An Analytic Expression for the Luminosity Function For Galaxies

Paul Schechter, 1976

Presentation by George Locke

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What is a Luminosity Function?

- Luminosity Distribution: $n_S\left(L\right)\Delta L$
  - The number of galaxies contained in a sample S in the luminosity interval *ΔL* about *L*
  - The sample S is over a volume V_S
- Luminosity Function: $\varphi_S\left(L\right)\Delta L \equiv \frac{n_S\left(L\right)\Delta L}{V_S\left(L\right)}$
  - Number density of galaxies at L per volume
  - Or: luminosity distribution per volume
- Gives expected distribution: $n_e\left(L\right) \equiv \varphi\left(L\right) V_S\left(L\right)$

So what is it good for?

- An accurate model can be used as a benchmark for theories of galaxy formation
  - Spatial covariance function
  - Cosmological corrections to the number magnitude relation
  - If luminosity can be expressed as a function of mass, then $\varphi(L)$ determines density
  - Extrapolate actual luminosity from observed luminosity in clusters
    - Can be used to estimate distance to cluster
  - Observed luminosity function can be used for above problems, but an analytic expression makes calculation easier

Outline

- Present expression for $\varphi(L)$
  - Derived from a “self-similar stochastic model” for galaxy formation
    - Press and Schecter 1974 *Ap. J.* **187**, 425
- Fit parameters to observations (by least squares)
- Test model against data

Note: analytic *expression*, not a *derivation*

The Model

$$\varphi(L)dL = \varphi^* \left(\frac{L}{L^*}\right)^\alpha \exp\left(-\frac{L}{L^*}\right) d\left(\frac{L}{L^*}\right)$$

- φ^*
  - Overall multiplicative factor – something like overall density
  - Units: Luminosity per volume, same as φ itself
- L^*
  - "Characteristic luminosity"
  - Sets location of corner in the luminosity distribution
  - There is an equivalent expression for M^*
- α
  - Controls slope at $L \ll L^*$
  - Pure number

Observations: General Distribution

- Used data from de Vacouleurs and de Vacouleurs (1964) *Reference Catalogue of Bright Galaxies* (Austin: University of Texas Press)
  - All galaxies brighter than $m_{B(0)lim}=11.75$ excluding Virgo cluster (anomalous velocity dispersion)
  - This sample comes to 184 galaxies
  - Red shift used as the only distance indicator here

Observations: Cluster Galaxies

- Used data from Oemler (1974) *Ap. J.* **194**, 1
  - Volume constant for all L
    - *n(L)* and *φ(L)* related by a constant
  - Just 13 clusters
- Funky rebinning procedure
- Looked at luminosities of galaxies within clusters
- Slightly different expression for model:

$$n_e(L)dL = n^* \left(\frac{L}{L^*}\right)^{\alpha} \exp\left(-\frac{L}{L^*}\right) d\left(\frac{L}{L^*}\right)$$

Fitting Procedure

$$n_e(L)dL = \varphi^* V^* \left(\frac{L}{L^*}\right)^{\alpha+3/2} \exp\left(-\frac{L}{L^*}\right) d\left(\frac{L}{L^*}\right)$$

- Model: Obtained from $n_e(L) = \varphi(L) V_S$
  - V^* is exactly determined by L^* by a lengthy formula
  - (expression shown here applies to the general case)

- Data: Correct for non-Hubble velocity dispersion
  - use $\sigma(L)$ the RMS uncertainty in red shift at L
  - $\sigma(L)$ depends on $\langle \Delta v^2 \rangle$

Fitting Procedure

- Minimize error $$\chi^2 \equiv \sum_i \frac{\left[n(M) - n_{e2}(M)\right]^2}{\sigma_i^2}$$
    - Used "unbiased estimates", see Wolberg (1967) *Prediction Analysis* (Princeton: D. van Nostrand), p 68
    - σ_i is just $\sqrt{N}$

– Nonlinear least squares fit – this problem not addressed in text

- Linear least squares fit gives closed form solution
- Non-linear fits must be solved numerically and can get caught in local minima

The Fit for General Observations

- $\varphi^* V^* = 216 \pm 6$
- $M^* = -20.60 \pm 0.11$
- $\alpha = -1.24 \pm 0.19$
- $\chi^2 = 0.53$

- Correlation Coefficients
  - $\rho_{\varphi^* V^*, M^*} = 0.240$
  - $\rho_{\varphi^* V^*, \alpha} = 0.231$
  - $\rho_{M^*, \alpha} = 0.973$

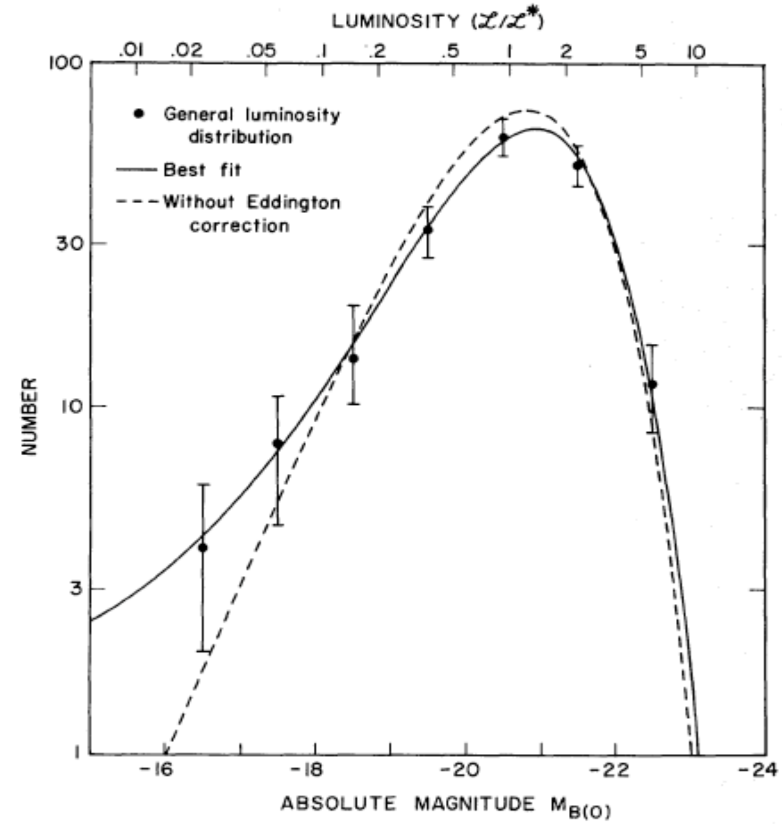


FIG. 1.—Best fit of analytic expression to observed general luminosity distribution. Broken line shows the effect of deleting the Eddington correction.

The Fit for Cluster Galaxies

- $n^* = 910 \pm 120$
- $M^*_{J(24.1)} = -21.41 \pm 0.10$
- $\alpha = -1.24 \pm 0.05$
- $\chi^2 = 16.4$
- Correlation Coefficients
  - $\rho_{n^*,M^*} = 0.928$
  - $\rho_{n^*,\alpha} = 0.939$
  - $\rho_{M^*,\alpha} = 0.823$
- Note cD galaxies excluded

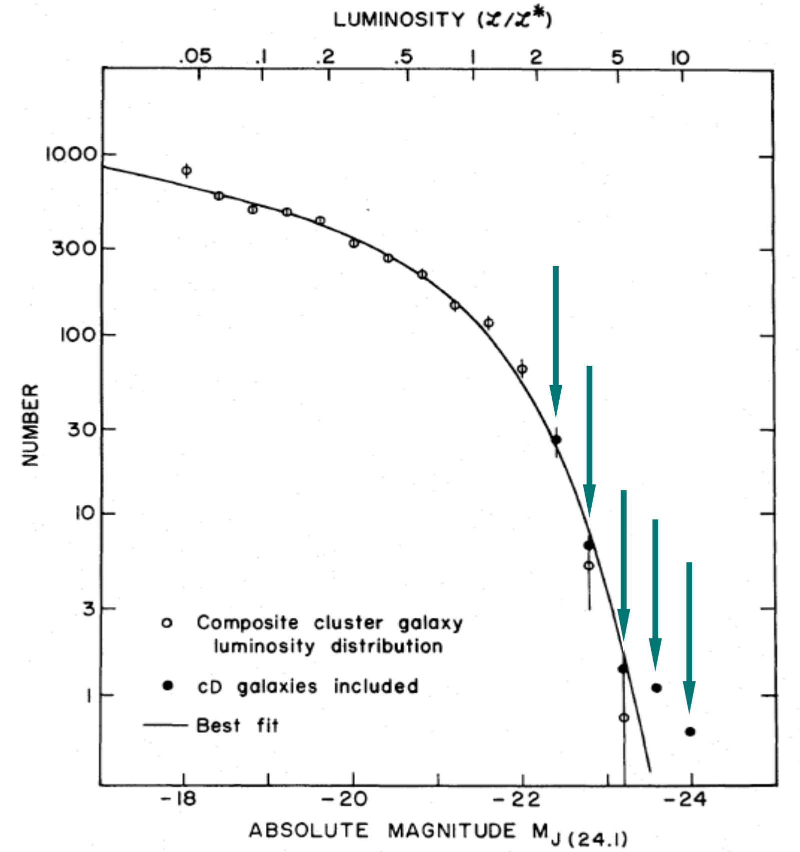


FIG. 2.—Best fit of analytic expression to observed composite cluster galaxy luminosity distribution. Filled circles show the effect of including cD galaxies in composite.

Testing the model

- Made separate fits for individual clusters
- One parameter fit
  - Set M^* and α to their values from the global fit and let n^* be a free parameter
  - Recall that n^* is simply an overall factor that does not contribute to structure
    - Further note: fit is now linear
- Two parameter fit
  - Free M^* as well

Testing the Model

LUMINOSITY FUNCTION FOR GALAXIES

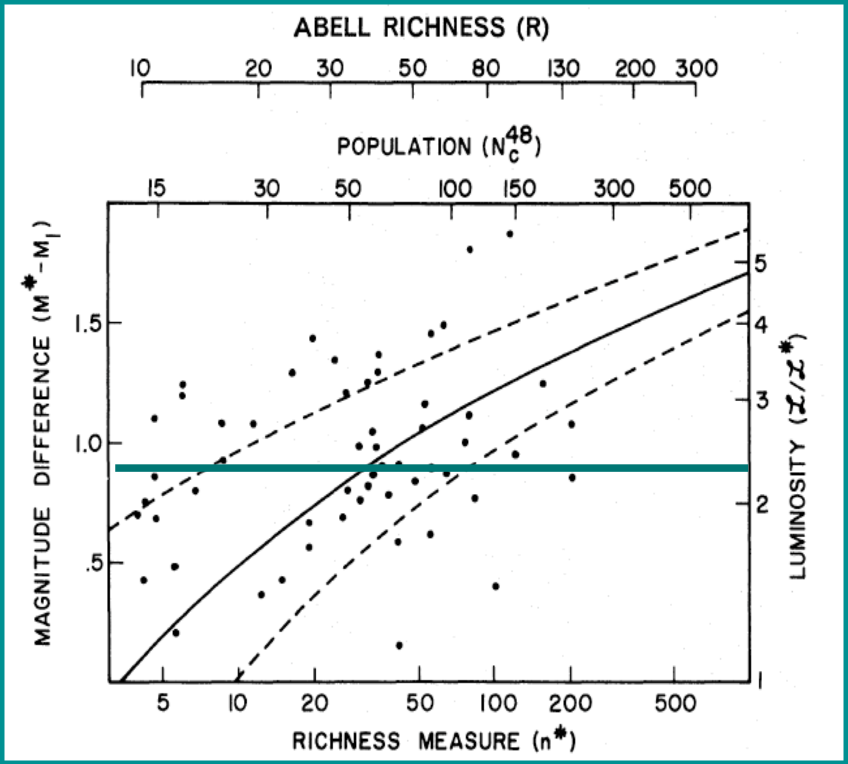
TABLE 2

PARAMETERS FOR FOURTEEN CLUSTERS ($\alpha = -5/4$)

	One-Parameter Fits ($M^* = 21.43$)			Two-Parameter Fits	
Cluster	n^*	$\sigma(n^*)$	χ^2/ν	M^*	$\sigma(M^*)$
A194	19	3	1.8	−22.32	0.64
A400	33	6	0.4	−21.41	0.33
A539	41	3	0.7	−21.26	0.22
A665	109	25	6.2	−22.52	0.66
A1228	39	4	0.7	−21.32	0.25
A1314	45	2	0.2	−21.41	0.11
A1367	46	5	1.8	−21.30	0.37
A1413	113	11	1.0	−21.36	0.24
A1656	107	8	2.8	−21.26	0.20
A1904	78	6	0.4	−21.50	0.17
A2151	60	11	0.5	−21.52	0.17
A2197	46	11	3.1	−20.63	0.44
A2199	75	6	1.3	−21.06	0.16
A2670	55	7	4.5	−22.09	0.53

Brightest Cluster Members

- If our assumption is correct, then brightness of galaxy should correlate with n^*
  - Expect larger differences simply from sampling more
- Very little correlation observed
  - Schechter also says that there is "reasonable agreement" within the expected uncertainty (?)



Expected correlation of absolute magnitude of brightest cluster galaxy with cluster richness. Broken lines show expected RMS dispersions. Data points from Sandage and Hardy (1973) *Ap. J.* **183**, 743

Conclusions

- Good fit over 6 magnitudes suggests that brightness of galaxies are drawn from a distribution
  - Fitted luminosity function for clusters shows variation only in the number of galaxies present there
- Poor prediction of brightest galaxy in cluster
- Two tested clusters exhibit distributions significantly divergent from prediction

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