COVARIANT QUANTIZATION OF SUPERSTRINGS

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We present a manifestly Lorentz covariant formulation of supersymmetric string theory In particular, we construct the fermion vertex and the supersymmetry generators in a BRST quantization using the techniques of superconformal field theory

1. Introduction. The fermionic string has been proposed as a unified theory of matter interacting through gauge and gravitational forces. Attractive features of string theory include the rich massless particle spectrum – containing a graviton, gauge vectors and chiral fermions – spacetime supersymmetry, and most likely finiteness as well. String theory is remarkably restricted. Internal consistency forces the string to be supersymmetric, the dimension of spacetime to be ten and the fundamental gauge group, if any, to be SO(32) or $E_8 \times E_8$.

The original formulation of fermionic string theory was Lorentz covariant [1,2]. The covariant formulation was used in a number of interesting applications, but it was never actually finished. The covariant formalism was used to discover the critical dimension $D = 10$, spacetime supersymmetry [3], to formulate the anomaly calculation in string theory [4] and to find the resulting restrictions on possible gauge groups [5]. The proof of supersymmetry and the explicit finiteness calculations were done in the light-cone gauge [6]. Supersymmetry has never been proved in the covariant formulation. The original covariant formalism gave scattering amplitudes only for processes involving zero or two spacetime fermions and an arbitrary number of bosons. The first calculation of a four-fermion amplitude used the light-cone gauge [7]. The four-fermion amplitude was eventually calculated using the covariant formalism[‡1], but there was no effective description of the multifermion amplitudes. In particular the vertex operator describing spacetime fermion emission was never fully constructed. A local field transforming as a spacetime spinor was constructed [10], but its scaling dimension or conformal weight was $D/16 = 5/8$, while a physical vertex operator must have dimension one.

In this letter we complete the covariant construction of the fermionic string following the program described in refs. [4,11]. We start with the Ramond–Neveu–Schwarz model, which describes the world-sheet of the string in terms of a superconformally invariant $(1 + 1)$-dimensional quantum field theory. Following a suggestion of Goddard and Olive, we use the Faddeev–Popov ghosts resulting from fixing the covariant super-

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[‡1] The four-fermion amplitude was developed in a series of calculations referred to in ref [8] For further references on earlier developments in dual theory see the reviews in ref [9]

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conformal gauge [12–14] to construct the BRST [15] operator, the fermion vertex and the spacetime supersymmetry generator. The resulting formalism is Lorentz invariant, reparametrization invariant, and supersymmetric. The choice of superconformal gauge allows us to utilize the techniques of two-dimensional conformal and superconformal field theory [16,17]. The advantage is that the theory is expressed in terms of an algebra of local conformal fields on the world surface, which is automatically defined on surfaces of arbitrary global topology by the principles of conformal field theory. The conformal algebra also provides powerful methods for calculating correlation functions. We will further explain and explore these aspects in a future communication [18].

2. *Fermionic strings.* A Lorentz covariant, reparametrization invariant action for the fermionic string is obtained by coupling string fields X^μ and their world-sheet superpartners ψ^μ to two-dimensional supergravity [19]

$$S=\int d^2z\, e\left[\tfrac{1}{2}\nabla_a X^\mu\cdot\nabla_a X^\mu-\tfrac{1}{2}i\bar{\psi}^\mu\gamma^a\nabla_a\psi^\mu + \tfrac{1}{2}i\left(\bar{\chi}_a\gamma^b\gamma^a\psi^\mu\right)\left(\partial_b X^\mu-\tfrac{1}{4}i\bar{\chi}_b\psi^\mu\right)\right], \tag{1}$$

where e_a^m is a two-dimensional vierbein and χ_a its gravitino partner. For the heterotic string [20], the pieces of the action involving the left-handed parts of χ_a and ψ^μ are replaced by an action for an $E_8\times E_8$ or SO(32) current algebra [‡2]. Reparametrization invariance and local supersymmetry allow the Lorentz covariant gauge choices

$$e_a^m=e^\phi\delta_a^m,\quad \chi_a=\gamma_a\zeta, \tag{2}$$

known as the superconformal gauge. One readily verifies that ϕ and ζ decouple from the action (1) due to its super-Weyl invariance, leaving the free-field action

$$S_{\text{gauge fixed}}=\int\left[\tfrac{1}{2}\left(\partial_a X^\mu\right)^2-\tfrac{1}{2}i\bar{\psi}^\mu\gamma\partial\psi^\mu\right]. \tag{3}$$

In the euclidean superspace defined by the complex coordinates $(z,\bar{z})$ and their anticommuting partners $(\theta,\bar{\theta})$, the action may be written in terms of scalar superfields $X^\mu(z,\bar{z},\theta,\bar{\theta})$ as

$$S_{\text{gauge fixed}}=\int d^2z\,d^2\theta\,\tfrac{1}{2}\bar{D}X^\mu\cdot DX^\mu, \tag{4}$$

where $D=\partial_\theta+\theta\partial_z$. For the heterotic string, $\bar{\theta}$ is absent and $\bar{D}$ is replaced by $\partial_{\bar{z}}$. The solution to the equations of motion is

$$X^\mu=X^\mu(z)+X^\mu(\bar{z})+\theta\psi^\mu(z)+\bar{\theta}\bar{\psi}^\mu(\bar{z}). \tag{5}$$

From now on we focus on the z-dependent fields, in particular $\psi^\mu(z)$. If needed, the entire construction may be duplicated for the $\bar{z}$-dependent pieces. The two-point function is

$$\langle X^\mu(z_1,\theta_1)X^\nu(z_2,\theta_2)\rangle=-g^{\mu\nu}\log(z_{12}), \tag{6}$$

where $z_{12}=z_1-z_2-\theta_1\theta_2$.

The time components X^0 are negative metric fields. The states created by X^0 decouple [22] due to the residual invariance of the gauge choice (2) under superconformal transformations $(z,\theta)\to f(z,\theta)$. These transformations are generated by the moments of the super stress–energy tensor

$$T(z,\theta)=-\tfrac{1}{2}DX^\mu D^2X^\mu=T_F+\theta T_B \tag{7}$$

$$=-\tfrac{1}{2}\left[\psi^\mu\partial_z X^\mu+\theta\left(\partial_z X^\mu\partial_z X^\mu+\partial_z\psi^\mu\psi^\mu\right)\right]. \tag{8}$$

T_B is the ordinary stress–energy tensor and T_F is its dimension-3/2 partner. The conformal properties of string operators are an essential element of our construction.

The coordinates (σ,τ), $\ln z=\tau+i\sigma$, describe a cylindrical string world sheet which admits two possible spatial (σ) boundary conditions for the fermion ψ^μ: anti-periodic (Neveu–Schwarz [2]) boundary conditions describe spacetime bosons and periodic (Ramond [1]) boundary conditions describe spacetime fermions. The emission of a string state which is a spacetime fermion changes the boundary condition on the world sheet from NS to R or vice versa. The fermion field ψ^μ must be double-valued around the operator describing this process, which therefore opens or closes a cut in ψ^μ. It is called a *spin* field [17] (for a single Majorana fermion ψ, the corresponding spin field S would be the dimension-1/16 order or disorder operator of the Ising model [23]). The basic spin field S^α has dimension $D/16=5/8$ and transforms as a 32-component spacetime

‡2 The use of rank 16 current algebras in incorporating gauge symmetries in string theories was also suggested by Freund [21]

Majorana spinor. On the other hand, the emission of spacetime bosons maintains the boundary condition on ψ^μ and can be described by operators which are the θ integrals of superfields. For instance, the z-dependent part of the emission vertex for a massless vector of momentum k and polarization ζ is [24]

$$V_{\mathrm{B}}(k,\zeta,z)=\int \mathrm{d}\theta\, \mathrm{e}^{\mathrm{i}k\cdot X}\mathrm{D}X_\mu \zeta^\mu(k)$$
$$=\mathrm{e}^{\mathrm{i}k\cdot X(z)}\left[\partial_z X_\mu+\mathrm{i}(k\cdot\psi)\psi_\mu\right]\zeta^\mu(k), \tag{9}$$

with $k\cdot\zeta=0$.

A subsector of this NS–R string theory is spacetime supersymmetric; it is obtained by projecting onto the states of even world surface fermion number $(-1)^F=1$. In the R sector, $(-1)^F$ acts as γ_{11}, so the even subspace corresponds to left-handed Majorana–Weyl spinors (we use the conventions $[\gamma_\mu,\gamma_\nu]_+=-g_{\mu\nu}$, $S_\beta=S^\alpha\epsilon_{\alpha\beta}$, $\epsilon_{\alpha\beta}=-\epsilon_{\beta\alpha}$, $\gamma_\mu^{\alpha\beta}=\gamma_\mu^{\beta\alpha}$). The lowest energy states now describe a massless ten-dimensional supermultiplet.

In order to calculate fermion scattering amplitudes we shall have to compute spin field correlation functions. In principle these can be calculated as the free energy of double-valued fields ψ^μ with cuts on the world surface connecting pairs of spin fields. More practical methods [25] employ the SO(1,9) currents $\psi^\mu\psi^\nu(z)$ and either the null vector [26] or vertex operator [27] construction of their spinor representation. For instance, in the vertex operator construction for the Wick rotated SO(10) currents, one begins by bosonizing the ten ψ^μ to obtain five scalar fields θ_i which parametrize the maximal torus of SO(10). The spin fields are then the 2^5 exponentials $:\mathrm{e}^{\mathrm{i}\alpha\cdot\theta}:c_\alpha$ with $\alpha=(\pm\frac{1}{2},\ldots,\pm\frac{1}{2})$ corresponding to the spinor weights of SO(10). c_α generates a two-cocycle in commutation relations. This construction is analogous to that used for the $\mathrm{E}_8\times\mathrm{E}_8$ and SO(32) currents of the heterotic string.

3. The ghosts. Vertex operators for physical state emission must have dimension one to make their z-integrals reparametrization invariant. The vertex operator $S_\alpha \mathrm{e}^{\mathrm{i}k\cdot X}(z)$ will not work since it has dimension $5/8+k^2/2=5/8$. We need a dimension-3/8 operator to complete the vertex. Goddard and Olive suggested that the operator might come from the Faddeev–Popov ghosts.

The Faddeev–Popov (super) determinant compensates for fixing the intrinsic super metric on the world surface. This superdeterminant is the jacobian for the change of variables

$$\delta e_a^m=\nabla_a\delta\xi^m,\quad \delta\chi_a=\nabla_a\delta\epsilon, \tag{10}$$

used to factor out the super-reparametrization group [12,14]. The jacobian may be represented by a path integral over a conjugate pair of free ghost superfields

$$C^z=c^z+\theta\gamma^\theta,\quad B_{z\theta}=\beta_{z\theta}+\theta b_{zz}, \tag{11}$$

whose action is

$$S_{\mathrm{ghost}}=\int \mathrm{d}^2z\,\mathrm{d}^2\theta\, B_{z\theta}\bar{\mathrm{D}}C^z. \tag{12}$$

From this action we find the ghost super stress–energy tensor

$$T_{\mathrm{ghost}}=-(\mathrm{D}^2B)C+\tfrac{1}{2}(\mathrm{D}B)(\mathrm{D}C)-\tfrac{3}{2}B(\mathrm{D}^2C), \tag{13}$$

and the two-point function $\langle C(z_1,\theta_1)B(z_2,\theta_2)\rangle=\theta_{12}/z_{12}$, where $\theta_{12}=\theta_1-\theta_2$. It is the full stress–energy tensor $T_{zz}=T_{\mathrm{matter}}(X,\psi)+T_{\mathrm{ghost}}$ which generates conformal transformations. In the algebra of conformal transformations resulting from the operator products [13]

$$T_{zz}T_{ww}\sim\frac{c/2}{(z-w)^4}+\frac{2T_{zz}}{(z-w)^2}+\frac{1}{z-w}\partial_z T_{zz}+\ldots, \tag{14}$$

the coefficient c of the Schwinger term receives contributions of

$$c_X=D,\quad c_\psi=D/2,\quad c_{b,c}=-26,\quad c_{\beta,\gamma}=11.$$

In $D=10$ spacetime dimensions $c=0$ so there is no anomaly in reparametrizations.

The key point to realize is that the world sheet gravitino χ_a and the spinor superconformal parameter ϵ must obey boundary conditions corresponding to the NS and R sectors in order that the action (eq. (1)) be well-defined and locally supersymmetric. Thus the correct fermion emission vertex introduces a cut in all the world sheet spinor fields, and will therefore involve a spin field Σ for the (commuting) spinor ghosts $\beta_{z\theta}$, γ^θ in addition to the spin field S^α. Σ is most conveniently constructed using an exponential representation of the β, γ algebra. In order to do this we reconstruct the algebra of ghost operators by decomposing the $c = 11$ spinor ghost system into two subsystems, one with $c = 13$ and the other with $c = -2$. The former is obtained by bosonizing the ghost current corresponding to ghost fermion number

$$j_z = -\gamma^\theta \beta_{z\theta}. \tag{15}$$

This current satisfies the anomalous conservation law

$$\partial_{\bar z} j_z = \tfrac{1}{8}\sqrt{g}\, Q R^{(2)}, \tag{16}$$

with $R^{(2)}$ the intrinsic scalar curvature on the world sheet and $Q = 2$. This anomaly is reflected in the operator product expansion

$$T_{zz}(\beta,\gamma) j_w \sim Q(z-w)^{-3} + (z-w)^{-2} j_z. \tag{17}$$

On the general world surface with g handles the integrated anomaly gives the Riemann–Roch index

$$\#\text{ zero modes }(\gamma) - \#\text{ zero modes }(\beta)$$
$$= \chi[\dim(\beta) - 1/2] = \chi \cdot Q/2, \tag{18}$$

where χ is the Euler characteristic $\chi = 2 - 2g$. On the sphere the anomaly is related to the two globally defined superconformal spinor zero modes which have been omitted from the Faddeev–Popov determinant. A careful treatment of ghost zero modes in the bosonic string theory is given in ref. [28]; the supersymmetric case will be sketched in ref. [18].

From the ghost current j_z we construct another stress–energy tensor

$$T_{13} = -\tfrac{1}{2}(j_z j_z - Q\,\partial_z j_z), \tag{19}$$

which has the same operator product (eq. (17)) with j_z but conformal anomaly $c = 13$. This current and stress-energy tensor can be represented by a scalar field[‡3] ϕ with two-point function $\langle\phi(z)\phi(w)\rangle = \ln(z-w)$.

$$j_z = -\partial_z\phi, \quad T_{zz} = -\tfrac{1}{2}(\partial_z\phi\,\partial_z\phi - Q\,\partial_z^2\phi). \tag{20}$$

The integrated anomaly can be interpreted as the existence of a background charge coupled to ϕ of magnitude $\chi \cdot Q/2$.

The difference $T_{-2} = T(\beta,\gamma) - T_{13}$ commutes with j_z and has $c = -2$. It may be written in terms of a pair of anticommuting free fields $\eta(z)$ of dimension 1 and $\xi(z)$ of dimension 0 as

$$T_{-2} = (\partial_z\xi)\eta. \tag{21}$$

The free field theory for $\eta(z)$, $\xi(z)$ has the two-point function $\langle\xi(z)\eta(w)\rangle = (z-w)^{-1}$. The exponential $e^{\alpha\phi}$ is a conformal field of dimension $-\frac{1}{2}\alpha(\alpha+2)$; the dimension-3/2 and $-1/2$ fields $\beta_{z\theta}, \gamma^\theta$ are represented by

$$\beta_{z\theta} = e^{-\phi}\partial_z\xi, \quad \gamma^\theta = e^{\phi}\eta. \tag{22}$$

Of course η and ξ can also be represented as exponentials of a free scalar field, of opposite signature to ϕ and with background charge $Q = -1$. We observe that the conjugate fields $e^{-\phi/2}$ and $e^{\phi/2}$ have dimensions 3/8 and $-5/8$, respectively, and are therefore candidates for inclusion in the fermion vertex as the spin field Σ.

Since ξ has dimension 0 there is always a single constant zero mode in ξ for any world sheet topology. Note that the zero mode of ξ does not appear in the β, γ algebra; only derivatives of ξ appear. This is important, because the irreducible algebra of BRST-invariant operators will only involve factors of $\partial_z\xi$. We are only interested in correlation functions of such physical operators, so we can dispense with the zero mode of ξ, and calculate expectation values in the vacuum $|0\rangle$ of the smaller algebra. Alternatively we can carry out the same calculation in the larger Hilbert space where the zero mode acts, but then the vacuum is doubly degenerate because of the zero mode. The two vacua can be written $|0\rangle$ and $\xi(0)|0\rangle$, with $\langle 0|0\rangle = 0$ but $\langle 0|\xi(z)|0\rangle = 1$. One consequence is that a factor of $\xi(z)$ must be included in correlation functions to absorb the zero mode. The location of this extra $\xi(z)$ on the world sheet is irrelevant since only the constant zero mode contributes.

The Hilbert space of the spinor ghosts β, γ is rather exotic because it derives from a first order

[‡3] This modification of the scalar field was first discussed in ref. [29]. It has been extensively developed in the context of conformal field theory in ref. [30]. D.F. would like to thank Vl. S. Dotsenko for pointing out the role of the anomalous current in these scalar field systems.

action for Bose fields. The SL(2, R) invariant NS vacuum state $|0\rangle_{\mathrm{gh}}$ of the ghosts is annihilated by the superconformal generators

$$L_n = \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} z^{n+1} T_{zz}^{\text{ghost}}, \quad n = 0, \pm 1,$$

$$G_n = \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} z^{n+1/2} T_{z\theta}^{\text{ghost}}, \quad n = \pm 1/2, \tag{23}$$

but is not a highest weight state (i.e., not annihilated by all the lowering operators of the fields b, c, β, γ. Rather the highest weight state is $\mathrm{e}^{-\phi(0)} c^z(0)|0\rangle$, which has L_0 eigenvalue (energy) $-1/2$; this is the tachyonic NS vacuum state. Similarly, the highest weight state in the Ramond sector is $\mathrm{e}^{-\phi/2(0)} c^z(0)|0\rangle$, which has energy $-5/8$. When we take the tensor product of this state with the states of energy $5/8$ created by the spin operator S_α, we find the massless R vacuum states. The coherent state operators $:\mathrm{e}^{-\alpha\phi(z)}:$ act to redefine the "Bose sea level" by filling a given set of energy levels of the β, γ system. For instance, $|0\rangle_{\mathrm{gh}}$ is annihilated by the Fourier components

$$\gamma_n = \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} z^{n-3/2} \gamma(z) \tag{24}$$

for $n \geqslant 3/2$ whereas $\mathrm{e}^{-\phi(0)}|0\rangle_{\mathrm{gh}}$ is annihilated by γ_n for $n \geqslant 1/2$. In contrast to the Fermi case, these different sea levels are unitarily inequivalent representations of the β, γ algebra. Note also that $\mathrm{e}^{-\phi(0)} c^z(0)|0\rangle$ carries one unit of ϕ-charge and therefore has $(-1)^F = -1$; the tachyonic NS ground state will be eliminated upon projecting onto even (total) sheet fermion number.

4. Vertices. We are now ready to assemble the various pieces of the fermion vertex. A physical vertex in either the NS or R sectors is one which respects BRST invariance, which is to say it commutes with the BRST charge

$$Q_{\mathrm{BRST}} = \frac{1}{2\pi\mathrm{i}} \oint \mathrm{d}z\, \mathrm{d}\theta\, C^z \left(T_{\text{matter}} + \tfrac{1}{2} T_{\text{ghost}}\right)$$

$$= \oint \frac{\mathrm{d}z}{2\pi\mathrm{i}} \left\{ c^z \left[T_{zz}(x, \psi) + \tfrac{1}{2} T_{zz}(b, c, \beta, \gamma)\right] + \gamma^\theta \left[T_{z\theta}(x, \psi) + \tfrac{1}{2} T_{z\theta}(b, c, \beta, \gamma)\right]\right\}. \tag{25}$$

Reparametrization invariance in the quantum string theory means that

$$Q_{\mathrm{BRST}}^2 = 0. \tag{26}$$

Physical states satisfy

$$Q_{\mathrm{BRST}}|\text{phys}\rangle = 0. \tag{27}$$

Operators such as $[Q_{\mathrm{BRST}}, O]$, O any operator, automatically commute with Q_{BRST} but create null states. The superspace integrals of dimension-1/2 superfields (e.g., eq. (9)) commute with Q_{BRST} and are appropriate vertices for boson emission. We find that the operator

$$V_1 = \mathrm{e}^{-\phi/2} S_\alpha \mathrm{e}^{\mathrm{i}k \cdot X} u^\alpha(k), \quad \gamma \cdot ku = 0, \tag{28}$$

is physical; this is to be expected since we have shown that it creates a physical R vacuum state. Since ϕ-charge is conserved in interactions we need a second piece of the fermion vertex with opposite ϕ-charge in order to have nonvanishing amplitudes; it is

$$V_2 = \left[\mathrm{e}^{\phi/2} S^\beta \gamma^\mu_{\beta\alpha} \left(\partial_z X^\mu + \tfrac{1}{4}\mathrm{i}k \cdot \psi \psi^\mu\right) + \mathrm{e}^{3\phi/2} \eta b_{zz} S_\alpha\right] \mathrm{e}^{\mathrm{i}k \cdot X} u^\alpha(k), \quad \gamma \cdot ku = 0. \tag{29}$$

The second term will not contribute to expectation values due to b-charge conservation. Note that although S^β has the opposite chirality compared to S_β, V_2 differs from V_1 by carrying one more unit of ϕ-charge so that the overall value of $(-1)^F$ remains the same. One might at first think that V_2 is a null operator since it can be written

$$V_2 = [Q_{\mathrm{BRST}}, \xi V_1], \tag{30}$$

by taking the contour integral of the BRST current over a contour which circles the operator $\xi V_1(z)$. However ξ is not part of the β, γ algebra and therefore this description is just an artifact of our representation. Nevertheless this presentation of V_2 will be a useful tool. In particular it immediately implies that V_2 is BRST invariant. The full fermion vertex is now

$$V_{\mathrm{F}} = \tfrac{1}{2}(V_1 + V_2). \tag{31}$$

This vertex is a local dimension-one conformal field so amplitudes constructed using it are manifestly dual. We can picture V_1 acting to create a cut in the world sheet spinors and V_2 acting to close a cut. Their operator product factorizes on the physical NS boson vertices [31]

$$V_1(k,z,u)V_2(p,w,v) \sim (z-w)^{k\cdot p-1}\bar{u}\gamma^\mu v \times [\partial_z X^\mu + i(k+p)\cdot\psi\psi^\mu] e^{i(k+p)\cdot X}(z) + \dots \quad (32)$$

Duality creates an apparent problem: how can we arrange the vertices so that they occur in the (time) order $V_1(z)V_2(w)V_1(t)V_2(u)\dots$, always taking us between the canonical NS and R Hilbert spaces? In the crossed channel we could not prevent the successive occurrence of V_1V_1 or V_2V_2. These operator products factorize the amplitude on what seem to be unphysical states associated with non-canonical Bose sea levels. We resolve this problem by the following argument.

Consider any correlation function of fermion vertices. It is a sum of correlation functions each containing an equal number of V_1 and V_2 vertices by ϕ charge neutrality. We show that any such correlation function satisfies the identity

$$\langle \cdots V_1(k_1,u_1,z_1)\cdots V_2(k_2,u_2,z_2)\cdots\rangle = \langle \cdots V_2(k_1,u_1,z_1)\cdots V_1(k_2,u_2,z_2)\cdots\rangle. \quad (33)$$

This allows us to rewrite all of the correlation functions of V_1 and V_2 in the canonical order $V_1V_2V_1V_2\cdots$, ensuring that multifermion amplitudes factorize on only physical poles.

To go from the right-hand side of eq. (33) to the left-hand side we calculate both in the large algebra which includes the ξ zero mode. Recall that a factor of $\xi(z)$ must be included in correlation functions to absorb the zero mode. The location of this extra $\xi(z)$ on the world sheet is irrelevant since only the zero mode contributes. Note that the calculation in the smaller algebra (without the zero mode) is manifestly BRST invariant, but in the large algebra it is not because $Q_{\mathrm{BRST}}\,\xi|0\rangle \neq 0$. Otherwise eq. (30) would imply that V_2 decouples and all fermion amplitudes would vanish.

To demonstrate the identity (33) perform the following manipulations. The left-hand side of eq. (33) is pictured in fig. 1a. First move $\xi(z)$ to z_1. Next write $V_2(z_2)=[Q_{\mathrm{BRST}}, 2\xi V_1(z_2)]$, expressing the commutator as a contour integral as in fig. 1b. Deform the contour to surround z_1 instead of z_2

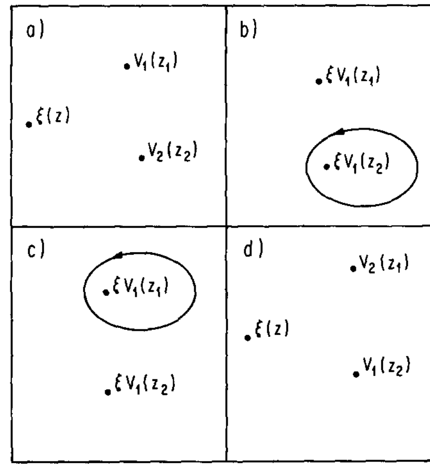

Fig 1 Manipulations necessary to demonstrate eq (33).

(fig. 1c). Finally replace the contour integral with $V_2(z_1)$ and move $\xi(z_1)$ back to z (fig. 1d) to obtain the right-hand side of eq. (33).

This manipulation shows that there is a remarkable redundancy in the representation of the fermion amplitude. In order to accommodate the fermion vertex we were forced to extend the Hilbert space to include an infinite number of inequivalent representations of the ghost algebra associated with the Bose sea levels. The fermion vertex generates a closed subalgebra of the enlarged matter–ghost system which describes all physical particles. As we have just seen all Bose levels are equivalent under the algebra of vertex operators. We also note that the picture changing operation of the old NS dual model [32] can be implemented by the above operations along with the identity

$$[Q_{\mathrm{BRST}}, \xi e^{-\phi}\psi^\mu e^{ik\cdot X}] = V_B. \quad (34)$$

The supersymmetry current is $V_F(k=0)$. The operator product eq. (32) shows that contour integrals of the supersymmetry current will change a fermion to a boson vertex as the contour is deformed through it. The potentially troublesome V_1V_1 and V_2V_2 operator products are removed by the above procedure. The supersymmetry algebra is made more transparent by interpreting $e^{\phi/2}S^\alpha(z)$ as the dimension-0 superspace partner θ^α to $X^\mu(z)$, with $e^{-\phi/2}S_\alpha(z)$ and $P^\mu=\partial_z X^\mu$ their canonical conjugates. Then we can write

$$V_{\mathrm{F}}(k=0) \sim \tfrac{1}{2}\left(\delta/\delta\theta^{\alpha} + \theta^{\beta}\gamma^{\mu}_{\alpha\beta}P^{\mu}\right) \tag{35}$$

for the supersymmetry current.

In conclusion, we have constructed the fermion vertex operator in a Lorentz covariant BRST quantization using conformal field theory techniques, completing a covariant quantization of the superstring. We have proven that the scattering amplitudes of superstring bosons and fermions are Lorentz invariant, BRST invariant, supersymmetric, and dual; further details will be presented elsewhere [18]. This formalism simplifies the calculation of fermion multipoint correlations; the four-fermion tree amplitude will be reproduced in ref. [25]. The method generalizes to multiloop corrections in a straightforward way; in particular; supersymmetry is apparent to all orders (at least to the extent that the expansion itself is sensible).

It should also be possible to give a superfield formulation of the superstring along the lines envisioned by Siegel [33], thus relating the NS–R quantization to the quantized version of the Green–Schwarz covariant theory [34]; the idea is that the two should be related by a version of Witten's non-abelian bosonization [35].

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