January, 1985 EFI 85-09

COVARIANT METHODS IN SUPERSTRING THEORY

D. Friedan and S.H. Shenker

Enrico Fermi and James Franck Institutes
and Department of Physics
University of Chicago, Chicago, IL 60637

ABSTRACT

We briefly describe some recent developments in string theory.

To appear in the Proceedings of the Santa Fe meeting of the APS Division of Particles and Fields, October 29 - November 3, 1984.

This work was supported in part by U.S. Department of Energy contract DE-AC02-81ER-10957 and the Alfred P. Sloan Foundation.

This is a summary of work on superstrings which we did in the summer of 1984 and presented at Aspen in August and ITP in August and September.[1]

Ten years ago Scherk and Schwarz[2] proposed that string theory could provide a fundamental theory of matter interacting through gauge and gravitational forces. By this summer three critical problems had emerged: to find string theories with anomaly free gauge symmetries, to find compactifications of string theories yielding chiral fermions in four dimensions, and to find a Lorentz covariant quantization of the fermionic string.

Mandelstam[3] and Witten[4] had especially emphasized the issue of gauge symmetry. Strings with fundamental gauge symmetries (type I) were thought to be anomalous because the massless spectrum gave both gauge and gravitational anomalies, assuming invariant low energy effective actions.[5] It appeared that gauge symmetry would have to be generated dynamically. Mandelstam suggested that the Foerster-Nielsen-Ninomiya-Shenker[6] mechanism would produce gauge symmetry in a strongly coupled string theory. Witten suggested generating gauge invariance by compactification of a closed string theory, using the construction of Frenkel, Kac and Segal[7] to replace D_c dimensions with a current algebra for a rank D_c gauge group.

It seemed to us that the strongly coupled string theory would be intractable, so we concentrated on calculating anomalies for type I strings and on applying the current algebra mechanism to closed fermionic strings. We showed that in compactifications of closed fermionic strings gauge particles come from supercurrent algebras. We proved that in such compactifications the fermions all have large masses if the gauge group is nonabelian.[8] Thus gauge symmetry must be fundamental.

To check the consistency of type I theories we worked out how to calculate anomalies directly in string theory. The potentially anomalous hexagon diagram is a straightforward one loop calculation in the covariant formulation of the fermionic string using the well-known vertices for emission of transverse and longitudinal massless spin one or spin two particles. Following this idea, Green and Schwarz[9] performed the calculation, with remarkable results. The anomaly cancelled only for gauge group $SO(32)$. Thus internal consistency dramatically constrains the fundamental gauge group.

Since a gauge invariant low energy effective action for the $SO(32)$ string would be anomalous it was clear there would have to be a Wess-Zumino term. Green and Schwarz[10] examined the structure of the string interactions responsible for the anomaly cancellation and used that information to write an anomaly-free field theory with gauge group $SO(32)$ and gauge-gravitational Wess-Zumino coupling. It was immediately noticed that the anomaly cancellation in the field theory also took place for gauge group $E_8 \times E_8$, more attractive than $SO(32)$ for phenomenological purposes. The problem became to find an $E_8 \times E_8$ string.

At the time the only known way to introduce gauge charges was to put them at the ends of open strings.[11] The gauge group in such theories had to be one of $SO(N)$ or $Sp(N)$. It was clear that to implement an $E_8 \times E_8$ gauge symmetry the gauge charges would have to be put in the interior of the string. Freund[12] suggested using the current algebra mechanism, noting that for both $E_8 \times E_8$ and $SO(32)$ the trace anomaly condition $26=10+16$ was satisfied, where 26 is the critical dimension of the bosonic string, 16 the rank of the group and 10 the space-time dimension of the compactified bosonic string. The problem became to combine the bosonic current algebra with the fermionic string. Gross, Harvey, Martinec and Rohm[13] found an elegant solution, the heterotic string, which works only for $E_8 \times E_8$ and $SO(32)$.

A fermionic string in D space-time dimensions is described in the superconformal gauge by D free scalar superfields $X^\mu(z,\theta,\bar{z},\bar{\theta})$ on the world surface.[14] For this discussion we focus on the (z,θ) dependence. The superconformal gauge requires free Fadeev-Popov superfields B and C, of spin $3/2$ and -1.[15] The action is $\int 1/2\bar{D}X^\mu DX_\mu + B\bar{D}C + \bar{B}D\bar{C}$, so $X^\mu = x^\mu + \theta\psi^\mu + \overline{\theta\psi}^\mu$, $B(z,\theta) = b_B + \theta b_F$, $C(z,\theta) = c_F + \theta c_B$. The super stress energy tensor is $-1/2DX^\mu D^2X_\mu - D^2BC + 1/2DBDC - 3/2BD^2C$. In the type I theory there are free Grassman degrees of freedom $\eta^i, \bar{\eta}_j$ on the ends of the open string, with $i,j=1 \cdots N$ for gauge group $SO(N)$ or $Sp(N)$.[16]

The single string Hilbert space splits into bosonic (Neveu-Schwarz) and fermionic (Ramond) sectors, characterized by boundary conditions $\psi^\mu(e^{2\pi i}z) = \pm\psi^\mu(z)$, and similarly for the half-integer spin bosonic ghosts b_B and c_B. The supersymmetric string theory is obtained by projecting on the subsector $(-1)^F=1$, where F is sheet fermion number.[17] In the functional integral the sum over Neveu-Schwarz and

Ramond sectors and the projection $(-1)^F=1$ are accomplished by summing over all boundary conditions for half-integer spin fields around all nontrivial closed curves on the world surface. In the type I theory a Lagrange multiplier is used to project on the sector $\overline{\eta}_i\eta^i=1$ and a "twist" projection onto the adjoint representation is accomplished by summing over orientable and nonorientable surfaces. For type II strings only orientable surfaces are included.[18]

Reparametrization invariance on the world surface requires the total trace anomaly $\hat{c}=\hat{c}_{\text{matter}}+\hat{c}_{\text{ghost}}$ to vanish.[19] Since $\hat{c}_{\text{matter}}=D$ and $\hat{c}_{\text{ghost}}=-10$, self-consistency fixes $D=10$.[15] Reparametrization invariance also implies the mass-shell conditions; physical states must have total energy $L_0=L_{0,\text{matter}}+L_{0,\text{ghost}}=0$. The fermionic ghost ground state contributes -1.[20] The bosonic ghost ground state contributes 1/2 in the N-S sector, 3/8 in the Ramond sector.[21] These values fix the lightest particles in both sectors to be massless.

In the functional integral representation the string anomalies come from zero modes of $\psi^\mu,\overline{\psi}^\mu$. On the torus (one loop) the Dirac operator has a zero mode only for periodic boundary conditions. On surfaces of genus $g>1$ there are 4^g boundary conditions. The Dirac operator has an odd number of zero modes (generically one) for $2^{2g-1}-2^{g-1}$ boundary conditions and an even number (generically zero) for the rest.

The major unsolved problem in the covariant approach to fermionic string theory is to construct the fermion vertex[22] and the space-time supersymmetry algebra. The matter contribution to the fermion vertex is a dimension $D/16=5/8$ conformal field $\Theta^\alpha(z)$, which transforms as a spinor under SO(1,9). To form a dimension 1 vertex we need an additional field $\Sigma(z)$ of dimension 3/8. Goddard and Olive[23] suggested that the ghosts might play a role. This motivated the calculation of the ground state energy of the bosonic ghosts. The value 3/8 for the Ramond sector implies that there is an operator $\Sigma(z)$ in the bosonic ghost sector.[21]

We have constructed a candidate $\Sigma(z)$ by exponentiating the line integral of the bosonic ghost current $j_B=c_Bb_B$. This "bosonized" bosonic ghost has trace anomaly $c=13$, leaving a residual system with $c=-2$, which can be represented in terms of new fermionic ghosts of dimensions 1 and 0. The nonzero ghost charge of $\Sigma(z)$ is an obstruction to constructing a covariant fermionic vertex operator.[21]

With J.D. Cohn and Z. Qiu[24] we have calculated correlation functions of Θ^α by three techniques: (1) null vectors for the $SO(1,9)$ Kac-Moody algebra and the corresponding differential equations,[25] (2) the spinorial extension of the Frenkel-Kac-Segal[7] vertex operator construction for $SO(1,9)$, and (3) the Luther-Peschel[26] doubling construction for D Ising spins.

Compactification consists of replacing six free superfields by a nontrivial superconformal field theory on the world surface. Superconformal invariance requires locality, supersymmetry and vanishing of the β-function. Reparametrization invariance requires $\hat{c}=6$. The most general renormalizable theory on the world surface can be interpreted as a condensate of the massless bosonic degrees of freedom of the string. There are at least two possibilities: (1) superconformal sigma models with Wess-Zumino terms and (2) supersymmetric sigma models in Ricci-flat six manifolds.[27]

In the first case, a massless vector particle is associated with each surface supercurrent. The mass operator for the fermions includes the supersymmetry generator G_0 of the Ramond sector of the sigma model. We proved from the super current algebra that $G_0>0$ for all nonabelian groups.[8] Therefore all fermions become massive. For case (2), the chiral fermion spectrum is given by Witten's index for the Ramond sector of the Ricci-flat sigma model, which is the index of a certain Dirac operator on the Ricci-flat manifold.

Another possibility, suggested by the form of the fermion vertex, is a compactification in which the bosonic ghosts couple to the matter fields.

Acknowledgments

We acknowledge interesting conversations with J. Cohn, P. Freund, D. Freedman, P. Goddard, M. Green, S. Mandelstam, D. Olive, Z. Qiu, J. Schwarz and E. Witten. We are grateful to the Aspen Center for Physics for its hospitality while most of this work was carried out.

References

1. The talk began with a discussion of two dimensional superconformal field theory. This material is contained in D. Friedan, Z. Qiu, and S.H. Shenker, EFI preprint 84-35 (1984), to appear in Physics Letters. We have substituted some historical comments.
2. J. Scherk and J.H. Schwarz, Nucl. Phys. B81 (1974) 118 and Phys. Lett. 52B (1974) 347.
3. S. Mandelstam, private communication and Berkeley preprint 84-1018 (1984).
4. E. Witten, private communication.
5. See L. Alvarez-Gaume and E. Witten, Nucl. Phys. B234 (1984) 269 and references therein.
6. D. Foerster, H.B. Nielsen and M. Ninomiya, Phys. Lett. 94B (1980) 135; S.H. Shenker, unpublished.
7. I.B. Frenkel and V.G. Kac, Invent. Math. 62 (1980) 23; G. Segal, Comm. Math. Phys. 80 (1981) 301.
8. D. Friedan and S.H. Shenker, in preparation.
9. M.B. Green and J.H. Schwarz, Cal Tech preprint CALT-68-1224 (1984).
10. M.B. Green and J.H. Schwarz, Phys. Lett. 149B (1984) 117.
11. H.M. Chan and J. Paton, Nucl. Phys. B10 (1969) 519.
12. P.G.O. Freund, private communication and ITP preprint NSF-ITP-84-157 (1984).
13. D. Gross, J. Harvey, E. Martinec and R. Rohm, Princeton preprint Print-85-0029 (1984).
14. P. Ramond, Phys. Rev. D3 (1971) 2493; A. Neveu and J. H. Schwarz, Nucl. Phys. B31 (1971) 86; J.-L. Gervais and B. Sakita, Nucl. Phys. B34 (1971) 477.
15. A.M. Polyakov, Phys. Lett. 103B (1981) 211.
16. A.P. Balachandran, P. Salomonson, B. Skagerstam and J. Winnberg, Phys. Rev. D15 (1977) 2308; D. Friedan and P. Windey, Nucl. Phys. B235 (1984) 395; L. Alvarez-Gaume, J. Phys. A16 (1983) 4177.
17. F. Gliozzi, J. Scherk and D. I. Olive, Phys. Lett. 65B (1976) 282 and Nucl. Phys. B122 (1977) 253; M.B. Green and J.H. Schwarz, Nucl. Phys. B181 (1981) 502.

18. See J.H. Schwarz, Phys. Rep. 89 (1982) 223 and references therein.

19. A.M. Polyakov, Phys. Lett. 103B (1981) 207; D. Friedan in *Recent Advances in Field Theory and Statistical Mechanics* (Les Houches, Summer 1982), J.-B. Zuber and R. Stora eds. (North-Holland, 1984).

20. M. Kato and K. Ogawa, Nucl. Phys. B212 (1983) 443; and S. Hwang, Phys. Rev. D28 (1983) 2614.

21. D. Friedan and S.H. Shenker, in preparation.

22. See D. Olive in *Dual Theory* ed. M. Jacob (North-Holland, 1974) and references therein.

23. P. Goddard and D. Olive, private communication.

24. J. D. Cohn, D. Friedan, Z. Qiu and S. H. Shenker, in preparation.

25. A. A. Belavin, A. M. Polyakov and A. B. Zamolodchikov, Nucl. Phys B241 (1984) 333; V.G. Knizhnik and A.B. Zamolodchikov, to appear in Nucl. Phys. B.

26. A. Luther and I. Peschel, Phys. Rev. B12 (1975) 3908.

27. Ricci-flat compactifications are investigated extensively in P. Candelas, G. Horowitz, A. Strominger and E. Witten, ITP preprint (1984).