{1-\sin\varphi}{1-\sin^2\varphi} = \boxed{\frac{v_0^2}{g(1+\sin\varphi)}}$$

1.40



$$\begin{cases} x = v_0 \cos\alpha \cdot t \\ y = v_0 \cdot \sin\alpha \cdot t - \frac{1}{2} g t^2 \end{cases}$$

$$r^2 = v_0^2 \cos^2\alpha \cdot t^2 + v_0^2 \sin^2\alpha \, t^2 - v_0 \sin\alpha \, g t^3 + \frac{1}{4} g^2 t^4 =$$

$$= v_0^2 t^2 - v_0 g \sin\alpha \cdot t^3 + \frac{1}{4} g^2 t^4$$

Criteria for r^2 to only increase: $\frac{d}{dt}(r^2) > 0$

$$2 v_0^2 t - 3 v_0 g \sin\alpha \, t^2 + g^2 t^3 > 0$$

$$g^2 t^2 - 3 v_0 g \sin\alpha \cdot t + 2 v_0^2 > 0 \rightarrow \text{no roots! } D < 0$$

$$D = 9 v_0^2 g^2 \sin^2\alpha - 4 \cdot g^2 \cdot 2 v_0^2 = g^2 v_0^2 \cdot 9 \left(\sin^2\alpha - \frac{8}{9}\right) < 0$$

$$\alpha_{max} = \operatorname{asin}\left(\frac{2\sqrt{2}}{3}\right)$$

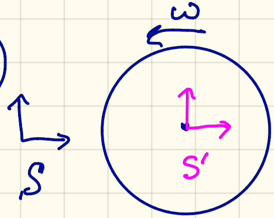
1.45 $\vec{v} = \vec{v}(t)$, but $|\vec{v}| = \text{const}$. Prove $\dot{\vec{v}} \perp \vec{v}$

$$\vec{v} \cdot \vec{v} = \text{const} \Rightarrow \frac{d}{dt}(\vec{v} \cdot \vec{v}) = 0 = 2\vec{v}\,\dot{\vec{v}} \Rightarrow \dot{\vec{v}} \perp \vec{v}$$

$\dot{\vec{v}} \perp \vec{v}$. Prove $|\vec{v}| = \text{const}$.

same idea: $\dot{\vec{v}}\,\vec{v} = 0 = \frac{1}{2}\frac{d}{dt}(\vec{v}\,\vec{v})$, $|\vec{v}| = \text{const}$

1.46



Assume that at $t=0$ puck is in the center and $S' \equiv S$ at $t=0$.

in S: trajectory is $\begin{cases} \varphi = 0 \\ r = v_0 \cdot t \end{cases}$

in S': $\begin{cases} r' = v_0 \cdot t \\ \varphi' = -\omega t \end{cases}$

$$\vec{a} = (\ddot{r} - \dot{\varphi}^2 \cdot r)\,\hat{r} + (r\ddot{\varphi} + 2\dot{r}\dot{\varphi})\,\hat{\varphi}$$

$$\left.\begin{array}{l} \varphi = \dot{\varphi} = \ddot{\varphi} = 0 \\ \dot{r} = v_0 \\ \ddot{r} = 0 \end{array}\right\} \vec{a} = 0$$

$\dot{r}' = v_0$; $\ddot{r}' = 0$

$\dot{\varphi}' = -\omega$; $\ddot{\varphi}' = 0$

$$\vec{a}' = -(\omega^2 v_0 t)\cdot\hat{r} - (2 v_0 \omega)\cdot\hat{\varphi}$$

↑ obviously not inertial.