Press-Schechter Formalism: Structure Formation by Self-Similar Condensation

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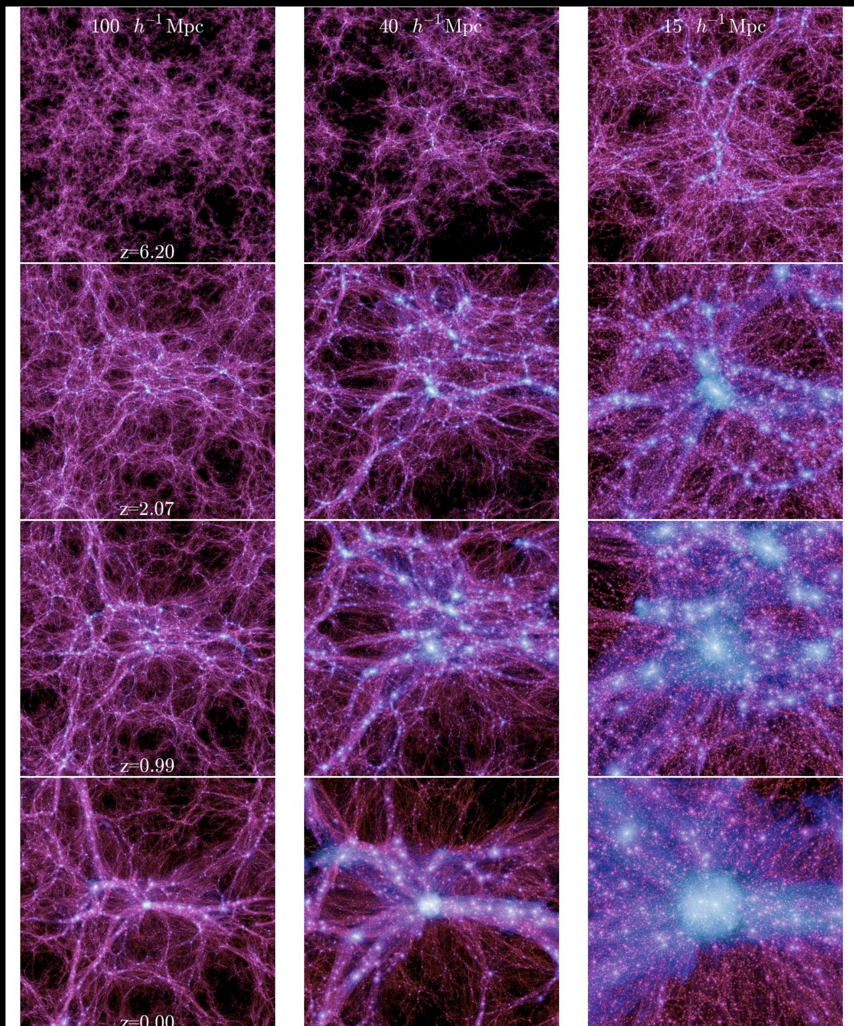
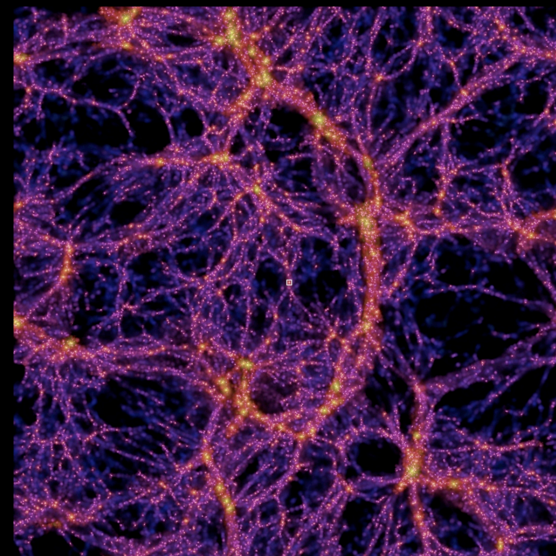
Based on Press & Schechter's 1974 Paper

Structure Formation in Simulations





Images Courtesy of the Max Planck Institute of Astrophysics & Virgo Consortium (Top, Millennium-II; Right, Aquarius Project).

Basic Definitions of Dark Matter

$$n_* \equiv \int_0^\infty n(m)dm$$

$$m_* \equiv n_*^{-1} \int_0^\infty mn(m)dm = \frac{\rho}{n_*}$$

$$q = \frac{4}{3}\pi m_* n_* G / H^2$$

$$N_J = n\left(\frac{v}{H}\right)^3$$

- Number Density
- Characteristic Particle Mass
- Deceleration Parameter: Describes the ability of the universe to inhibit condensation.
- Jean's Number: Describes the number of particles needed to begin condensation. See Eqn. (3) for an empirical formula.

Intuitions for Self-Similarity

- Gravitational Collapse: “Large correlations in the gas must be interperated as changes in n(m), by treating highly correlated groups of particles as single more massive particles.” (Pg. 427)
- Non-Linearity of Equations: If we expect n(m) to depend only on the statistics of the current scale, then the evolutions of n(m) will depend on the statistical properties of the non-linear differential equations. (Pg. 428)

Preparation for Derivation

- Define Mass Variance inside of Volume V.

$$\Sigma^2(V) = \langle M \rangle_V^2 - \langle M^2 \rangle_V$$

- An upper bound on the variance can be found by taking the dark matter particles to be distributed uniformly, so that the variance is linear in volume.

$$\Sigma^2(V) = \sigma^2 V$$

Derivation

- We define the probability of finding a fractional mass deviation between δ and δ+dδ in volume V as P(δ,V).

$$p(\delta, V) = \frac{1}{\sqrt{2\pi\delta_*^2}} e^{-\frac{\delta^2}{2\delta_*^2}}$$

$$\delta = \frac{(M - \langle M \rangle)}{M}$$

$$\delta_* \equiv \frac{\sqrt{\Sigma^2(V)}}{M} = \frac{\sigma V^{\frac{-1}{2}}}{\rho}$$

Derivation (Cont'd)

$$P = \int_{\delta = R_1/R_2}^{\infty} p(\delta, V)\, d\delta = \frac{erfc\left(\frac{R_1 \rho \sqrt{V}}{\sqrt{2} R_2 \sigma}\right)}{2}$$

$$R_2 = R_1 / \delta$$

- The turn-around scale, R_2, for R_1 is found for spherical collapse in the Appendix.

- We can define the probability of having an overdensity δ collapse before R_2 as P.

Derivation (Cont'd)

- The number density distribution is found by multiplying the percentage of collapsed mass with the mass density of the second scale and dividing by M.

- The factor of 2 was added to take into account the underdensities around the collapsed objects (Improved explanation can be found in Bond et al. 1991).

$$\frac{dP}{dM} = 2^{\frac{-3}{2}} \pi^{\frac{-1}{2}} \frac{R_1}{R_2} \frac{\rho_1^{\frac{1}{2}}}{\sigma_1} M^{\frac{-1}{2}} e^{\left(\frac{-1 R_1^2 \rho_1 M}{2 R_2^2 \sigma_1^2}\right)}$$

$$n_2(m) = \frac{2}{M} \rho_1 \left(\frac{R_1}{R_2}\right)^3 \frac{dP}{dM}$$

$$\rho_2 = \rho_1 \left(\frac{R_1}{R_2}\right)^3$$

$$\sigma_2^2 = \sigma_1^2 \left(\frac{R_1}{R_2}\right)$$

Summary

- Press-Schechter Formalism allows us to analytically recreate the linear evolution of dark matter perturbations without the need of resource exhaustive computer simulations.

- Press-Schechter Formalism describes a universe where different scales collapse in a similar manner without a dependence on the scale size (Self-similar condensation).

- The Formalism has been extensively used and compared to simulations to confirm its accuracy.

- An improved Sheth-Tormen Formalism, which uses ellipsoidal collapse and excursion set theory, reproduces the main results of Press-Schechter while correcting the discrepancies at the high and low mass extremes of the halo mass function.