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Abstract
I suggest a pragmatic formal structure for (more) fundamental theories and describe a mechanism that produces such a structure and possibly produces exotic non-canonical low-energy phenomena in $SU(2)$ and $SU(3)$ gauge theories that might allow conditional predictions that can be tested. These slides are on my web page http://www.physics.rutgers.edu/~friedan/#Perimeter

While visiting PI this week, I would also like to discuss informally a separate project in pure QFT, a scheme to construct a new kind of QFT of extended objects (also on my web page).
The most fundamental theory at present is the Standard Model + General Relativity.

SM+GR is an effective QFT with UV cutoff distance \((10^3 \text{GeV})^{-1}\).

GR can be treated as an effective QFT because quantum corrections in GR are completely negligible on this huge distance scale \((10^3 \text{GeV})^{-1} = 10^{16} \ell_P\).

SM+GR describes almost all physics at distances \(> (10^3 \text{GeV})^{-1}\).

Only dark matter, neutrino mixing, and some CP violation are left unexplained.
I am interested in the possibility of formal fundamental physics:

1. hypothesize a “more fundamental” formal machinery that can “produce” the Standard Model + General Relativity

2. predict consequences beyond the SM+GR that can be checked experimentally

Some prototypes:

- GR from Newtonian Gravity + Special Relativity
- Grand Unification from the SM

Notice that the testable prediction of Grand Unification is *conditional*. The RG acting on the space of grand unified theories *can* produce the SM. *If* the RG produces the SM, then it also produces proton decay (which unfortunately has not been seen).
None of the attempts at formal fundamental physics have worked in the 45 years since the SM was finished.

One reasonable response is to give up, perhaps hoping that experiment will eventually give more guidance.

Alternatively, we might reconsider the assumptions that led to the present situation.

An analogy: hiking up a mountain without a map or a GPS. If after 45 years no measurable altitude has been gained, maybe it is time to backtrack and reconsider the choices of direction.

We should especially question the truisms and mathematical idealizations that have governed the enterprise.

This is not easy. Physicists are human. Collective adherence to truisms provides psychological and social comfort. But it is not the business of theoretical physicists to be comfortable before experiment shows them to be right.
Truism: We need Quantum Gravity

On the contrary,

1. $L_{\exp} \approx 10^{16} \ell_P$ so it is implausible to check experimentally.

2. It is presumptuous to assume Quantum Mechanics to be valid over 16 orders of magnitude in distance/energy beyond where there is evidence.

<table>
<thead>
<tr>
<th>energy in GeV</th>
<th>$10^{18}$</th>
<th>$10^{15}$</th>
<th>$10^{12}$</th>
<th>$10^9$</th>
<th>$10^6$</th>
<th>$10^3$</th>
<th>$10^0$</th>
<th>$10^{-3}$</th>
<th>$10^{-6}$</th>
<th>$\cdots$</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance in $\ell_P$</td>
<td>$10^1$</td>
<td>$10^4$</td>
<td>$10^7$</td>
<td>$10^{10}$</td>
<td>$10^{13}$</td>
<td>$10^{16}$</td>
<td>$10^{19}$</td>
<td>$10^{22}$</td>
<td>$10^{25}$</td>
<td>$\cdots$</td>
</tr>
</tbody>
</table>

$L_{\exp}$

$\cdots$ ???? ?? ?? ? Quantum Mechanics
Effective QFT is a minimal description of large distance physics.

A minimal description of short distance physics is an effective S-matrix.

Consider an experiment of size \( \gtrsim L \) that probes distances \( < L \). The preparation of the initial scattering states and the detection of the final scattering states are described by the effective QFT at distance \( L \). The scattering amplitudes describe the short distance physics, at distances smaller than \( L \).

This is an *effective* S-matrix: an S-matrix with IR cutoff \( L \).

What we mean by “short distance physics” is relative. \( L \) is pushed smaller and smaller as science progresses. \( L \) is a sliding scale.
A pragmatic formalism

\[ \begin{array}{cccccccc}
\text{distance in } \ell_P & 10^1 & 10^4 & 10^7 & 10^{10} & 10^{13} & 10^{16} & 10^{19} & 10^{22} & 10^{25} & \ldots \\
\end{array} \]

\[ L \]

\[ \begin{array}{c}
\text{S-matrix}(L) \quad \text{QFT}(L) \\
\end{array} \]

For every \( L \gg 1 \) there is

- an effective QFT with UV cutoff \( L \) describing all physics at distances \( > L \),

- an effective S-matrix with IR cutoff \( L \) describing physics at distances \( < L \),

such that \( \text{QFT}(L) \) and \( \text{S-matrix}(L) \) agree at distances \( \approx L \).
and satisfy consistency conditions for $L' < L$

\[
\begin{align*}
L' & \xrightarrow{\text{QFT}(L)} L \\
& \xrightarrow{\text{QFT}(L')} \\
& \xleftarrow{\text{S-matrix}(L)} \\
& \xleftarrow{\text{S-matrix}(L')}
\end{align*}
\]

- $\text{QFT}(L') \supset \text{QFT}(L)$ (i.e., via the RG from $L'$ to $L$)
- $\text{S-matrix}(L) \supset \text{S-matrix}(L')$ (via an “S-matrix RG”?)
A mechanism is needed that produces this formal structure.

I came to these ideas during the period 1977 – 2002 in the process of formulating such a mechanism.

The mechanism is a certain mathematically natural 2d nonlinear model (2d-NLM), called the lambda model, whose target manifold is the space of classical space-time fields. The lambda model produces at every $L \gg 1$ an effective QFT which is the quantum string background for an effective string S-matrix.

I suggest that the lambda model is worth exploring because

1. It produces consistent realizations of the formal structure described above: $\text{QFT}(L) + \text{S-matrix}(L)$ for $L \gg 1$.

2. The mechanism for producing QFT does not necessarily correspond to canonical quantization. There are concrete possibilities of producing non-canonical degrees of freedom and interactions, so there are possibilities of conditional predictions that might be interesting and testable.
The 2d-RG as a mechanism for space-time physics (1977-79)

The coupling constants of the general 2d-NLM are given by a Riemannian metric $g_{\mu\nu}(X)$ on the target manifold $M$,

$$\int D X \ e^{-\int d^2z \ g_{\mu\nu}(X) \partial X^\mu \bar{\partial} X^\nu} \quad X(z, \bar{z}) \in M$$

The 2d-RG drives to solutions of $R_{\mu\nu} = 0$

$$\Lambda \frac{\partial}{\partial \Lambda} \ g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2)$$

The 2d-RG is a mechanism that produces solutions of a GR-like classical space-time field equation.

This suggested the possibility of actually answering

*Where does space-time field theory come from?*

or even

*Where do the laws of physics come from?*

with an unexpected mechanism, the 2d-RG.
The 2d-RG incorporated into string theory (1983-85)

- 2d scale invariance (the RG fixed point equation $\beta = 0$) as consistency condition for the string S-matrix recipe

- a string background = a 2d-NLM of the world-sheet with degrees of freedom $X^\mu(z, \bar{z}) + \cdots$ such that the 2d coupling constants are the metric $g_{\mu\nu}(X)$ plus some space-time non-abelian gauge fields, scalar fields, fermion fields, etc.

- the $\beta = 0$ equation of this 2d-NLM (generalizing $R_{\mu\nu} = 0$) is a semi-realistic classical field equation which includes GR and potentially the SM

- The string S-matrix at low momentum agrees with the S-matrix of the perturbative canonical quantization of the classical field equation $\beta = 0$. 
Questions (1987)

1. How does the 2d-RG act in string theory as a mechanism? \( \beta = 0 \) is a only consistency condition.

2. Where does quantum field theory come from? What produces a functional integral over the space-time fields?

3. What is the quantum string background, given by a quantum state of a QFT? as opposed to a classical field (\( R_{\mu\nu} = 0 \))?

4. What produces an effective string S-matrix with IR cutoff in a background described by an effective QFT with UV cutoff?

These questions rejected several truisms about string theory that were cemented in the mid-1980s:

1. The string S-matrix is an idealized S-matrix of everything.

2. A string background is a 2d-CFT, a solution of \( \beta = 0 \).

3. The low energy limit of string theory is the QFT that happens to have the same low momentum scattering amplitudes.
The lambda model (1988-2002)

Consider a world-sheet given by a 2d-CFT with

\[ \lambda^i = \text{the 2d coupling constants} \]

\[ \phi_i(z, \bar{z}) = \text{the 2d scaling fields} \]

\[ |\phi_i\rangle = \text{the states on the circle} \]

The \( \phi_i \) correspond to the modes of the space-time fields, e.g.,

\[ \phi_i(z, \bar{z}) = e^{ip(i)_\mu X^\mu} h^{(i)}_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu \quad i \leftrightarrow p(i), h^{(i)}_{\mu\nu} \]

The 2d-scaling-dimensions are

\[ \text{dim}(\phi_i) = \Delta(i) = 2 + \delta(i) \quad \text{dim}(\lambda^i) = -\delta(i) \quad \delta(i) = p(i)^2 \]

The \( \lambda^i \) form a system of local coordinates on the space of 2d-QFTs by the insertions

\[ e^{\int d^2z \lambda^i \phi_i} \]

The marginal couplings, \( \delta(i) = 0 \), parametrize the \( \beta = 0 \) submanifold.
Let \((ds)^2 = \mu^2|dz|^2\) be the world-sheet 2d metric.

Suppose a 2d UV cutoff \(\Lambda \gg \mu\).

(The 2d-QFT is “realistic” from the point of view that considers a continuum QFT to be a mathematical idealization.)

The cutoff string propagator (the cutoff 2d-cylinder) is

\[
\sum_i |\phi_i\rangle \frac{1 - \left(\frac{\Lambda}{\mu}\right)^{-\delta(i)}}{\delta(i)} \langle \phi_i|
\]

Only the modes with \(\left(\frac{\mu}{\Lambda}\right)^\delta(i) < 1\) propagate. Writing

\[
L^2 = \ln \left(\frac{\Lambda}{\mu}\right)
\]

The condition for propagation is

\[
L^2 \delta(i) > 1 \quad \delta(i) = p(i)^2
\]

Thus the 2d UV cutoff \(\Lambda\) is an IR cutoff \(L\) on the S-matrix.
Effective perturbations $\lambda^i \phi_i$ at the 2d UV scale $\Lambda$ are suppressed by the 2d-RG running from $\Lambda$ to $\mu$

$$\lambda^i(\mu) = \left(\frac{\Lambda}{\mu}\right)^{-\delta(i)}\lambda^i \quad \text{dim}(\lambda^i) = -\delta(i)$$

If $\left(\frac{\Lambda}{\mu}\right)^{-\delta(i)} \ll 1$ then $\lambda^i$ is effectively irrelevant. The effectively marginal couplings are those satisfying $\left(\frac{\Lambda}{\mu}\right)^{-\delta(i)} \approx 1$, which is

$$L^2 \delta(i) < 1 \quad \delta(i) = p(i)^2$$

Thus the 2d UV cutoff $\Lambda$ is a UV cutoff $L$ on the space-time fields describing the classical string background.
Now let the $\lambda^i$ vary on the world-sheet, becoming sources $\lambda^i(z, \bar{z})$.

Make the $\lambda^i(z, \bar{z})$ fluctuate at 2d distances $< \Lambda^{-1}$ with the fluctuations governed by the 2d-NLM

$$\int \mathcal{D}\lambda \ e^{-\int d^2z \ g_{\text{str}}^{-2} G_{ij}(\lambda) \partial \lambda^i \bar{\partial} \lambda^j} \ e^{\int d^2z \ \lambda^i(z, \bar{z}) \phi_i(z, \bar{z})}$$

where

- $G_{ij}(\lambda)$ = the natural metric on the manifold of 2d-QFTs
- $g_{\text{str}}$ = the string coupling constant
- the $\lambda^i(z, \bar{z})$ fluctuate at 2d distances $< \Lambda^{-1}$

This 2d-NLM is the lambda model.
on the 2d distance scale

\[
\begin{array}{ccc}
\text{\textbf{\lambda-model}} & \text{\textbf{effective 2d-QFT}} & \text{\textbf{\mu^{-1}}} \\
0 & \Lambda^{-1} & \mu^{-1}
\end{array}
\]

\[
L^2 = \ln\left(\frac{\Lambda}{\mu}\right)
\]

The \(\lambda\)-model fluctuations at 2d distances \(< \Lambda^{-1}\) produce an effective 2d-QFT with UV cutoff \(\Lambda\) and thus an effective string S-matrix with IR cutoff \(L\).

The \(\lambda\)-model is designed precisely so that the effective S-matrices it generates are consistent. It implements the “S-matrix RG”. Roughly: the \(\lambda\)-model replicates the froth of tiny handles. [The detailed calculation can be explained perhaps at a later time.]

As \(\lambda\)-fluctuations are integrated out, i.e., as the 2d-RG operates on the \(\lambda\)-model, from smaller 2d distance \(\Lambda'^{-1}\) to larger \(\Lambda^{-1}\),

\[
\Lambda' > \Lambda \quad L' > L \quad S\text{-matrix}(L') \supset S\text{-matrix}(L)
\]
Its target manifold is the manifold of effective string backgrounds at distance $L$, which is the manifold of classical space-time fields of wave number $> L^{-1}$. 
The QFT is the \textit{a priori} measure of the lambda model, which is a measure on the target manifold, i.e., a functional integral on the classical space-time fields. There are possibilities of non-canonical effects

- winding modes associated to $\pi_1$ of the manifold of space-time fields
- 2d instantons associated to $\pi_2$ of the manifold of space-time fields
This is a modest, pragmatic version of the S-matrix philosophy (which was first proposed by Heisenberg in 1943-48, then re-appeared in the 1950’s, then in the 1960’s became the Church of the S-matrix with Chew as Pope).

The pure S-matrix philosophy proposed the S-matrix as a complete replacement for quantum mechanics.

This was a grandiose, foolish idealization. Physics in the real world is done with quantum mechanics (which in some regimes is accurately approximated by classical mechanics) I’m not sure the S-matrix would be enough, even in principle.

**Truism:** An S-matrix is a form of quantum mechanics.

No. An S-matrix is derived from a quantum mechanics. A quantum mechanics gives an S-matrix, but an S-matrix does not give a quantum mechanics.

Quantum mechanics gives a time evolution operator $U(t, t_0)$. The S-matrix is the asymptotic limit $U(+\infty, -\infty)$.

The string theory S-matrix as a theory of everything is a version of the idealized S-matrix philosophy.

Here I adopt a pragmatic version. Our knowledge of short distance physics, where we do not yet have a quantum mechanical
String theory truisms that were adopted in the 1980s that I think have been misleading for fundamental physics (although fruitful for mathematics).

**Truism: The low momentum/large distance limit of string-theory is a quantum field theory.**
No: the low momentum S-matrix amplitudes correspond to those of a quantum field theory. This is very suggestive, but string theory does not produce an actual QFT — a hamiltonian or a functional integral over fields.

**Truism: A solution of $R_{\mu\nu} = 0$ (or its generalizations) is a string background**
No. need QFT as background, not classical fields.

**Truism: A string S-matrix is potentially a TOE.**
No, an idealized S-matrix, but need effective S-matrix at scale $L$
This is a \textit{specific} effective qft!
The space of effective QFTs is too big. Naturalness does not seem to work.

$R_{\mu\nu} = 0$ is not quite Einstein’s equation. How to produce a realistic field theory? (with gauge, fermion, scalar fields)

The fixed points $R_{\mu\nu} = 0$ have unstable directions (relevant couplings). How to get stability?

String theory makes S-matrices without taking an asymptotic limit in a QFT.

string theory = a 2d recipe for a perturbative S-matrix containing massless particles, including a spin-2 graviton, gauge bosons, fermions, and scalars — low-momentum scattering states potentially enough to capture the high momentum S-matrix of SM+GR.
a continuum qft is a mathematical idealization
realistic qfts have UV cutoffs (including 2d-qfts of the string world-surface)
an asymptotic S-matrix is a mathematical idealization
realistic S-matrices have IR cutoffs