A pragmatic approach to formal fundamental physics (more)

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Abstract

I continue discussing a long-running project to construct a mechanism to produce the fundamental laws of physics.

These slides and related writings can be found at

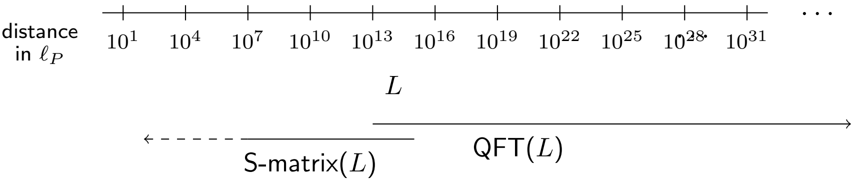
http://www.physics.rutgers.edu/~friedan/#munich

I have a strong interest in the flow of ideas in physics, so there is a fair amount of that in these slides.

But I was asked to elaborate in more technical detail on yesterday's colloquium talk.

So I will skip over most of the non-technical slides, leaving them to be read offline by anyone interested.

The minimal practical formalism for fundamental physics:



For observers at every distance scale $L \gg 1$,

- an effective QFT(L) with UV cutoff L describing physics at distances $> L$
- an effective S-matrix(L) with IR cutoff L describing physics at distances $< L$

L is a sliding distance scale. What we mean by "short distance physics" is relative.

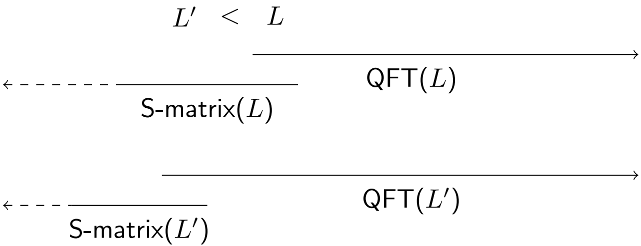
Progress pushes L to smaller L' (but still $\gg 1$)

Consistency conditions have to hold:

(1) The renormalization group makes QFT(L) from QFT(L').

(2) The "S-matrix RG" makes S-matrix(L') from S-matrix(L), using the scattering states at the larger L to make those at smaller L'.

(3) S-matrix(L) agrees with the scattering amplitudes derived from QFT(L') where both apply, i.e. between L' and L.



The λ-model is a mechanism that *produces* such a formal structure.

The λ-model is a mathematically natural 2d nonlinear model (2d-NLM) whose target manifold is the space of classical space-time fields which describe the classical string backgrounds = the space of 2d coupling constants of the string worldsheet.

At every $L \gg 1$, the λ-model produces

- a *quantum* string background — an effective 2d-QFT of the string worldsheet giving an effective string S-matrix(L) with IR cutoff L.
- an effective QFT(L) in the form of a functional measure on the manifold of space-time fields with UV cutoff L

The 2d-RG as a mechanism for space-time physics (1977–79)

In the general renormalizable 2d-NLM

$$\int \mathcal{D}X \; e^{-\int d^2z \; g_{\mu\nu}(X)\partial X^\mu \bar{\partial} X^\nu} \qquad X(z,\bar{z}) \in M$$

the coupling constants are given by a Riemannian metric $g_{\mu\nu}(X)$ on a manifold M.

The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda} \, g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2)$$

drives the 2d-NLM to a solution of $R_{\mu\nu} = 0$

This was extremely exciting (at least for me).

The 2d-RG is a *mechanism* that *produces* solutions of a GR-like space-time field equation $R_{\mu\nu} = 0$.

It suggested the possibility of actually answering the question

Where does space-time field theory come from?

or even

Where do the laws of physics come from?

with a quite unexpected mechanism: the 2d-RG.

The 2d-RG seemed promising in that it was a mechanism that at least produced *classical* field theory.

By the late 1970s it had become clear that there are too many effective QFTs. A mechanism was needed that would *produce* effective QFTs more selectively than the QFT RG.

The 2d-RG incorporated into string theory (1981–85)

- The 2d-RG fixed point equation $\beta = 0$ as consistency condition for the string S-matrix recipe (2d scale invariance)

- A string background as a 2d-NLM of the worldsheet with degrees of freedom $X^\mu(z, \bar{z})$ etc. such that the 2d coupling constants are the space-time metric $g_{\mu\nu}(X)$ plus non-abelian gauge fields, scalar fields, fermion fields, etc.

- The $\beta = 0$ equation of this 2d-NLM (generalizing $R_{\mu\nu} = 0$) as a semi-realistic classical field equation which includes GR and potentially the SM

- The string S-matrix at low momentum agrees with the S-matrix of the perturbative canonical quantization of the classical field equation $\beta = 0$.

In the 1980s, there were several motivations for using the string theory S-matrix for short distance physics.

1. String theory constructs S-matrices without assuming a short distance QFT.

2. The string scattering states include massless particles, in particular a spin-2 graviton, so would be suitable for short distance scattering in backgrounds that include SM+GR.

3. The $\beta = 0$ equation of the original general 2d-NLM, $R_{\mu\nu} = 0$, was not quite Einstein's equation. The $\beta = 0$ equation for the 2d-NLM of the string worldsheet was a potentially realistic space-time field equation.

4. The RG fixed points, $\beta = 0$, of the general 2d-NLM have unstable directions. The 2d supersymmetry of the string worldsheet 2d-NLM eliminates the unstable directions (tachyons in the S-matrix).

Questions (1987)

1. $\beta = 0$ is a only consistency condition for the string recipe. How does the 2d-RG act in string theory as a *mechanism*?

2. Where does *quantum* field theory come from?

   What produces a functional integral over space-time fields?

3. What is the *quantum* string background, which should be given by a quantum state of a QFT?

   (as opposed to the classical string backgrounds given by classical fields solving $R_{\mu\nu} = 0$.)

4. What can produce an *effective* string S-matrix with IR cutoff in an *effective* quantum background described by an effective QFT with UV cutoff?

Reject mathematical idealizations that were adopted as truisms in the mainstream of string theory:

1. The string S-matrix as an asymptotic, idealized S-matrix without IR cutoff — a "theory of everything".

2. The string backgrounds as the backgrounds for such asymptotic string S-matrices: the solutions of $\beta = 0$, i.e., the Calabi-Yau manifolds ($R_{\mu\nu} = 0$) and generalizations.

3. The assumption that the low momentum physics of string theory *is* the (supersymmetric) QFT that happens to have the same low momentum scattering amplitudes as the asymptotic string S-matrix.

The λ-model (1988-2002)

Consider a 2d-NLM of the string worldsheet, with

$$\begin{aligned} \lambda^i &= \text{the 2d coupling constants,} \\ \phi_i(z,\bar{z}) &= \text{the corresponding 2d scaling fields,} \\ |\phi_i\rangle &= \text{the corresponding states on the circle.} \end{aligned}$$

The index i labels the modes of the space-time fields, e.g.,

$$\phi_i(z,\bar{z}) = e^{ip_\mu(i)X^\mu}\, h_{\mu\nu}(i)\, \partial X^\mu \bar{\partial} X^\nu \qquad i \;\leftrightarrow\; p_\mu(i),\, h_{\mu\nu}(i)$$

Inserting the perturbation

$$e^{\int d^2z\; \lambda^i \phi_i(z,\bar{z})}$$

makes $\{\lambda^i\}$ a system of local coordinates on the space of 2d-QFTs.

The 2d scaling-dimensions and the 2d β-function are

$$\dim(\phi_i) = 2 + \delta(i) \qquad \dim(\lambda^i) = -\delta(i) \qquad \dim(\lambda^i \phi_i) = 2$$

$$\beta^i(\lambda) = -\delta(i)\lambda^i + O(\lambda^2)$$

where

$$\delta(i) = p(i)^2$$

The marginal couplings

$$\dim(\lambda^i) = -\delta(i) = 0$$

parametrize the $\beta = 0$ submanifold of 2d-QFTs.

The 2d-RG drives the worldsheet towards the $\beta = 0$ submanifold.

$(ds)^2 = \mu^2 |dz|^2$ is the 2d metric. $\quad \Lambda^{-1} \ll \mu^{-1}$ is a 2d UV cutoff.

The cutoff string propagator (the cutoff 2d-cylinder) is

$$\int_0^{\ln(\Lambda/\mu)} d\tau \left(\sum_i \; |\phi_i\rangle \; e^{-\tau\delta(i)} \; \langle\phi_i| \right) = \sum_i \; |\phi_i\rangle \; \frac{1 - e^{-L^2\delta(i)}}{\delta(i)} \; \langle\phi_i|$$

where

$$e^{-L^2\delta(i)} = (\Lambda/\mu)^{-\delta(i)} \qquad L^2 = \ln(\Lambda/\mu)$$

The only propagating modes are those satisfying

$$\delta(i) > L^{-2} \qquad \text{which is} \quad p(i)^2 > L^{-2}$$

So the 2d UV cutoff Λ^{-1} is an IR cutoff L on the string S-matrix.

An effective 2d-QFT with 2d UV cutoff Λ^{-1} gives an effective string S-matrix(L) with $L^2 = \ln(\Lambda/\mu)$.

What are the effective 2d coupling constants at 2d scale Λ^{-1}?

Microscopic coupling constants $\lambda^i(\Lambda)$ at 2d scale Λ^{-1} parametrize the effective 2d QFT.

Their effects are suppressed by the 2d-RG running from Λ^{-1} to μ^{-1}

$$\lambda^i(\mu) = (\Lambda/\mu)^{-\delta(i)}\,\lambda^i(\Lambda) = e^{-L^2\delta(i)}\lambda^i(\Lambda) \qquad \dim(\lambda^i) = -\delta(i)$$

$L^2\delta(i) > 1 \implies \lambda^i(\Lambda)$ is effectively irrelevant

The only $\lambda^i(\Lambda)$ that matter have

$$\delta(i) < L^{-2} \qquad \text{which is} \quad p(i)^2 < L^{-2}$$

These are the *effectively marginal* couplings.

So there is a UV cutoff L on the modes of the space-time fields that describe the effective 2d QFT.

The λ-model

Now let the λ^i vary on the worldsheet, becoming sources $\lambda^i(z,\bar{z})$.

Make the $\lambda^i(z,\bar{z})$ fluctuate at 2d distances $< \Lambda^{-1}$, governed by the 2d-NLM

$$\int \mathcal{D}\lambda \;\; e^{-\int d^2z \; g_{\rm str}^{-2} G_{ij}(\lambda)\partial\lambda^i\bar{\partial}\lambda^j} \;\; e^{\int d^2z \; \lambda^i(z,\bar{z})\phi_i(z,\bar{z})}$$

where

- $G_{ij}(\lambda) =$ the natural metric on the manifold of 2d-QFTs
- $g_{\rm str} =$ the string coupling constant
- $\lambda^i(z,\bar{z}) \in \mathcal{M} =$ the target space
  $=$ the manifold of classical string backgrounds
  $=$ the manifold of worldsheet 2d-QFTs
  $=$ the manifold of classical space-time fields.

This 2d-NLM is the λ-model.

The λ-model as S-matrix RG

Implement the S-matrix RG by integrating out small handles. These are the handles that contribute to the effective local worldsheet dynamics.

Make a small handle by identifying the boundaries of two holes of radius r around points z_1, z_2 close in the worldsheet.

$$(z - z_1)(z - z_2) = q = r^2 e^{i\theta}$$

Integrate over the moduli z_1, z_2, q, summing over states on the boundary circles (the θ integral projecting on the spin-0 states).

$$\int d^2 z_1 \int d^2 z_2 \int_{\Lambda^{-1}}^{\frac{1}{2}|z_1 - z_2|} \frac{dr}{r}$$

$$\sum_{i_1, i_2} r^{-\delta(i_1)} \phi_{i_1}(z_1, \bar{z}_1) \;\; G^{i_1 i_2} \;\; r^{-\delta(i_2)} \phi_{i_2}(z_2, \bar{z}_2)$$

The cutoff-dependence comes from the approximately marginal fields, the ϕ_i with $\delta(i) \sim 0$.

$$\int d^2 z_1 \int d^2 z_2 \; \ln\left(\Lambda |z_1 - z_2|\right) G^{i_1 i_2} \; \phi_{i_1}(z_1, \bar{z}_1) \; \phi_{i_2}(z_2, \bar{z}_2)$$

Cancel the small handle with the λ-model 2-point function

$$\int d^2 z_1 \int d^2 z_2 \; \langle \, \lambda^{i_1}(z_1, \bar{z}_1) \; \lambda^{i_2}(z_2, \bar{z}_2) \, \rangle \phi_{i_1}(z_1, \bar{z}_1) \; \phi_{i_2}(z_2, \bar{z}_2)$$

This works to first order in the sum over handles in any classical background λ, therefore the full interacting 2d-NLM with target metric $G_{ij}(\lambda)$ removes the cutoff dependence to all orders, therefore replicates exactly the sum over small handles.

on the 2d distance scale:

λ-model ... effective 2d-QFT

0 ... Λ^{-1} ... μ^{-1}

$$L^2 = \ln\left(\Lambda/\mu\right) \gg 1$$

The λ-fluctuations at 2d distances $< \Lambda^{-1}$ produce an effective 2d-QFT with UV cutoff Λ^{-1}.

This effective 2d-QFT in turn gives an effective string S-matrix(L) with IR cutoff L.

The λ-model is designed precisely to implement the “S-matrix RG”.

The λ-fluctuations are designed precisely to replicate the froth of small handles.

Integrating out the λ-fluctuations at 2d scales from Λ'^{-1} to Λ^{-1}

$$\Lambda'^{-1} < \Lambda^{-1} \qquad\qquad L' > L$$

takes the effective S-matrix(L') with the larger IR cutoff L' to the effective S-matrix(L) with the smaller IR cutoff L

$$\text{S-matrix}(L') \supset \text{S-matrix}(L)$$

by, in effect, integrating out the froth of small handles at 2d scales between Λ'^{-1} and Λ^{-1}, thereby integrating out the string modes with $p(i)^2$ from L'^{-2} up to L^{-2}.

(The basic calculation is shown in the Appendix.)

The λ-model (like any 2d-NLM) is specified by two pieces of data

- the metric $g_{\mathrm{str}}^{-2} G_{ij}(\lambda)$ on the target manifold
- a measure $d\lambda\,\rho(\lambda)$ on the target manifold $\mathcal{M}$ which gives the functional volume element

$$\int \mathcal{D}\lambda = \prod_{(z,\bar{z})} \int_{\mathcal{M}} d\lambda(z,\bar{z})\,\rho(\lambda(z,\bar{z}))$$

$d\lambda\,\rho(\lambda)$ is called the *a priori* measure.

At 2d scale Λ^{-1}, a point $(z,\bar{z})$ represents a 2d block $\Lambda^{-1} \times \Lambda^{-1}$. The measure $d\lambda\,\rho(\lambda)$ summarizes the fluctuations inside a block.

$d\lambda\,\rho(\lambda)$ evolves under the 2d-RG, diffusing in the target manifold $\mathcal{M}$ due to the λ-fluctuations. At the same time, it is driven by the beta-function $\beta^i(\lambda)$ because the λ^i are not exactly marginal, they flow with the 2d scale Λ^{-1} towards the $\beta = 0$ submanifold.

$d\lambda\,\rho(\lambda)$ evolves by the driven diffusion equation

$$\Lambda \frac{\partial}{\partial \Lambda}\,\rho(\lambda) = \nabla_i \left(g_{\mathrm{str}}^2 G^{ij} \partial_j + \beta^i \right) \rho(\lambda)$$

(taking $d\lambda$ to be the metric volume element on $\mathcal{M}$).

$d\lambda\,\rho(\lambda)$ at scale Λ^{-1} is produced by integrating out the λ-fluctuations from 2d distance ~ 0 up to Λ^{-1}, driving it to the equilibrium measure

$$d\lambda\,\rho(\lambda) \to d\lambda\ e^{-\frac{1}{g_{\mathrm{str}}^2} S(\lambda)} \qquad \text{where} \quad \beta^i = G^{ij} \partial_j S$$

Recall that the λ^i are the spacetime field modes with UV cutoff L.

So $d\lambda\,\rho(\lambda)$ is the functional integral of an effective QFT(L) with classical action $\frac{1}{g_{\mathrm{str}}^2} S(\lambda)$.

Thus the λ-model *produces* a QFT(L) at every $L \gg 1$.

The consistency conditions are satisfied:

S-matrix(L) $\supset$ S-matrix(L') for $L > L'$ by design of the λ-model.

QFT(L') $\supset$ QFT(L) via the QFT RG because of the 2d RG — the decoupling of irrelevant operators.

S-matrix(L) and QFT(L) agree on amplitudes at scale $\sim L$ because the scattering amplitudes of S-matrix(L) near the IR cutoff L are given by the 2d correlation functions near the 2d UV cutoff Λ^{-1}, which are determined by the *a priori* measure $d\lambda\,\rho(\lambda)$ which is QFT(L).

The λ-model is a nonperturbative 2d-NLM, with possibilities of nonperturbative semi-classical effects:

- winding modes associated to π_1 of the target manifold $\mathcal{M}$
- 2d instantons associated to $\pi_2(\mathcal{M})$

where $\mathcal{M}$ = the manifold of space-time fields

π_k(the manifold of $SU(N)$ gauge fields in $\mathbb{R}^4$) = $\pi_{k+3}(SU(N))$

So there are winding modes ($k = 1$) when $\pi_4(SU(N)) \neq 0$

$$\pi_4(SU(2)) = \mathbb{Z}_2$$

and there are 2d instantons ($k = 2$) when $\pi_5(SU(N)) \neq 0$

$$\pi_5(SU(2)) = \mathbb{Z}_2 \qquad \pi_5(SU(3)) = \mathbb{Z}$$

The winding modes for $SU(2)$ and the 2d-instantons for $SU(2)$ and $SU(3)$ offer possibilities of conditional predictions.

If the λ-model produces SM+GR then it also produces

- *non-canonical degrees of freedom from the $\mathbb{Z}_2$ winding mode in the manifold of $SU(2)$ gauge fields on $\mathbb{R}^4$*
- *non-canonical interactions from the 2d instanton in the manifold of $SU(2)$ gauge fields and the 2d instantons in the manifold of $SU(3)$ gauge fields on $\mathbb{R}^4$*

$\mathbb{Z}_2$ winding mode in the manifold of $SU(2)$ gauge fields on $\mathbb{R}^4$

Let $A(x_+, x_-, u)$ be a zero-size instanton at x_+ and a zero-size anti-instanton at x_- with relative orientation $u \in SU(2)/\mathbb{Z}_2$.

$$\mathcal{L}(x) = \frac{1}{8\pi}\mathrm{tr}\left(F_{\mu\nu}(x)F^{\mu\nu}(x)\right) = \delta^4(x - x_+) + \delta^4(x - x_-)$$

$$\mathcal{L}^{\mathrm{top}}(x) = \frac{1}{8\pi}\mathrm{tr}\left(F_{\mu\nu}(x){*F}^{\mu\nu}(x)\right) = \delta^4(x - x_+) - \delta^4(x - x_-)$$

The winding mode representing the nontrivial element in $\pi_1 = \mathbb{Z}_2$ is the nontrivial closed geodesic loop in $SU(2)/\mathbb{Z}_2$

$$\theta \mapsto A(x_+, x_-, u(\theta)) \qquad u(0) = 1 \quad u(2\pi) = -1$$

This loop in $\mathcal{M}$ has zero length, so the winding mode will be a 2d field of scaling dimension = 0 + quantum corrections, so it has a chance of participating in the *a priori* measure which is the space-time QFT.

The $\mathbb{Z}_2$ winding mode is *bi-local* in space-time, depending on the two space-time points x_+, x_-.

The 2d instantons for $SU(2)$ and $SU(3)$ gauge fields are nontrivial 2-spheres in slightly more complicated configurations of zero-sized instantons and anti-instantons.

To do:

A huge amount of foundational technical work is still to be done.

More urgent is to find out if the λ-model can in fact make conditional predictions of observable non-canonical effects in $SU(2)$ and $SU(3)$ gauge theory in 4 dimensions.

1. Figure out how to calculate semi-classical corrections to the *a priori* measure of a 2d-NLM coming from winding modes and 2d instantons.

2. Calculate the corrections to the canonical SM

   - from the bi-local winding mode in the manifold of $SU(2)$ gauge fields.

   - from the multi-local 2d instantons in the manifolds of $SU(2)$ and $SU(3)$ gauge fields

Especially tantalizing is the top-down construction of QFT(L).

The λ-model operates from 2d distance ~ 0 up to Λ^{-1}.

Recall that $L^2 = \ln(\Lambda/\mu)$.

So the λ-model builds QFT(L) from space-time distance $\sim \infty$ *down* to L.

Unnaturalness could be natural in QFT(L).