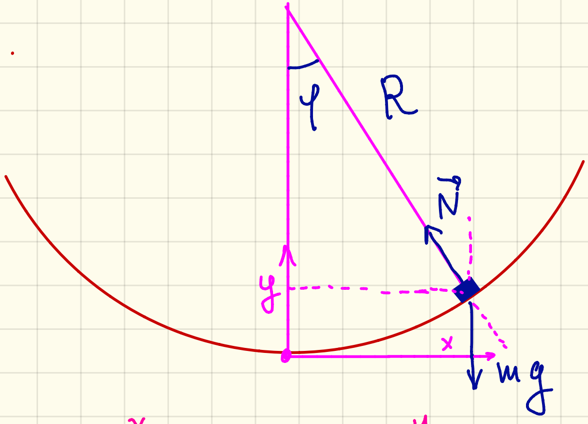



$$\sin\varphi = \frac{x}{R} \qquad 1-\cos\varphi = \frac{y}{R}$$

$$\begin{cases} x = R\sin\varphi \\ y = R(1-\cos\varphi) \end{cases}$$

$$F_x = -N\cdot\sin\varphi$$

$$F_y = N\cos\varphi - mg$$

$$N - mg\cos\varphi = m\frac{\dot{x}^2+\dot{y}^2}{R}$$

centripital force!

$$m\ddot{x} = -mg\cos\varphi\sin\varphi - m\frac{\dot{x}^2+\dot{y}^2}{R}\cdot\sin\varphi = -mg\frac{x}{R}\sqrt{1-\frac{x^2}{R^2}} - m\frac{\dot{x}^2+\dot{y}^2}{R}\cdot\frac{x}{R}$$

$$m\ddot{y} = -mg + mg\cos^2\varphi + m\frac{\dot{x}^2+\dot{y}^2}{R}\cos\varphi = -mg(1-\cos^2\varphi) + m\frac{\dot{x}^2+\dot{y}^2}{R}\sqrt{1-\frac{x^2}{R^2}}$$

As before, simplify for $x, y, \dot{x}, \dot{y} \ll 1$

keep first and second orders

$$m\ddot{x} = -mg\frac{x}{R}\sqrt{1-\frac{x^2}{R^2}} - m\frac{\dot{x}^2+\dot{y}^2}{2}\cdot\frac{x}{R} = -mg\frac{x}{R} + \underbrace{mg\frac{x^3}{2R^3} - \frac{m}{2R}\dot{x}^2 x - \frac{m}{2R}\dot{y}^2 y}_{\text{third order: neglect}}$$

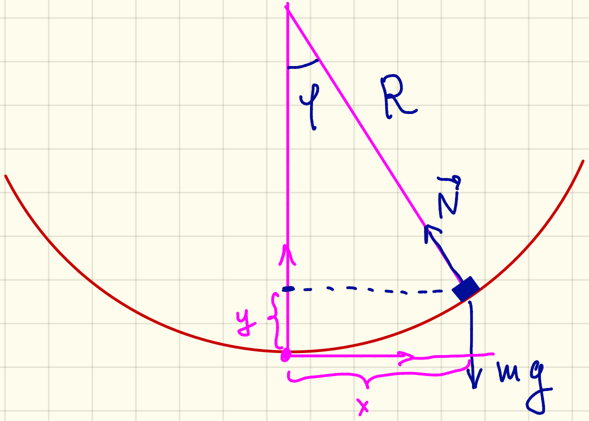
$$\ddot{x} = -\frac{g}{R}x \quad \Rightarrow \quad \boxed{x = A\cos(\omega t+\delta)\ ,\ \omega = \sqrt{\frac{g}{R}}}$$

$\sin\varphi = \frac{x}{R}$ $\quad 1-\cos\varphi = \frac{y}{R}$

$$m\ddot{y} = -mg\left(1-\left(1-\frac{y}{R}\right)^2\right) + m\frac{\dot{x}^2+\dot{y}^2}{R} + \underbrace{m\frac{\dot{x}^2+\dot{y}^2}{2R^2}\cdot x^2}_{\text{fourth order}} =$$

$$m\ddot{y} = -mg\left(\frac{2y}{R} - \frac{y^2}{R^2}\right) + m\frac{\dot{x}^2+\dot{y}^2}{R}$$

↑ ugly! But, can substitute solution for x and make sure it's a solution of this one




$$\sin\varphi = \frac{x}{R} \qquad 1-\cos\varphi = \frac{y}{R}$$

$$x^2 + (R-y)^2 = R^2 \quad \leftarrow \quad \text{not } x^2+y^2 = R^2 \,!!$$

$$y^2 - 2Ry + x^2 = 0$$

$$y = R \pm \sqrt{R^2 - x^2} \approx R \pm R\left(1 - \frac{x^2}{2R}\right)$$

$$\boxed{y = \frac{x^2}{2R}} \quad \leftarrow \text{ satisfies } y \ll 0$$

discard $2R - \frac{x^2}{2R}$ solution

Now let's take $x$ solution $x = x_0 \cos\omega t$ and use $y = x^2/2R$ to see if $y$ equation is satisfied

$$x = x_0 \cdot \cos\omega t \qquad y = \frac{x^2}{2R} = \frac{x_0^2}{2R}\cos^2\omega t \qquad y^2 = \frac{x_0^4}{4R^2}\cos^4\omega t$$

$$\dot{x} = -x_0\omega \sin\omega t \qquad \dot{y} = -\frac{x_0^2}{R}\omega \cdot \cos\omega t \cdot \sin\omega t = -\frac{x_0^2}{2R}\omega \cdot \sin 2\omega t$$

$$\dot{x}^2 = x_0^2\omega^2 \sin^2\omega t \qquad \dot{y}^2 = \frac{x_0^4}{R^2}\omega^2 \cos^2\omega t \cdot \sin^2\omega t$$

$$\ddot{y} = -\frac{x_0^2}{R}\omega^2 \cos 2\omega t$$

$$\frac{\dot{x}^2+\dot{y}^2}{R} = \frac{1}{R}\left(\underbrace{x_0^2\,\omega^2 \sin^2\omega t}_{\text{second order}} + \underbrace{\frac{x_0^4}{R^2}\,\omega^2 \cos^2\omega t \cdot \sin^2\omega t}_{\text{fourth order}}\right) =$$

$$g\left(\frac{2y}{R} - \frac{y^2}{R^2}\right) = g\left(\overbrace{\frac{x_0^2}{R^2}\cos^2\omega t}^{\text{second order}} - \overbrace{\frac{x_0^4}{4R^4}\cos^4\omega t}^{\text{fourth order}}\right) \qquad \omega^2 = \frac{g}{R}$$

$$-\frac{x_0^2\omega^2}{R}\cos 2\omega t \stackrel{?}{=} -g\,\frac{x_0^2}{R^2}\cos^2\omega t + \frac{x_0^2\omega^2}{R}\sin^2\omega t$$

$$-\frac{x_0^2 g}{R^2}\cos 2\omega t \stackrel{?}{=} -\frac{x_0^2 g}{R^2}\cos^2\omega t + \frac{x_0^2 g}{R^2}\sin^2\omega t$$

$$\cos 2\omega t \stackrel{?}{=} \cos^2\omega t - \sin^2\omega t$$

true!