Rutgers Physics 381 Mechanics I (Fall'18/Gershtein)

Class Exam - October 12, 2017

This is a closed book/notes exam. A one-sided 8.5x11 sheet with only formulae is allowed. Please attach the sheet to your solutions. Calculators are not needed. Exam duration - 1hr 20min.

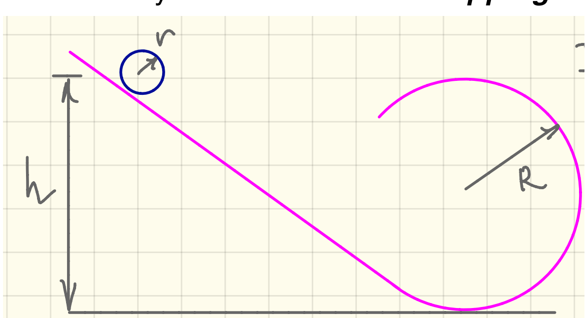
Operation	Cartesian coordinates (x, y, z)	Cylindrical coordinates (ρ, φ, z)	Spherical coordinates (r, θ, φ), where θ is the polar and φ is the azimuthal angle[α]
A vector field $\mathbf{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_\rho\hat{\boldsymbol{\rho}} + A_\varphi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\theta\hat{\boldsymbol{\theta}} + A_\varphi\hat{\boldsymbol{\varphi}}$
Gradient ∇f	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\hat{\boldsymbol{\varphi}}$
Divergence $\nabla \cdot \mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$
Curl $\nabla \times \mathbf{A}$	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\mathbf{z}}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\varphi}}$

1. Potential energy of a particle is given by $U(\rho,\varphi,z)=a\cdot\rho\cdot z\cdot\cos\varphi$.

a) what is the force on the particle?

b) what work need to be done on the particle to move it from a point (1,0,0) to $(3,\pi/3,1)$?

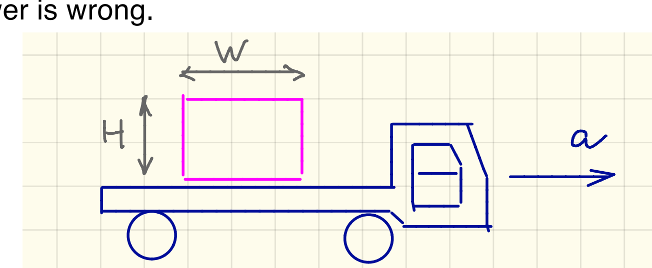
2. Rolling loop the loop. A rubber cylinder of radius r is released from rest along the straight track that ends in a loop of radius R (R>>r) as shown in the figure below. Moment of inertia of a cylinder is $I=\frac{1}{2}mr^2$. Find the minimum height h needed for the cylinder to make a loop without leaving the track. Assume that the cylinder rolls ***without slipping***



3. A box of height ***H*** and width ***W*** is on a flatbed truck. The friction coefficient between the box and the truck bed is μ. The trucks accelerates with constant acceleration a. Depending on values of H, W, a, and μ the box may do one of four things: stay in place, slip, tumble, and both slip and tumble.

a) Using torques ***relative to the center of mass of the box***, find the conditions for each of the four outcomes.

b) Consider torques around the bottom left corner of the box. The answer will be different. Explain why that answer is wrong.



$$U = U(r, \varphi, z) = a r z \cos\varphi$$

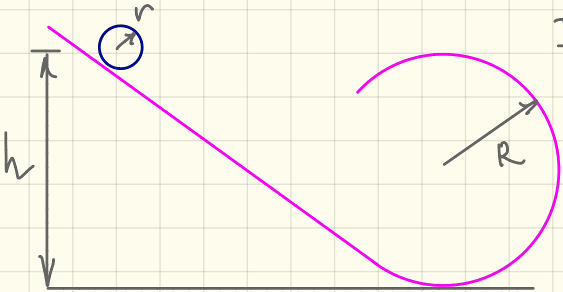
$$\vec{\nabla} U = \hat{r}\frac{\partial U}{\partial r} + \hat{z}\frac{\partial U}{\partial z} + \hat{\varphi}\frac{1}{r}\frac{\partial U}{\partial \varphi} =$$

$$F = -\hat{r}\, a z \cos\varphi - \hat{z}\, 2 a r \cos\varphi + \hat{\varphi}\, a z \sin\varphi$$

$$W\left[(1, 0, 0) \to \left(3, \frac{\pi}{3}, 1\right)\right] =$$

$$= U\left(3, \frac{\pi}{3}, 1\right) - U(1, 0, 0) = 3a \cdot \frac{1}{2} - 0 = \frac{3}{2} a$$

work that an external force has to do to move the particle from $(1,0,0)$ to $\left(3, \frac{\pi}{3}, 1\right)$



$r \ll R$

$I = \frac{1}{2} m r^2$

Energy conservation:

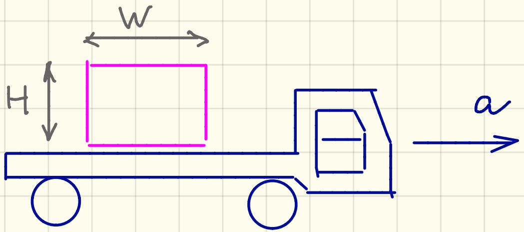
$$mgh = mg \cdot 2R + \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

$\omega r = v$, so $\quad gh = 2gR + \frac{1}{2} v^2 + \frac{1}{4} r^2 \omega^2$

$$h = 2R + \frac{3}{4} \frac{v^2}{g}$$

For the critical h, normal force at the top of the loop is zero, so $\frac{v^2}{R} = g \Rightarrow v^2 = gR$

$$h = 2R + \frac{3}{4} R = \frac{11}{4} R$$

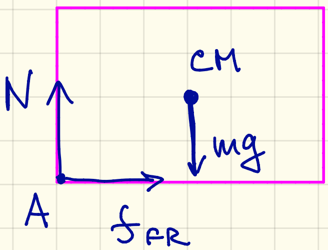


1) To not slip $(f_{FR})_{max} > ma \Rightarrow \mu m g > ma$

if $a > \mu g$ the box slips, else does not

2) torques around $\underline{\underline{CM}}$. Box tumbles if

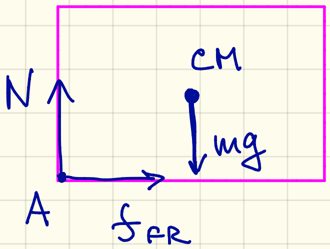
$$\tau = N\frac{W}{2} - f_{FR}\cdot\frac{H}{2} < 0$$



2a) box is also slipping: $a > \mu g$

$f_{FR} = \mu m g$, $\quad mgW - \mu m g H < 0$

if $a > \mu g$ and $W < \mu H$ box slips and tumbles.



2b) box is not slipping:

$$a < \mu g$$

$$f_{FR} = ma$$

tumbles if $mgW - maH < 0$

if $W < \frac{a}{g} H$ box tumbles.

3) Torque around point A (corner)

→ if $mg\frac{W}{2}$ is the only torque, box <u>never</u> tumbles.

→ systems with origin @ A or @ CM are both <u>not inertial</u>. Can only trust torques around CM (more when we study non-inertial systems)