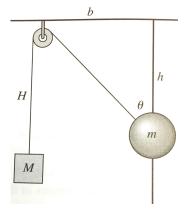
Rutgers Physics 381 Mechanics I (Fall'17/Gershtein)

Class Exam - October 13, 2017

This is a closed book/notes exam. A one-sided 8.5x11 sheet with only formulae is allowed. Please attach the sheet to your solutions. Calculators are not needed. Exam duration - 1hr 20min.

Operation	Cartesian coordinates (x, y, z)	Cylindrical coordinates $(ho, arphi, z)$	Spherical coordinates (r,θ,φ) , where θ is the polar and φ is the azimuthal angle $^{\alpha}$
A vector field A	$A_x\hat{f x}+A_y\hat{f y}+A_z\hat{f z}$	$A_{ ho}\hat{oldsymbol{ ho}}+A_{arphi}\hat{oldsymbol{arphi}}+A_{z}\hat{f z}$	$A_{r}\hat{f r}+A_{ heta}\hat{m heta}+A_{arphi}\hat{m arphi}$
Gradient ∇f	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$rac{\partial f}{\partial ho}\hat{oldsymbol{ ho}} + rac{1}{ ho}rac{\partial f}{\partial arphi}\hat{oldsymbol{arphi}} + rac{\partial f}{\partial z}\hat{oldsymbol{z}}$	$rac{\partial f}{\partial r}\hat{\mathbf{r}} + rac{1}{r}rac{\partial f}{\partial heta}\hat{oldsymbol{ heta}} + rac{1}{r\sin heta}rac{\partial f}{\partialarphi}\hat{oldsymbol{arphi}}$
Divergence $\nabla \cdot \mathbf{A}$	$rac{\partial A_x}{\partial x} + rac{\partial A_y}{\partial y} + rac{\partial A_z}{\partial z}$	$rac{1}{ ho}rac{\partial\left(ho A_{ ho} ight)}{\partial ho}+rac{1}{ ho}rac{\partial A_{arphi}}{\partialarphi}+rac{\partial A_{z}}{\partial z}$	$rac{1}{r^2}rac{\partial\left(r^2A_r ight)}{\partial r}+rac{1}{r\sin heta}rac{\partial}{\partial heta}\left(A_ heta\sin heta ight)+rac{1}{r\sin heta}rac{\partial A_arphi}{\partialarphi}$
Curi $\nabla \times \mathbf{A}$	$egin{aligned} \left(rac{\partial A_z}{\partial y} - rac{\partial A_y}{\partial z} ight)&\hat{\mathbf{x}} \ + \left(rac{\partial A_x}{\partial z} - rac{\partial A_z}{\partial x} ight)&\hat{\mathbf{y}} \ + \left(rac{\partial A_y}{\partial x} - rac{\partial A_x}{\partial y} ight)&\hat{\mathbf{z}} \end{aligned}$	$egin{aligned} \left(rac{1}{ ho}rac{\partial A_z}{\partial arphi}-rac{\partial A_{arphi}}{\partial z} ight)\!\hat{oldsymbol{ ho}} \ +\left(rac{\partial A_{ ho}}{\partial z}-rac{\partial A_z}{\partial ho} ight)\!\hat{oldsymbol{arphi}} \ +rac{1}{ ho}\left(rac{\partial \left(ho A_{arphi} ight)}{\partial ho}-rac{\partial A_{ ho}}{\partial arphi} ight)\!\hat{f z} \end{aligned}$	$egin{aligned} &rac{1}{r\sin heta}\left(rac{\partial}{\partial heta}\left(A_{arphi}\sin heta ight)-rac{\partial A_{ heta}}{\partialarphi} ight)\hat{\mathbf{r}}\ &+rac{1}{r}\left(rac{1}{\sin heta}rac{\partial A_{r}}{\partialarphi}-rac{\partial}{\partial r}\left(rA_{arphi} ight) ight)\hat{oldsymbol{ heta}}\ &+rac{1}{r}\left(rac{\partial}{\partial r}\left(rA_{ heta} ight)-rac{\partial A_{r}}{\partial heta} ight)\hat{oldsymbol{arphi}} \end{aligned}$

- **1.** Potential energy of a particle is given by $U(r,\theta,\phi)=a\cdot r\cdot sin\ \theta\cdot cos\ \phi$.
 - a) what is the force on the particle?
 - **b)** what work need to be done on the particle to move it from a point $(1,\pi/4,\pi/4)$ to $(2,\pi/2,\pi)$?
- **2.** A metal ball (mass m) with a hole through it is threaded on a frictionless vertical rod. A massless string (length L) attached to the ball runs over a small, massless, frictionless pulley and supports a block of mass M. Horizontal distance between the rod and the pulley is b (see the figure). The position of the two masses can be uniquely specified by the one angle θ .
- **a)** write down gravitational potential energy of the system $U(\theta)$. (It is given easily in terms of heights shown in **H** and **h** in the figure. Express **H** and **h** in terms of θ , **L** and **b**)
- b) By differentiating $U(\theta)$ find whether the system has an equilibrium position, and for what values of m and M equilibrium can occur. Discuss stability of any equilibrium positions.



3. A box of height \boldsymbol{H} and width \boldsymbol{W} is dropped from small height on a conveyer belt that runs with speed \boldsymbol{V} . The friction coefficient between the box and the belt is μ . The box will slip right after it's dropped, but may or may not tumble (obviously if H is much larger then W the box will tumble). For a given \boldsymbol{H} and $\boldsymbol{\mu}$, find the minimum \boldsymbol{W} for which the box does not tumble.

