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PHENOMENOLOGY AND CONFORMAL FIELD THEORY OR CAN STRING THEORY PREDICT THE WEAK MIXING ANGLE?*

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We show that the weak mixing angle θ_w is the same for continuously connected classical vacua of the heterotic string which have chiral fermions in their massless spectra. We also show that the world-sheet quantum field theory for any classical vacuum with spacetime supersymmetry possesses an $N=2$ superconformal invariance.

1. Introduction

The initial indications that the theory of superstrings [1] would lead to a unique and direct prediction of the low energy particle spectrum have proved to be misleading. It now appears [2] that every two-dimensional quantum field theory with $(0,1)$ superconformal invariance is a classical vacuum state for the heterotic superstring [3]. The nonperturbative physics that would tell us which of these vacua is stable and/or show that superstrings are truly consistent only on a subset of them (determined perhaps by some topological invariant of the space of all field theories)

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is at present beyond our comprehension. The possible responses to this situation seem to fall into three general categories. The first is to gleefully declare that superstrings clearly have nothing to do with experimentally accessible physics, and save oneself the bother of plumbing their intricate depths. A second response is to assert that contact with experiment can only come after the true structure of superstring theory is fully understood; i.e. that it is premature to try to relate the real world to the crude semiclassical approximation to a presently nonexistent theory.

The practical consequences of the second attitude are remarkably similar to those of the first. Superstring theory will become the province of a small group of mathematically sophisticated adepts with little contact with the rest of particle physics. We believe that although this is a possible avenue for development of the theory and its relationship to the rest of physics, it is not the most desirable one. Rather, there is a third response to the situation, which is to retain hope that important low energy physics can be extracted from the semiclassical approach, and that important clues to the structure of string theory may be gleaned from the requirement that some classical vacuum obey the constraints of phenomenology. Most of the extant work on string phenomenology uses the low energy effective Lagrangian for the string propagating in a flat ten-dimensional background, sometimes supplemented with a hint from Kaluza-Klein theories (which are never a very good approximation to a consistent superstring picture of the real world [4]). We have no quarrel with this admirable method but we believe that the time has come to try to make a closer connection between phenomenological questions and the real formalism of string theory. Desirable phenomenological properties should be formulated as constraints on two-dimensional superconformal field theories. The advantages of this approach are twofold. First it enables us to answer questions about the assumptions that go into the effective lagrangian method. Do superstrings really predict extra low energy gauge structure? Do they naturally predict the value of the weak mixing angle? Can one find vacua with discrete symmetries which constrain fermion mass matrices and couplings in the manner required by experiment? The second desirable feature of the "phenomenology of conformal field theory" is a sociological one. The techniques that one must employ to answer the sort of questions we wish to pose are not so far removed from the standard repertory of gauge theory model builders. An emphasis on this aspect of string theory will enable a wider class of physicists to come to grips with the guts of string theory. It is clear that this will lead to further progress in the field.

The present paper is intended as a modest beginning of the program outlined above. Indeed it is not even that, for results of the type that we are envisaging have already been obtained by Friedan, Qiu and Shenker [5], by Boucher et al. [6], by Hull and Witten [7], and by several other authors [8–11]. We are attempting to continue this line of reasoning, which unfortunately has not obtained a very wide audience. More specifically, we will prove two general results about heterotic superstring vacua, and the properties they must have to approximate the real world. The first has to do with the weak mixing angle θ_w. For grand unified theories with a unification scale of order the Planck mass, the standard GUT prediction for value of θ_w at low energies generally fails due to excessive running of the coupling constants. On the other hand, nonrenormalizable interactions, between the Higgs fields responsible for GUT breaking and the gauge bosons, can invalidate the standard prediction of θ_w even at the tree level (i.e. at the Planck scale). Witten has argued [12] that Wilson-line symmetry breaking in Kaluza-Klein theories nonetheless makes the same prediction of θ_w at the Planck scale as do standard GUTs. The point is essentially that (at least in one gauge) the relationship between the symmetric and spontaneously broken vacua is merely through a change of boundary conditions for the fields on the internal manifold. The kinetic energy terms for the unbroken gauge fields are precisely what they were in the symmetric theory. This observation however, is far from answering the question of whether string theory itself makes a firm prediction of the mixing angle. First of all, many superstring vacua are not related in any obvious way to simple Wilson-line breaking of a model with grand unified symmetry. Secondly, even if we have found a vacuum with the right value of θ_w, we must deal with the problem of flat directions. Many superstring vacua are continuously connected to other solutions of the string equations of motion, and as we move through one of these families of solutions we can easily imagine that although the standard model gauge group remains unbroken, the value of θ_w changes continuously. In such a situation one could hardly claim that string theory predicts the value of the mixing angle.

We will formulate a criterion for the two-dimensional field theory, which guarantees that such a theory will incorporate the standard *tree-level* prediction of θ_w; i.e. its value just below the "compactification" scale, which we presume to be of order the Planck scale, following refs. [4]. The value of θ_w at, say, the weak scale is of course sensitive to the spectrum of light particles via renormalization group effects. We further show, for any vacuum which has four-dimensional chiral fermions, that the value of the weak mixing angle does not change as we move to continuously connected solutions of the equations of motion. Actually, one can show that those string vacua for which a ratio of gauge couplings can change continuously, must actually have $N=2$ spacetime SUSY (if they have any at all). Furthermore, these vacua can be shown to have $N=4$ world-sheet SUSY, which is a severe restriction on the 2d conformal field theory. The techniques that are used to prove these latter results are similar to those we will present here, and the details will appear in another paper [13].

The second general result that we will obtain has to do with the two-dimensional criterion for four-dimensional $N=1$ SUSY. Boucher et al. [14] have already argued that a sufficient condition for this is the existence of a chiral world-sheet U(1) current which promotes the $(0,1)$ superconformal symmetry of the heterotic string to $(0,2)$. In addition one must constrain all physical vertex operators to have integer

U(1) charges. We will prove the converse of this result: The existence of spacetime SUSY implies the existence of the extra U(1).

Sect. 2 of this paper briefly outlines the way to treat gauge bosons in heterotic string theory and recalls some results that have been obtained in other work. We then give the proof that the weak mixing angle indeed takes on discrete values in interesting vacuum states. In sect. 3 we prove our result about the relation between spacetime and world-sheet SUSY. We conclude with a list of other problems that should be attacked with the sort of methods employed in this paper.

2. The weak mixing angle

It is by now well known that the vertex operator of a zero momentum gauge boson in string theory is obtained by multiplying the right- or left-moving piece of the four-momentum density on the world-sheet into a left- or right-moving world-sheet current density, which transforms according to the adjoint representation of the gauge group. In the heterotic string theory the right-moving sector has $N=1$ world-sheet supersymmetry, and in order to generate a physical vertex operator the current must belong to a dimension-$\frac{1}{2}$ superfield. The existence of such an operator puts strong constraints on the spectrum of massless spacetime fermions. In particular, it is shown in ref. [11] that the fermions cannot be chiral. Since we will use this result at a later point in our argument, it is worthwhile outlining the proof.

The proof depends crucially on the fact that the superfield to which the current belongs obeys a super-Kac-Moody algebra. Furthermore, the conformal generators of the full theory take the form:

$$L_0 = L_0^{\mathrm{SKM}} + L_0^{\mathrm{C}},$$

where L_0^{SKM} is the natural conformal generator constructed from bilinears in the super-Kac-Moody generators, and L_0^{C} is an additional generator commuting with the super-Kac-Moody algebra and having a non-negative spectrum in the Ramond sector of the theory. Friedan, Qiu and Shenker [5] have shown that whenever the super-Kac-Moody algebra is nonabelian the spectrum of L_0^{SKM} is strictly positive. This proves that if the model has massless fermions transforming in a chiral representation of a nonabelian gauge group, the nonabelian currents must be left-moving, i.e. come from the bosonic part of the heterotic string. However, the existence of even an *abelian* super-current algebra is enough to destroy chirality. The point is that the superpartners of the currents are world-sheet fermions. In the Ramond sector they have zero modes and the entire Hilbert space of the theory is a tensor product, with one factor being a representation of the Clifford algebra formed by the zero modes of the spacetime fermions plus the zero modes of the fermionic SKM currents. Since this representation is larger than that of the four-dimensional Dirac algebra, it contains both chiralities of four-dimensional fermions. This argument remains valid when restricted to the subspace of states transforming in a given representation of the left-moving gauge group, so we conclude that no such representation can be chiral, as claimed above.

An immediate consequence of this result is that in any realistic heterotic string vacuum, all gauge bosons must arise from currents in the left-moving (bosonic) sector of the theory. These currents, denoted by $J^a(\bar{z}) = \sum_n J_n^a \bar{z}^{-n-1}$, generate a Kac-Moody algebra:

$$[J_n^a, J_m^b] = \tfrac{1}{2}kn\delta^{ab}\delta_{n,-m} + if^{abc}J_{n+m}^c .$$

The Schwinger term in this algebra obeys a quantization condition for all simple nonabelian factors in the gauge group. If we normalize the generators so that $f^{abc}f^{dbc} = N\delta^{ad}$ for SU(N), etc., then the condition is simply that k be an integer; k is called the level of the algebra. If we have obtained our gauge group by Wilson-line breaking of a simple grand unified group, then the levels of all factors in the current algebra (including the abelian ones) will be the same. If we have no a priori connection between our representation and one in which the currents all belong to a simple group, then there is no reason for the Schwinger term of the U(1) current to be the same as that of the nonabelian factors, nor for it to be quantized. It is clear however, that the condition for the weak mixing angle to be the same as that predicted by grand unification, is that the U(1) current which is normalized to give the correct value for all quark, lepton and Higgs charges must have a Schwinger term identical to that of the SU(2) current algebra. This Schwinger term determines the coupling of two gauge bosons to a graviton. Requiring it to be the same for all gauge bosons completely determines the normalization of the U(1) boson's vertex operator. If this operator has the standard coupling to quarks and leptons, then the weak mixing angle takes on its canonical value (at the Planck scale).

We will not enter here into the question of how to find such a vacuum. Witten has shown that Wilson-line symmetry breaking certainly is a sufficient condition, but we do not know whether it is the only way to guarantee the result. The question we wish to ask here is whether, once we have found such a ground state, we can find others nearby which have continuously variable values of θ_{w}.

Ground states of string theory are superconformally invariant field theories. To leading order, we have a one parameter family of such theories for every $(1,1)$ operator in the original model which is the highest component of a superfield. Actually these are only true flat directions in the potential if the operator is exactly marginal. We will not, however, have to worry about this extra constraint. The question is thus whether, when we add a particular $(1,1)$ operator to the lagrangian, we can change the U(1) Schwinger term. In order for this to happen, the addition to the lagrangian must not commute with the U(1) current $j(\bar{z})$. However, since $j(\bar{z})$ is the derivative of a free chiral scalar field, $j(\bar{z}) = i\partial_{\bar{z}}\phi(\bar{z})$, we can easily determine the j-dependence of *all* operators in the conformal field theory. They have the form*:

$$\Phi(z,\bar{z}) = :\exp(i\alpha\phi(\bar{z}))P(j(\bar{z})):\tilde{\Phi}(z,\bar{z}), \tag{2.1}$$

where $P(j(\bar{z}))$ is a polynomial in $j(\bar{z})$ and its derivatives with definite integral conformal dimension, say m, and $\tilde{\Phi}$ is an operator which commutes with $j(\bar{z})$. If α is different from zero, the operator carries non-zero U(1) charge. Charged operators, if they exist, are of no interest for our current inquiry. They represent string vacua in which the U(1) symmetry is spontaneously broken. There may indeed be flat directions along which this is true. (There certainly are in the low energy effective field theory: they correspond to VEV's for combinations of squark and/or slepton fields.) We are interested in values of the weak mixing angle in vacua that preserve the U(1). Thus we may take $\alpha = 0$.

The neutral operators in (2.1) have dimension $(m+h_{\mathrm{L}}, h_{\mathrm{R}})$ if $\tilde{\Phi}$ has dimension $(h_{\mathrm{L}}, h_{\mathrm{R}})$. We are in a model with positive metric, and Φ should not commute with j, so the dimension can only be $(1,1)$ if $m=1$, $h_{\mathrm{L}}=0$, $h_{\mathrm{R}}=1$. Thus $\tilde{\Phi}(z,\bar{z})$ must in fact be a $(0,1)$ operator, i.e. a right-moving chiral current, say $O(z)$. In order for the new lagrangian to preserve superconformal invariance, $O(z)$ must be the highest component of a superfield. We have already argued however, that the existence of such an operator precludes the existence of chiral spacetime fermions. Thus, we arrive at the result announced in the introduction: In any vacuum with chiral fermions, there are no superconformally invariant perturbations which preserve a U(1) gauge symmetry but change its Schwinger term. The weak mixing angle cannot be changed continuously. Note that if we allow perturbations by $(1,1)$ operators of the form $j(\bar{z})O(z)$, then we can indeed change the angle. The operator $j(\bar{z})O(z)$ is like an abelian Thirring coupling and is exactly marginal. It will of course shift the Schwinger term in the $j(\bar{z})j(\bar{w})$ operator product. Thus $j(\bar{z})$ must be renormalized in order to be a proper gauge boson vertex operator. This will change its coupling to charged states and will shift the weak mixing angle. Thus the discreteness of the weak mixing angle in superstring vacua is directly connected to the chirality of spacetime fermions. In fact, all vacua, which contain such a U(1) super-Kac-Moody algebra, have $N=2$ spacetime SUSY if they have any at all. This is simply because, given a massless gravitino state, one can construct a second gravitino by applying the zero modes $\psi_0^{\mu}\psi_0$ to the state. Here $\psi^{\mu}(z)$ is the usual Neveu-Schwarz-Ramond field with four-dimensional Minkowski index μ, and $\psi(z)$ is the dimension-$\frac{1}{2}$ world-sheet superpartner of $\mathcal{O}(z)$; the pair $\psi^{\mu}\psi$ has even total fermion number. We will show in a future paper [13] that the above vacua actually have $N=4$ SUSY on the world sheet, which is a significant step towards a complete classification of them [15].

* These operators are not all conformal fields, i.e. they are not all primary with respect to the Virasoro algebra, but they can be chosen so that they all have definite conformal dimension (L_0 eigenvalue), as well as definite charge (j_0 eigenvalue).

3. Spacetime and world-sheet supersymmetry

The importance of preserving spacetime supersymmetry in a classical superstring ground state has been stressed by many authors. It seems to be the only hope of solving the hierarchy problem in a weakly coupled theory. It is clearly of interest to have a simple criterion for checking whether a two-dimensional field theory has this property. A necessary and sufficient criterion is the existence of an $N=2$ supersymmetry current algebra on the world sheet, plus a charge quantization condition on the U(1) current contained in this algebra.

Witten and Hull [7] showed that in any spacetime supersymmetric classical vacuum of the heterotic string which is described by a nonlinear sigma model, the *local* $N=1$ superconformal invariance [16] of the two-dimensional (world-sheet) field theory extends to a *global* $N=2$ superconformal invariance [17]. Boucher et al. [14] found quite generally that $(0,2)$ world-sheet SUSY ensures the existence of $N=1$ spacetime SUSY. If the two-dimensional field theory has $(0,2)$ world-sheet SUSY, then it contains a right-moving U(1) current $J(z)$ which transforms the two supersymmetry charges into each other. Note that the two world-sheet supercharges are conceptually very different from each other. The first is the remnant of a local gauge symmetry of string theory, while the second is an accidental global symmetry of a particular string vacuum. The U(1) current, which transforms the two into each other (and therefore does not commute with the gauge SUSY generator) is not a physical operator. It does not commute with the BRST charge and is not the highest component of an $N=1$ superfield. It has no free fermionic superpartner and the arguments of the previous section to not apply. Boucher et al. showed how to construct the spacetime SUSY generators from this U(1) current. We will review this construction below.

Our purpose here is to show in a general context that spacetime SUSY implies $N=2$ superconformal invariance on the world-sheet. The starting point is to write the spacetime supersymmetry current in the $-\frac{1}{2}$ picture [18]:

$$V^{\alpha}_{-1/2}(z)=\mathrm{e}^{-\phi/2}\mathscr{S}_{\alpha}\Sigma(z),$$
$$V^{\dot\alpha}_{-1/2}(z)=\mathrm{e}^{-\phi/2}\mathscr{S}_{\dot\alpha}\Sigma^{\dagger}(z),\tag{3.1}$$

where $\mathrm{e}^{-\phi/2}$ is a spin field for the (β,γ) superconformal ghost system, $\mathscr{S}_{\alpha}$ and $\mathscr{S}_{\dot\alpha}$ are spin fields for the (free) world-sheet fermions ψ^{μ} with four-dimensional Minkowski indices, and Σ and $\Sigma^{\dagger}$ are fields in the Ramond sector of the internal $N=1$ superconformal field theory. The supersymmetry charges are given by $Q_{\alpha}=\oint \mathrm{d}z\, V^{\alpha}_{-1/2}(z)$, $Q_{\dot\alpha}=\oint \mathrm{d}z\, V^{\dot\alpha}_{-1/2}(z)$. All the fields here have conformal dimension zero with respect to the antiholomorphic stress-energy tensor $\overline{T}(\bar z)$, so their correlation functions have no antiholomorphic dependence; hence "dimension" will always refer to the holomorphic conformal dimension.

The relevant operator product expansions (OPE's) for the spin fields are

$$e^{q_1\phi}(z)e^{q_2\phi}(w) \sim (z-w)^{-q_1q_2}e^{(q_1+q_2)\phi}(w) + \cdots ,$$

$$\mathscr{S}_\alpha(z)\mathscr{S}_{\dot\beta}(w) \sim (z-w)^0 \sigma^\mu_{\alpha\dot\beta}\psi_\mu(w) + \cdots ,$$

$$\mathscr{S}_\alpha(z)\mathscr{S}_\beta(w) \sim (z-w)^{-1/2}\eta_{\alpha\beta} I + \cdots , \tag{3.2}$$

where I is the identity operator. If four-dimensional spacetime is taken to have a Minkowski signature, then $V^\alpha_{-1/2}$ and $V^{\dot\alpha}_{-1/2}$ are hermitian conjugates of each other. The powers of $z-w$ appearing in the $\mathscr{S}_\alpha$, $\mathscr{S}_{\dot\alpha}$ OPE's (3.2) are more easily derived, however, by using an euclidean signature and representing [19] the spin fields as exponentials of two free bosons $H^{1,2}(z)$: $\mathscr{S}_\alpha = e^{i\alpha\cdot H}$, $\mathscr{S}_{\dot\alpha} = e^{i\dot\alpha\cdot H}$, where $\alpha = (\pm\frac{1}{2}, \pm\frac{1}{2})$, $\dot\alpha = (\pm\frac{1}{2}, \mp\frac{1}{2})$. Note that the dimension of $e^{q\phi}$ is $-\frac{1}{2}q(q+2)$ and that of $\mathscr{S}_\alpha$ and $\mathscr{S}_{\dot\alpha}$ is $\frac{1}{4}$. Since the currents $V^{\alpha,\dot\alpha}_{-1/2}(z)$ have dimension 1, the fields Σ and $\Sigma^\dagger$ must have dimension $\frac{3}{8}$. The supersymmetry algebra

$$\{Q_\alpha, Q_{\dot\beta}\} = \sigma^\mu_{\alpha\dot\beta}P_\mu , \qquad \{Q_\alpha, Q_\beta\} = 0$$

leads to the following OPE's for Σ, $\Sigma^\dagger$:

$$\Sigma(z)\Sigma^\dagger(w) \sim (z-w)^{-3/4} I + \cdots ,$$

$$\Sigma(z)\Sigma(w) \sim (z-w)^{3/4}\mathcal{O}(w) + \cdots ,$$

$$\Sigma^\dagger(z)\Sigma^\dagger(w) \sim (z-w)^{3/4}\mathcal{O}^\dagger(w) + \cdots , \tag{3.3}$$

where $\mathcal{O}$ is some dimension-$\frac{3}{2}$ operator (whose coefficient could be zero a priori) and locality requires the subleading terms in (3.3) to be less singular by integer powers of $z-w$. The first of these relations follows from the fact that the OPE for the supersymmetry currents of opposite spacetime chirality must have a pole with residue equal to the momentum current. The other two follow from the requirement that two SUSY currents of the same chirality have no singularity in their OPE. Note that the translation current appears in the -1 picture here, as $e^{-\phi}\psi_\mu$, in place of the more familiar 0 picture current ∂X_μ.

In any classical vacuum for the heterotic string which incorporates four-dimensional Minkowski space, the six internal supercoordinates $X^i + \theta\psi^i$ are replaced by an "internal" superconformal field theory [2]. The associated $N=1$ superconformal algebra [16] for the "internal" stress tensor $T(z)$ and its superpartner $T_F(z)$ (the world-sheet supersymmetry current) is

$$T(z)T(w) \sim \frac{\frac{3}{4}\hat{c}}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial_w T(w)}{z-w} + \cdots ,$$

$$T(z)T_{\mathrm{F}}(w) \sim \frac{\frac{3}{2}T_{\mathrm{F}}(w)}{(z-w)^2} + \frac{\partial_w T_{\mathrm{F}}(w)}{z-w} + \cdots ,$$

$$T_{\mathrm{F}}(z)T_{\mathrm{F}}(w) \sim \frac{\frac{1}{4}\hat{c}}{(z-w)^3} + \frac{\frac{1}{2}T(w)}{z-w} + \cdots , \tag{3.4}$$

with central charge $\hat{c}=6$. A *conformal* superfield $\Phi(z,\theta)=\Phi_0(z)+\theta\Phi_1(z)$ of dimension h is *primary* with respect to this algebra; it satisfies

$$T_{\mathrm{F}}(z)\Phi_0(w) \sim \frac{\frac{1}{2}\Phi_1(w)}{z-w} + \cdots ,$$

$$T_{\mathrm{F}}(z)\Phi_1(w) \sim \frac{h\Phi_0(w)}{(z-w)^2} + \frac{\frac{1}{2}\partial_w\Phi_0(w)}{z-w} + \cdots ,$$

$$T(z)\Phi_0(w) \sim \frac{h\Phi_0(w)}{(z-w)^2} + \frac{\partial_w\Phi_0(w)}{z-w} + \cdots ,$$

$$T(z)\Phi_1(w) \sim \frac{(h+\frac{1}{2})\Phi_1(w)}{(z-w)^2} + \frac{\partial_w\Phi_1(w)}{z-w} + \cdots . \tag{3.5}$$

On the other hand, the fields Σ and $\Sigma^\dagger$ are nonlocal with respect to T_{F}, because they make states in the Ramond sector of the theory:

$$\Sigma(z)T_{\mathrm{F}}(w) \sim (z-w)^{-1/2}, \qquad \Sigma^\dagger(z)T_{\mathrm{F}}(w) \sim (z-w)^{-1/2}. \tag{3.6}$$

The absence of more singular terms in (3.6) (also the square root branch cut) follows from dimensional analysis plus BRST invariance of the gravitino vertex operator: The OPE $(\mathrm{e}^{-\phi/2}\mathscr{S}_\alpha\Sigma)(z)\cdot(\mathrm{e}^{\phi}T_{\mathrm{F}})(w)$ must have no single pole term.

The $N=2$ superconformal algebra we wish to construct contains in addition to T and T_{F} a dimension 1 U(1) current $J(z)$, conventionally normalized by

$$J(z)J(w) \sim \frac{\frac{1}{2}\hat{c}}{(z-w)^2} + \cdots . \tag{3.7}$$

The $N=1$ supersymmetry current T_{F} splits into two terms,

$$T_{\mathrm{F}}=\frac{1}{\sqrt{2}}\left(T_{\mathrm{F}}^{+}+T_{\mathrm{F}}^{-}\right), \tag{3.8}$$

which have charge ± 1 under J,

$$J(z)T_{\mathrm{F}}^{\pm}(w)\sim \pm\frac{T_{\mathrm{F}}^{\pm}(w)}{z-w}+\cdots, \tag{3.9}$$

and also satisfy

$$\begin{aligned} &T_{\mathrm{F}}^{+}(z)T_{\mathrm{F}}^{+}(w)\sim T_{\mathrm{F}}^{-}(z)T_{\mathrm{F}}^{-}(w)\sim \mathcal{O}(1),\\ &T_{\mathrm{F}}^{+}(z)T_{\mathrm{F}}^{-}(w)\sim \frac{\frac{1}{4}\hat{c}}{(z-w)^3}+\frac{\frac{1}{2}J(w)}{(z-w)^2}+\frac{\frac{1}{2}T(w)+\frac{1}{4}\partial_w J(w)}{z-w}+\cdots; \end{aligned} \tag{3.10}$$

these additional OPE's complete the $N=2$ superalgebra. We will first construct J from the fields Σ and $\Sigma^{\dagger}$, then derive (3.8), (3.9) and (3.10).

Consider the four-point function

$$f(z_i)=\left\langle \Sigma(z_1)\Sigma^{\dagger}(z_2)\Sigma(z_3)\Sigma^{\dagger}(z_4)\right\rangle .$$

Using $\mathrm{SL}_2(C)$ invariance [20] it can be written as

$$f(z_i)=\left(\frac{z_{13}z_{24}}{z_{12}z_{34}z_{14}z_{23}}\right)^{3/4}\tilde{f}(x), \tag{3.11}$$

where $z_{ij}\equiv z_i-z_j$ and $x\equiv z_{12}z_{34}/z_{13}z_{24}$. The singularities of this function as pairs of z_i approach each other are determined by the OPE's (3.3). Using these constraints one finds that $\tilde{f}(x)$ is an analytic function and $\tilde{f}(x)\to$ constant as $x\to 0,1,\infty$; hence $\tilde{f}$ is a constant, in fact $\tilde{f}=1$ using the normalization $\langle I\rangle=1$. Now expand (3.11) as $z_1\to z_2$:

$$f(z_i)\sim z_{12}^{-3/4}z_{34}^{-3/4}\left\{1+\frac{3}{4}\frac{z_{12}z_{43}}{z_{23}z_{24}}+\cdots\right\}. \tag{3.12}$$

The second term in the expansion indicates the presence of a dimension 1 field in the $\Sigma\Sigma^{\dagger}$ channel, which we identify as $J(z)$. Given the normalization (3.7) of J's two-point function for $\hat{c}=6$, eq. (3.12) shows that its three-point function with Σ and $\Sigma^{\dagger}$ is

$$\left\langle \Sigma(z_1)\Sigma^{\dagger}(z_2)J(z_3)\right\rangle=\tfrac{3}{2}z_{12}^{1/4}z_{13}^{-1}z_{23}^{-1}. \tag{3.13}$$

(The overall *sign* of (3.13) is conventional, as it can be changed by redefining $J \to -J$.) Eq. (3.13) is equivalent (given (3.3) and (3.7)) to the OPE's

$$\Sigma(z)\Sigma^{\dagger}(w) \sim (z-w)^{-3/4}I + (z-w)^{1/4}\tfrac{1}{2}J(w) + \cdots ,$$

$$J(z)\Sigma(w) \sim \frac{\frac{3}{2}\Sigma(w)}{z-w} + \cdots ,$$

$$J(z)\Sigma^{\dagger}(w) \sim \frac{-\frac{3}{2}\Sigma^{\dagger}(w)}{z-w} + \cdots . \tag{3.14}$$

At this stage we will use the fact that the U(1) current $J(z)$ can be decoupled from the other fields in the theory [14,10,21]. First write $J(z) = i\sqrt{3}\,\partial_z H(z)$, where $H(z)$ is a canonically normalized free scalar field, $H(z)H(w) = -\ln(z-w)$. Then any operator $\Phi(z,\bar{z})$ with U(1) charge q, i.e. such that

$$J(z)\Phi(w,\overline{w}) \sim \frac{q\Phi(w,\overline{w})}{z-w} + \cdots , \tag{3.15}$$

can be written as $\Phi =: \exp[i(q/\sqrt{3})H]P(J)\!:\, \tilde{\Phi}$, where $\tilde{\Phi}$ commutes with H, and $P(J)$ is a polynomial in $J(z)$ and its derivatives, cf. eq. (2.1).

In particular, (3.14) implies that Σ has charge $\frac{3}{2}$, and so $\Sigma = e^{i\sqrt{3}H/2}\tilde{\Sigma}$. But $\tilde{\Sigma}$ has dimension $\frac{3}{8} - \frac{1}{2}(\frac{1}{2}\sqrt{3})^2 = 0$, so it must be the identity operator. Similarly one finds that $\Sigma^{\dagger} = e^{-i\sqrt{3}H/2}$. These relations and eq. (3.1) express the spacetime supersymmetry currents $V^{\alpha,\dot{\alpha}}_{-1/2}$ in terms of the U(1) current $J(z)$ [14,10,21].

Similarly one can decouple J from the world-sheet supersymmetry generator $T_{\mathrm{F}}(z)$, which does not have a definite charge but can be decomposed as $T_{\mathrm{F}} = \Sigma_q \exp[\sqrt{\frac{1}{3}}\, iq/H]\tilde{T}^{q}_{\mathrm{F}}$. Inserting these representations of Σ, $\Sigma^{\dagger}$, and T_{F} into (3.6) and contracting the H exponentials, one finds that only the charges $q = \pm 1$ can be present in the above expansion of T_{F}, thus reproducing (3.8), (3.9).

Also because of (3.6), the operator product of $J(z)$ with $T_{\mathrm{F}}(w)$ can be no more singular than $(z-w)^{-1}$, that is

$$J(z)T_{\mathrm{F}}(w) \sim \frac{T'_{\mathrm{F}}(w)}{z-w} + \cdots , \tag{3.16}$$

where $T'_{\mathrm{F}} = \sqrt{\frac{1}{2}}\,(T^{+}_{\mathrm{F}} - T^{-}_{\mathrm{F}})$. According to eq. (3.5), the dimension-$\frac{3}{2}$ field $-2T'_{\mathrm{F}}(z)$ defined by (3.16) is the upper component of a dimension-1 $N=1$ superfield whose lower component is $J(z)$, and consequently one obtains the OPE

$$T_{\mathrm{F}}(z)T'_{\mathrm{F}}(w) \sim -\frac{\frac{1}{2}J(w)}{(z-w)^2} - \frac{\frac{1}{4}\partial_w J(w)}{z-w} \cdots . \tag{3.17}$$

Note also that

$$J(z)T_F'(w) \sim \frac{T_F(w)}{z-w} + \cdots . \tag{3.18}$$

Finally we will show that

$$T_F'(z)T_F'(w) \sim -T_F(z)T_F(w) + \mathcal{O}(1). \tag{3.19}$$

When combined with (3.17) and the third line of (3.4), this equation yields the remaining OPE's (3.10) of the $N=2$ superalgebra. Eq. (3.19) is most easily derived using the Laurent expansions $T_F(z) = \frac{1}{2}\Sigma_r G_r z^{-r-3/2}$, $T_F'(z) = \frac{1}{2}\Sigma_r G_r' z^{-r-3/2}$, and $J(z) = \Sigma_n J_n z^{-n-1}$. In terms of modes, the OPE's (3.7), (3.16), (3.18) and (3.17) read respectively $[J_m, J_n] = \frac{1}{2}\hat{c} m \delta_{m+n,0}$, $[J_m, G_r] = G_{m+r}'$, $[J_m, G_r'] = G_{m+r}$ and $\{G_r, G_s'\} = -(r-s)J_{r+s}$. Therefore

$$\begin{aligned} \{G_r', G_s'\} &= \{[J_0, G_r], G_s'\} = -\{G_r, [J_0, G_s']\} + [J_0, \{G_r, G_s'\}] \\ &= -\{G_r, G_s\} - (r-s)[J_0, J_{r+s}] = -\{G_r, G_s\}, \end{aligned} \tag{3.20}$$

which yields (3.19) when translated back into OPE's.

4. Conclusion

The methods that we have used to demonstrate that string theory gives discrete predictions for the weak mixing angle and that $N=1$ spacetime SUSY implies $(0,2)$ superconformal invariance depended crucially on the existence of certain holomorphic fields in the vacuum conformal field theory. We should emphasize strongly that this in no way implies that the entire theory splits into holomorphic and antiholomorphic sectors. Rather, spacetime gauge symmetries are connected with holomorphic fields, and these will always exist if the vacuum state preserves the relevant gauge symmetry. We expect that more results of the type we have described can be obtained by using stronger hypotheses about the spacetime symmetries of the required vacuum state.

For example, two of us (T.B. and L.D.) have recently shown [13] that $N=2$ spacetime SUSY of a heterotic string vacuum implies $N=4$ SUSY on the worldsheet. Indeed, the constraint is even stronger than this. The right-moving degrees of freedom of the "internal" superconformal field theory can be broken up into the product of two free $N=1$ superfields, each with $\hat{c}=1$, and a representation of the $N=4$ superconformal algebra with $\hat{c}=4$. If one assumes $N=4$ spacetime SUSY, then the right-moving modes are just six free superfields. It therefore may be possible to show that all $N=4$ supersymmetric heterotic string vacua are generalized toroidal compactifications [22]. Of course, this class of vacua is probably not too relevant to the real world, but it indicates the power of the type of analysis we have proposed.

There are several problems of more phenomenological relevance that can probably be attacked by these methods. One might hope for example that one could rule out the possibility of a string vacuum with only the supersymmetric standard model in its low energy spectrum. Alternatively, one might hope to classify such vacua completely. One can study the general criteria for charge quantization and its connection with the existence of magnetic monopoles. One can study general constraints on the operator product coefficients of quark and lepton vertex operators and attempt to find mechanisms for suppressing baryon number violation, flavor changing neutral currents and the like. We believe that we have just scratched the surface of what can be done.

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