A pragmatic approach to formal fundamental physics

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Abstract

A minimal practical formal structure is suggested for a comprehensive fundamental theory. A mechanism that produces such a structure is reviewed. The proposed mechanism has possibilities of producing non-canonical phenomena in SU(2) and SU(3) quantum gauge theories. One possibility is a natural vacuum condensate coupled to the SU(2) gauge fields. Such non-canonical effects might provide conditional predictions that can be tested.

Contents

1 Formal fundamental physics

For 45 years, the most fundamental theory of physics has been the Standard Model combined with General Relativity, describing almost everything known at distances larger than about $(10^3\text{GeV})^{-1}$. Only dark matter, neutrino mixing, and some CP violation are left out. The project of formal fundamental physics is to hypothesize a more comprehensive formal machinery that includes the SM+GR and makes predictions that can be checked against experiment. It seems overreaching to attempt at predicting the SM+GR itself. It would be enough to make conditional predictions of the form *if the formalism produces the SM+GR then it also produces such and such testable phenomena beyond the SM+GR*. An example is the conditional prediction of proton decay in Grand Unification. If a conditional prediction can be checked successfully against experiment, then the proposed formalism becomes a serious candidate for a more fundamental theory. Especially useful would be specific conditional predictions of dark matter, neutrino mixing, and CP violation.

SM+GR is an effective quantum field theory with short distance cutoff on the order of $(10^3\text{GeV})^{-1} = 10^{16}\ell_P$ where ℓ_P is the Planck distance. GR can be treated as an effective QFT because quantum effects are negligible at distances $\gg \ell_P$. The effective QFT with short distance cutoff $\gg \ell_P$ is indistinguishable from classical GR.

A more comprehensive formalism should be capable of producing an effective QFT such as the SM+GR and should make definite conditional predictions of observable phenomena beyond the SM+GR.

In the 45 years since the SM was verified, none of the attempts at formal fundamental physics have worked. One reaction is to give up on the project, perhaps hoping that experiment will eventually provide more guidance. Alternatively, it might be useful to reexamine the assumptions that have guided the formal fundamental physics enterprise and reconsider paths not taken. The SM+GR already encodes very much experimental evidence. New high energy experimental discoveries will most likely lead to a new effective QFT that improves incrementally on the SM. The question for formal fundamental physics is how to use a specific effective QFT such as the SM+GR as guidance towards a formalism more comprehensive and more predictive than effective QFT in general.

1.1 Against Quantum Gravity

It might be useful to question the truism that General Relativity and Quantum Mechanics must be reconciled in a theory of Quantum Gravity. On the contrary, there is no appreciable conflict at distances $\gg \ell_P$. The smallest distance presently accessible to experiment is roughly $L_{\text{exp}} = (10^3\text{GeV})^{-1} = 10^{16}\ell_P$. To suppose a conflict is to extrapolate the validity of both Quantum Mechanics and General Relativity over 16 orders of magnitude from $L_{\text{exp}} = 10^{16}\ell_P$ down to ℓ_P. Nothing is known about physics at such small distances.

energy (GeV)	10^{18}	10^{15}	10^{12}	10^{9}	10^{6}	10^{3}	10^{0}	10^{-3}	10^{-6}	10^{-9}	10^{-12}	⋯⋯
distance (ℓ_P)	10^{1}	10^{4}	10^{7}	10^{10}	10^{13}	10^{16}	10^{19}	10^{22}	10^{25}	10^{28}	10^{31}	

ℓ_P L_{exp} 1mm

- $\longrightarrow$

???? ??? ?? ? Quantum Mechanics

Such a presumptuous extrapolation beyond the physical evidence would be justifiable if it yielded a testable prediction, as for example the extrapolation of Grand Unification yielded the conditional prediction of proton decay. But it is implausible that any proposed theory of Quantum Gravity can be checked experimentally given that the smallest distance presently accessible to experiment is $10^{16}\ell_P$. There is no practical possibility of checking whether any proposed theory of Quantum Gravity actually describes the real world. Without the possibility of an experimental test, except in fantasy, such extreme extrapolation beyond the experimental evidence is unlikely to be useful for fundamental physics.

1.2 Against mathematical idealizations

Formal structures are used in physics for practical purposes, not as ideal mathematical forms. A quantum field theory is used as an *effective* theory describing physics at distances greater than some UV cutoff at the short distance limit of the evidence. An effective QFT says nothing about distances smaller than the UV cutoff. It does not even suppose the existence of a space-time continuum. A continuum QFT is a mathematical idealization which extrapolates far beyond the practical use of the formalism.

An S-matrix is used as an *effective* theory that describes physics at distances smaller than the scattering region. The asymptotic S-matrix is a mathematical idealization which supposes ingoing scattering states are produced infinitely early in time and infinitely far from the scattering region, and outgoing scattering states are detected infinitely later in time and infinitely far away. Actual scattering experiments take place within a finite region of space over a finite period of time. Again, the idealized asymptotic S-matrix extrapolates far beyond the practical use of the formalism.

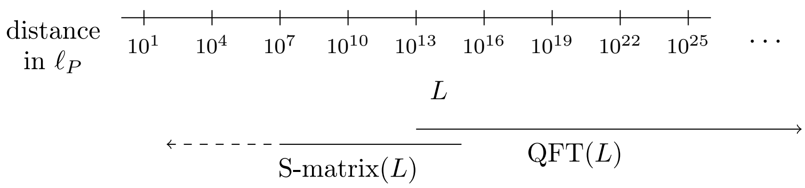
The mathematical idealizations do serve mathematical purposes. But the practical limits of physical knowledge are encoded in the *effective* QFT with a UV cutoff and the *effective* S-matrix with an IR cutoff.

2 A minimal practical formal structure

2.1 An effective QFT for distances $> L$ and an effective S-matrix for distances $< L$, for observers at every distance scale $L \gg \ell_P$

A leading edge high energy experiment of size L probes for new physics at distances $< L$. The observer uses an effective QFT to describe physics at distances $> L$. The observer measures physics at distances $< L$ as scattering amplitudes between ingoing and outgoing scattering states in the effective QFT. Short distance physics is probed by sending things in and measuring what comes out. For short distance physics there is only an effective S-matrix.

Prudence suggests a formalism that corresponds with what is observable. For every distance $L \gg \ell_P$ there is an effective QFT(L) with UV cutoff L and an effective S-matrix(L) with IR cutoff L. L is a sliding distance scale. The meaning of "short distance physics" depends on the scale L of the observer. The condition $L \gg \ell_P$ expresses the impracticality of Planck scale experiments.

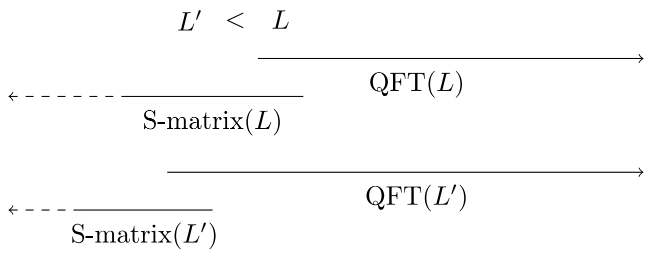


2.2 An S-matrix does not imply a microscopic hamiltonian

There is no presumption of QFT or any quantum mechanical hamiltonian all the way down to ℓ_P. Having an effective S-matrix does not require or imply that there must be a microscopic hamiltonian. The implication is in the other direction. Given a microscopic hamiltonian, scattering amplitudes can be derived from it. Given an S-matrix, there is no necessity that it is or can be derived from a microscopic quantum mechanical hamiltonian.

2.3 QFT renormalization group operates from smaller distance L to larger; S-matrix renormalization group operates from larger L to smaller

As progress pushes to a shorter distance L', the descriptions of physics must be consistent:



C1 QFT(L) must derive from QFT(L') by the QFT renormalization group.

C2 The scattering amplitudes derived from QFT(L') must agree with S-matrix(L) at intermediate distances between L' and L.

C3 S-matrix(L') must derive from S-matrix(L) by the "S-matrix renormalization group".

The S-matrix RG is the operation on effective S-matrices that takes an effective S-matrix with IR cutoff distance L to an effective S-matrix with a smaller cutoff L' by using the scattering states at distance L to make the scattering states at the smaller distance L'. The S-matrix RG and the QFT RG operate in opposite directions.

This formal structure is local in L. An observer at scale L makes only a modest extrapolation by supposing that at a somewhat smaller distance scale $L' < L$ there will be a somewhat more fundamental effective QFT(L').

3 A mechanism that produces such a formal structure

A search for a mechanism that would produce a realistic QFT began with [1, 2]. A mechanism was finally proposed in [3]. The mechanism produces a formal structure such as described above. The line of thought is sketched in the Appendix.

3.1 Summary

The argument for the proposed mechanism is summarized:

1. String theory provides a way to construct a self-consistent S-matrix for short distance physics without requiring a short distance QFT.

2. When the string worldsheet is described by an effective 2d-QFT with 2d cutoff distance Λ^{-1}, the string S-matrix constructed is an effective S-matrix(L) where the IR cutoff L is given in dimensionless units by $L^2 = \ln(\Lambda/\mu)$ where $\mu^2|dz|^2$ is the worldsheet

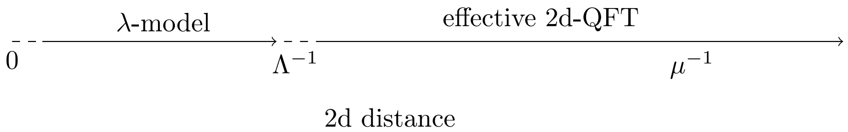
metric. The condition $L \gg 1$ is the requirement $\Lambda^{-1} \ll \mu^{-1}$, i.e., the requirement that the 2d cutoff distance Λ^{-1} be insignificant at the 2d distance scale μ^{-1} used in the S-matrix calculation.

3. The string background is encoded in the local worldsheet physics at 2d distance Λ^{-1}.

4. The S-matrix RG acts on S-matrix(L) by integrating out the froth of small handles in the worldsheet. Decreasing IR cutoff L is equivalent to increasing 2d UV cutoff Λ^{-1}.

5. The effects of the froth of small handles are replicated by a certain mathematically natural local 2d nonlinear model (2d-NLM) called the λ-*model*. Its target manifold is the space of effective 2d-QFTs of the worldsheet, parametrized by the effective 2d coupling constants $\lambda^i(\Lambda)$ at 2d scale Λ^{-1}. These $\lambda^i(\Lambda)$ are the modes of the classical background space-time fields with UV cutoff L. The target manifold of the λ-model is thus the space of classical space-time fields with UV cutoff L.

6. The froth of small handles is replaced by the λ-model acting at 2d distances $< \Lambda^{-1}$. The λ-model produces an effective worldsheet QFT with 2d UV cutoff Λ^{-1}.



7. Integrating out the λ-fluctuations has the same effect as integrating out the froth of small handles so the 2d-RG of the λ-model implements the S-matrix RG.

8. The 2d-RG of the λ-model produces a measure on the target manifold. This is the *a priori* measure of the 2d-NLM. A measure on the target manifold is a functional integral over the space-time fields with UV cutoff L, i.e., an effective quantum field theory QFT(L).

9. The quantum states of QFT(L) are the quantum string backgrounds.

10. The consistency conditions **C1**, **C2**, **C3** are automatically satisfied on QFT(L) and S-matrix(L). So the λ-model produces a consistent realization of the minimal practical formal structure described above.

11. The effective QFT(L) is produced by a 2d mechanism that does not necessarily correspond to canonical quantization. There are concrete possibilities of nonperturbative semi-classical 2d effects which could produce non-canonical degrees of freedom and non-canonical interactions in QFT(L). These 2d effects are the 2d winding modes and 2d instantons coming from nontrivial homotopy groups π_1 and π_2 of the target manifold of the λ-model, which is the space of space-time fields. These homotopy groups are nontrivial when the space-time fields include SU(2) and SU(3) gauge fields in four space-time dimensions.

The last point is the main reason for investigating the λ-model as a formalism for fundamental physics. There are concrete possibilities of testable conditional predictions of the form *if QFT(L) contains SM+GR, then it predicts certain specific non-canonical degrees of freedom and interactions beyond those of the canonically quantized effective quantum field theory.*

3.2 2d-QFT of the string worldsheet

In the general renormalizable 2d nonlinear model

$$\int e^{-\int d^2z\, g_{\mu\nu}(X)\partial X^\mu \bar{\partial} X^\nu} \mathcal{D}X \qquad X(z) \in M \tag{1}$$

the field $X(z)$ takes values in a target manifold M. The 2d coupling constants are given by a Riemannian metric $g_{\mu\nu}(X)$ on M. The manifold M is taken to be compact and the metric $g_{\mu\nu}(X)$ is taken to have euclidean signature in order that the 2d-QFT will be well defined. The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda}\, g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2) \tag{2}$$

drives the 2d-NLM to a solution of $R_{\mu\nu} = 0$.

The 2d-QFT of the string worldsheet is a elaboration of the general 2d-NLM in which the target manifold M is space-time and the 2d coupling constants consist of the space-time metric $g_{\mu\nu}(X)$ as well as some non-abelian gauge fields, scalar fields, fermion fields, etc. on the space-time M. The equation $\beta = 0$ generalizing $R_{\mu\nu} = 0$ is a semi-realistic supersymmetric classical field equation which includes GR and potentially the SM.

In abstract notation,

$$\begin{aligned}
\lambda^i &= \text{the 2d coupling constants,}\\
\phi_i(z) &= \text{the corresponding spin-0 scaling fields of the 2d-QFT,}\\
|\phi_i\rangle &= \text{the corresponding radial quantization states on the unit circle in 2d,}\\
G_{ij} &= \text{the natural metric } \langle\phi_i|\phi_j\rangle.
\end{aligned}$$

The index i labels the modes of the space-time fields. For example, a mode $\delta_i g_{\mu\nu}(X)$ of the space-time metric with euclidean wavenumber $p_\mu(i)$ corresponds to 2d field $\phi_i(z)$

$$\delta_i g_{\mu\nu}(X) = h_{\mu\nu}(i) e^{ip_\mu(i)X^\mu} \quad i \leftrightarrow p_\mu(i),\, h_{\mu\nu}(i) \qquad \phi_i(z) = h_{\mu\nu}(i)\partial X^\mu \bar{\partial} X^\nu\, e^{ip_\mu(i)X^\mu(z)} \tag{3}$$

For each value of the λ^i there is a perturbed 2d-QFT made by inserting in the worldsheet

$$e^{\int d^2z\, \lambda^i \phi_i(z)} \tag{4}$$

and every nearby 2d-QFT is made this way. The λ^i thus form a system of local coordinates on the space of 2d-QFTs.

The 2d scaling-dimensions are

$$\dim(\phi_i) = 2 + \delta(i) \qquad \dim(\lambda^i) = -\delta(i) \qquad \delta(i) = p(i)^2 \tag{5}$$

The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda}\, \lambda^i = \beta^i(\lambda) \qquad \beta^i(\lambda) = -\delta(i)\lambda^i + O(\lambda^2) \tag{6}$$

drives the worldsheet 2d-QFT towards the $\beta = 0$ submanifold. The $\beta = 0$ submanifold is parametrized by the marginal coupling constants

$$\dim(\lambda^i) = -\delta(i) = -p(i)^2 = 0 \tag{7}$$

which correspond to the zero-modes of the space-time fields. The string worldsheet has no relevant operators, no λ^i with $\delta(i) < 0$. There are no unstable directions of the 2d-RG.

3.3 Effective string S-matrix with IR cutoff L

Let $\mu^2|dz|^2$ be the 2d metric of the worldsheet. Impose a 2d UV cutoff $\Lambda^{-1} \ll \mu^{-1}$. The cutoff string propagator (the cutoff integral over 2d-cylinders) is

$$\int_0^{\ln(\Lambda/\mu)} \sum_{i,j} \; |\phi_i\rangle \; G^{ij} e^{-\tau\delta(i)} d\tau \; \langle\phi_j| = \sum_{i,j} \; |\phi_i\rangle \; \frac{1-e^{-L^2\delta(i)}}{\delta(i)} G^{ij} \; \langle\phi_j| \tag{8}$$

where

$$L^2 = \ln(\Lambda/\mu) \qquad e^{-L^2\delta(i)} = (\Lambda/\mu)^{-\delta(i)} \tag{9}$$

The only modes that propagate are those that satisfy

$$\delta(i) > L^{-2} \qquad \text{which is} \qquad p(i)^2 > L^{-2} \tag{10}$$

So the 2d UV cutoff Λ^{-1} puts an IR cutoff L on the string S-matrix. An effective 2d-QFT of the worldsheet gives an effective string S-matrix(L) with L given by $L^2 = \ln(\Lambda/\mu)$.

3.4 Effective 2d coupling constants $\lambda^i(\Lambda)$

The effects of the 2d coupling constants $\lambda^i(\Lambda)$ at 2d scale Λ^{-1} are suppressed by the 2d-RG running from Λ^{-1} up to μ^{-1}

$$\lambda^i(\mu) = (\Lambda/\mu)^{-\delta(i)} \, \lambda^i(\Lambda) = e^{-L^2\delta(i)} \lambda^i(\Lambda) \tag{11}$$

If $L^2\delta(i) > 1$ then $\lambda^i(\Lambda)$ is effectively irrelevant; its effects on the worldsheet are negligible. The only $\lambda^i(\Lambda)$ that matter are the effectively marginal couplings

$$\delta(i) < L^{-2} \qquad \text{which is} \qquad p(i)^2 < L^{-2} \tag{12}$$

So there is a UV cutoff distance L on the modes of the space-time fields that are the coupling constants of the effective 2d-QFT of the worldsheet.

The 2d UV cutoff Λ^{-1} separates the 2d coupling constants and fields into two subsets. The effectively marginal λ^i with $\delta(i) < L^{-2}$ are the perturbations of the classical string background. The effectively irrelevant $\phi_i(z)$ with $\delta(i) > L^{-2}$ are the vertex operators describing the propagating string modes in the effective S-matrix.

3.5 Implement the S-matrix renormalization group

Consider the effect of a small handle in the worldsheet. A small handle is made by identifying the boundaries of two holes of radius r around two points z_1, z_2 which are close together in the worldsheet. The identification is

$$z \leftrightarrow z' \qquad (z - z_1)(z' - z_2) = r^2 e^{i\theta} \tag{13}$$

Insert a sum over radial quantization states on each boundary circle. Integrate over the moduli z_1, z_2, r, θ. The integral over θ projects on the spin-0 states. The effect of the small handle becomes the bi-local insertion

$$\frac{1}{2} \sum_{i_1,i_2} \int d^2z_1 \, \phi_{i_1}(z_1) \int d^2z_2 \, \phi_{i_2}(z_2) \int_{\Lambda^{-1}}^{\frac{1}{2}|z_1-z_2|} dr \; r^{-1-\delta(i_1)-\delta(i_2)} \; g_{\text{str}}^2 G^{i_1 i_2} \tag{14}$$

where $G_{i_1 i_2}$ is the natural metric on the space of 2d-QFTs and $g_{\rm str}$ is the string coupling constant. The cutoff dependent contribution comes from the effectively marginal fields

$$\frac{1}{2} \sum_{\delta(i_1)\sim 0} \sum_{\delta(i_2)\sim 0} \int d^2z_1\, \phi_{i_1}(z_1) \int d^2z_2\, \phi_{i_2}(z_2)\; g_{\rm str}^2 G^{i_1 i_2} \ln\left(\Lambda|z_1 - z_2|\right) \tag{15}$$

The integration region of interest here is $\Lambda^{-1} < |z_1 - z_2| \ll \mu^{-1}$ where the small handle contributes to the local worldsheet physics.

The cutoff dependence of the small handle expressed by (16) can be canceled by letting the effectively marginal 2d coupling constants λ^i fluctuate locally on the worldsheet. Make the λ^i into sources $\lambda^i(z)$ so the worldsheet insertion becomes

$$e^{\int d^2z\, \phi_i(z) \lambda^i(z)} \tag{16}$$

Then set the $\lambda^i(z)$ fluctuating with 2-point correlation function

$$\langle\, \lambda^{i_1}(z_1)\; \lambda^{i_2}(z_2)\, \rangle = -g_{\rm str}^2 G^{i_1 i_2} \ln\left(\Lambda|z_1 - z_2|\right) \tag{17}$$

The cancellation of the single small handle (16) by the gaussian λ-fluctuations (18) holds in every background 2d-QFT. Therefore the cutoff dependence of the entire nongaussian froth of small handles is canceled by λ-fluctuations governed by the 2d-NLM

$$\int e^{-\int d^2z\; g_{\rm str}^{-2} G_{ij}(\lambda) \partial\lambda^i \bar\partial\lambda^j}\; e^{\int d^2z\, \phi_i(z)\lambda^i(z)} \mathcal{D}\lambda \qquad \lambda(z) \in \mathcal{M} \tag{18}$$

$\mathcal{M}$ = the manifold of effective worldsheet 2d-QFTs with 2d cutoff Λ^{-1}

= the space of classical space-time fields with UV cutoff L given by $L^2 = \ln(\Lambda/\mu)$

This 2d-NLM is the λ-model. $\mathcal{M}$ is the target manifold.

The sum over handles does not depend on the arbitrary choice of Λ^{-1} so the sum over λ-fluctuations at 2d distances $< \Lambda^{-1}$ has the same effect as the sum over small handles, since either of these sums cancels the Λ^{-1} dependence of the sum over handles at 2d distances $> \Lambda^{-1}$. Therefore integrating out the λ-fluctuations at 2d distances $< \Lambda^{-1}$ is equivalent to integrating out the froth of small handles. Integrating out the λ-fluctuations at 2d distances $< \Lambda^{-1}$ produce an effective 2d-QFT with UV cutoff Λ^{-1} giving an effective string S-matrix(L) with IR cutoff L.

$$0 \;\xrightarrow{\quad \lambda\text{-model} \quad}\; \Lambda^{-1} \;\xrightarrow{\qquad\qquad \text{effective 2d-QFT} \qquad\qquad}\; \mu^{-1}$$

2d distance

The 2d-RG of the λ-model operates from smaller 2d distance Λ'^{-1} up to larger 2d distance Λ^{-1} by integrating out the λ-fluctuations at 2d distances between Λ'^{-1} and Λ^{-1}. This is equivalent to integrating out the small handles, taking the effective S-matrix(L') with larger IR cutoff L' to the effective S-matrix(L) with smaller IR cutoff L. Thus the 2d-RG of the λ-model implements the S-matrix RG.

3.6 Production of an effective QFT with UV cutoff L

Like any 2d-NLM, the λ-model is specified by two pieces of data

- the metric $g_{\rm str}^{-2} G_{ij}(\lambda)$ on the target manifold $\mathcal{M}$

- the *a priori* measure $\rho(\lambda)d\lambda$ on the target manifold $\mathcal{M}$

The *a priori* measure on $\mathcal{M}$ determines the functional volume element on the fields $\lambda^i(z)$

$$\mathcal{D}\lambda = \prod_z \rho(\lambda(z))\, d\lambda(z) \tag{19}$$

A point z in the effective worldsheet represents a 2d block of dimensions $\Lambda^{-1} \times \Lambda^{-1}$. The measure $\rho(\lambda(z))\, d\lambda(z)$ summarizes the integrated-out short distance λ-fluctuations inside the block, hence *a priori* which means *from the earlier* or *from what has gone before*.

The *a priori* measure $\rho(\lambda)d\lambda$ evolves under the 2d-RG. It diffuses in $\mathcal{M}$ because of the λ-fluctuations. Simultaneously, the λ^i flow along $\beta^i(\lambda)$ towards the $\beta = 0$ submanifold. So $\rho(\lambda)d\lambda$ evolves under a driven diffusion process. Taking $d\lambda$ to be the metric volume element, $\rho(\lambda)$ becomes a function on $\mathcal{M}$. The driven diffusion equation is

$$\Lambda \frac{\partial}{\partial \Lambda}\, \rho(\lambda) = \nabla_i \left(g_{\mathrm{str}}^2 G^{ij} \partial_j + \beta^i \right) \rho(\lambda) \tag{20}$$

Integrating out the λ-fluctuations up to Λ^{-1} drives $\rho(\lambda)d\lambda$ to the equilibrium measure

$$\rho(\lambda)d\lambda \to e^{-g_{\mathrm{str}}^{-2} S(\lambda)} d\lambda \qquad \text{where} \quad \beta^i = G^{ij}\partial_j S$$

The λ^i are the space-time field modes with UV cutoff L so the *a priori* measure $\rho(\lambda)d\lambda$ is the functional integral of an effective QFT(L) with classical action $g_{\mathrm{str}}^{-2} S(\lambda)$.

The λ-model thus produces an effective S-matrix(L) and an effective QFT(L) at every $L \gg 1$. The consistency conditions **C1, C2, C3** are automatically satisfied. The S-matrix RG acts on the S-matrix(L) by design. The QFT RG acts on the QFT(L) because of the decoupling ofeffectively irrelevant couplings $\lambda^i(\Lambda)$. This decoupling is a fundamental property of the 2d-RG. Agreement between S-matrix(L) and QFT(L) on scattering amplitudes at scales $\sim L$ is guaranteed because the scattering amplitudes of S-matrix(L) near the IR cutoff L are given by the 2d correlation functions of vertex operators near the 2d UV cutoff Λ^{-1} which are determined by the *a priori* measure $\rho(\lambda)d\lambda$ which is QFT(L).

3.7 Possible non-canonical degrees of freedom and couplings in SU(2) and SU(3) quantum gauge theory

The λ-model is a nonperturbative 2d-NLM with possibilities of nonperturbative semi-classical 2d effects [4]. These are 2d winding modes associated to $\pi_1(\mathcal{M})$ and 2d instantons associated to $\pi_2(\mathcal{M})$, where the target manifold $\mathcal{M}$ is the manifold of space-time fields. Suppose as a conditional that the λ-model produces an effective quantum string background with four macroscopic space-time dimensions and SU(N) gauge fields. Then the mathematical results

$$\pi_k \text{ of the manifold of } SU(N) \text{ gauge fields on } \mathbb{R}^4 \cup \{\infty\} = \pi_{k+3}(SU(N)) \tag{21}$$

$$\pi_4(SU(2)) = \mathbb{Z}_2 \qquad \pi_5(SU(2)) = \mathbb{Z}_2 \qquad \pi_4(SU(3)) = 0 \qquad \pi_5(SU(3)) = \mathbb{Z} \tag{22}$$

imply that there are 2d winding modes when the gauge group is SU(2) and 2d-instantons when the gauge group is SU(2) or SU(3).

These nonperturbative semi-classical effects in the λ-model offer possibilities of conditional predictions of the form *if the λ-model produces SM+GR then it also produces non-canonical degrees of freedom from the $\mathbb{Z}_2$ winding mode in the SU(2) gauge fields and non-canonical interactions from the 2d instantons in the SU(2) and SU(3) gauge fields.*

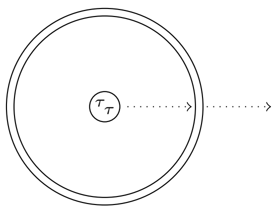
3.8 Winding mode in the SU(2) gauge fields

Let $A_{+-}(u,0)$ be an SU(2) instanton-anti-instanton configuration on $\mathbb{R}^4$ in the limit where one or both of the instanton sizes goes to zero. The relative orientation u between the instanton and the anti-instanton lies in the adjoint representation $\mathrm{SU}(2)/\mathbb{Z}_2$. The remaining moduli of the instanton-anti-instanton pair are left implicit. $A_{+-}(u,0)$ is a solution of the Yang-Mills equation which is the 2d-RG equation $\beta = 0$. The gauge fields $A_{+-}(u,\rho)$ parametrized by u and by a small size ρ form a neighborhood of the origin in a $\mathbb{R}^4/\mathbb{Z}_2$ orbifold. The 2d winding mode is the $\mathbb{Z}_2$ twist field of this orbifold. The moduli of the $\mathbb{Z}_2$ twist field are the remaining moduli of the instanton-anti-instanton pair, including fermionic zero-modes.

The 2d instantons in the SU(2) and SU(3) gauge fields on $\mathbb{R}^4$ are also found in zero-size instanton-anti-instanton configurations. For SU(3) the relative orientation of a zero-size instanton-anti-instanton pair is parametrized by SU(3)/U(1) which has a topologically nontrivial 2-sphere. For SU(2) the minimal nontrivial 2-sphere is in the zero-size configurations of two instantons and two anti-instantons.

3.9 Vacuum condensate of SU(2) Yang-Mills flow defects

Again suppose that the target manifold includes SU(2) gauge fields on $\mathbb{R}^4$. Suppose that some nonzero portion of the *a priori* measure fall in the $\mathbb{Z}_2$-odd sector. Then the λ-model will contain a gas of $\mathbb{Z}_2$ twist fields. If this gas is a plasma then the system is pinned near the orbifold point(s) far from the classical ground state at the flat SU(2) gauge field. So suppose that the twist fields bind into Z_2-neutral pairs or larger neutral collections. These neutral molecules start near an orbifold point and then flow under the 2d-RG flow down to the flat SU(2) gauge field. The 2d-RG generated by $\beta^i(\lambda)$ on the modes of the SU(2) gauge field is to first approximation the Yang-Mills flow — the gradient flow of the Yang-Mills action. There is a unique downward trajectory $A_{\mathrm{YMF}}(t)$ of the Yang-Mills flow starting from an orbifold point, $A_{\mathrm{YMF}}(-\infty) = A_{+-}(\mathbf{1},0)$. The instanton and anti-instanton grow, merge, and annihilate, ending with the flat gauge field $A_{\mathrm{YMF}}(+\infty)$. A neutral molecule will appear in the worldsheet as a Yang-Mills flow defect operator.



The inner dotted arrow represents the slow flow near the orbifold point. The fast part of the flow is between the solid circles. The outer dotted arrow represents the slow flow near the flat gauge field.

The 2d gas of defects will appear as a vacuum condensate in the effective QFT(L) produced by the λ-model. The nature of the condensate will depend on the details of the downward Yang-Mills trajectories and on the initial measure on the neighborhood of the orbifold points. The trajectories are controllable. The slow segments are controlled by perturbing around $\beta = 0$. The fast intermediate segments can be calculated numerically. The initial measure might be driven by the slow action of the Yang-Mills flow near the orbifold points to a more or less universal measure. What remains undetermined will depend on the microscopic 2d physics of the $\mathbb{Z}_2$ twist fields.

4 To do

Most urgent is to determine if the λ-model can in fact make conditional predictions of observable non-canonical effects in SU(2) and SU(3) gauge theory in 4 dimensions. This requires figuring out how to calculate semi-classical corrections to the *a priori* measure of the λ-model coming from the 2d winding modes and instantons. Most promising seems to be the calculation sketched just above of the vacuum condensate of the winding modes.

If such conditional predictions can be made and if they can be checked, there will be motivation for further investigation of the λ-model. There are also many basic questions about the λ-model, but the most interesting of these seem difficult and open-ended. Expending effort does not seem worthwhile unless and until a successful conditional prediction provides motivatation. In particular, the technical foundations all remain to be built.

The top-down construction of effective QFT is tantalizing. The λ-model operates at 2d distances from near 0 up to Λ^{-1}. The corresponding space-time distance is given by $L^2 = \ln(\Lambda/\mu)$. So the λ-model builds effective QFT from the largest distances in space-time *down* to L (nevertheless guaranteeing that the QFT RG is satisfied).

The *a priori* measure $\rho(\lambda)d\lambda$ of the λ-model is a measure on the target manifold. The target manifold is the space of effective 2d-QFTs of the string worldsheet. This is the space of effective classical string backgrounds. In the extreme large distance limit $L \to \infty$, only the marginal 2d couplings λ^i fluctuate. The target manifold becomes the space of worldsheet 2d conformal field theories. These are the idealized classical string backgrounds. The λ-model dynamically produces a measure on this huge space of exact backgrounds. The *a priori* measure $\rho(\lambda)d\lambda$ is the functional integral of an effective QFT only locally when it concentrates near a background with large space-time dimensions. Exploring the dynamics of such concentration all the possible variety of places the measure might concentrate is an intimidating task. A more practical approach is to assume that some part of the measure concentrates near a large 4-manifold space-time with space-time fields including those of the SM+GR. If conditional on this assumption predictions can be made of testable non-canonical effects, and if they can be checked against experiment, then it might be worth asking such questions as, is there anything in the λ-model mechanism that would cause the dynamically generated *a priori* measure $\rho(\lambda)d\lambda$ to concentrate near backgrounds with macroscopic space-time dimensions, four in particular?

Wick rotation is an after-thought. Space-time has been taken to be compact with euclidean signature in order that the 2d-NLM of the worldsheet be a well defined effective 2d-QFT. Only finitely many modes $\lambda^i(\Lambda)$ of the space-time fields fluctuate and their fluctuations are governed by a positive definite metric $G_{ij}(\lambda)$. The λ-model is thus an effective mechanism. Wick rotation is to be carried out in the effective QFT(L) and in the effective S-matrix(L) if and when the *a priori* measure concentrates at a macroscopic space-time. Wick rotation is then only approximate, up to corrections that are small at distance scales well away from L. There is no explanation of Wick rotation.

How is cosmology to be done? Perhaps the space-time scale L can be related (inversely) to cosmological time, so cosmology at later times and larger length scales would be governed by QFT(L), while cosmology at earlier times and smaller length scales would be described by an outgoing scattering state in S-matrix(L).

Appendix. Notes on the line of thought

A.1 Search for a mechanism that produces QFT

The ideas expressed in this note were developed during the period 1977–2002 in the process of searching for and eventually formulating a mechanism that would produce quantum field theory.

The 2d-RG as a mechanism for space-time physics (1977–79)

The line of thought began with the renormalization of the general renormalizable 2d non-linear model (2d-NLM)

$$\int e^{-\int d^2z\, g_{\mu\nu}(X)\partial X^\mu \bar{\partial} X^\nu} \mathcal{D}X \qquad X(z) \in M \tag{23}$$

whose coupling constants are given by a Riemannian metric $g_{\mu\nu}(X)$ on a manifold M. The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda} g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2) \tag{24}$$

drives the 2d-NLM to a solution of $R_{\mu\nu} = 0$. This was very exciting (at least for me). The 2d-RG appeared as a *mechanism* that *produces* solutions of a GR-like space-time field equation $R_{\mu\nu} = 0$. It suggested the possibility that a mechanism — the 2d-RG — might actually answer questions like *Where does space-time field theory come from?* or even *Where do the laws of physics come from?* The goal became a mechanism that produces the actual laws of physics, rather than fundamental principles that would determine the laws of physics.

It had become clear by the late 1970s that there are too many effective QFTs. A mechanism was needed that would produce effective QFT more selectively than the QFT renormalization group. The 2d-RG seemed promising for the purpose since it at least produced classical field theory. The goal then became a mechanism that produces *quantum* field theory and that has the 2d-RG as its classical limit. The 2d-NLM had two other shortcomings as a mechanism for space-time physics: the $\beta = 0$ equation $R_{\mu\nu} = 0$ is not quite Einstein's equation; and the RG fixed points, the solutions of $\beta = 0$, have unstable directions along which the RG flow diverges from the fixed point rather than converges.

The 2d-RG incorporated into string theory (1981–85)

In the early 1980s it was realized that the 2d-RG fixed point equation $\beta = 0$ (i.e., 2d scale invariance) is a consistency condition for calculating the string S-matrix from a worldsheet 2d-QFT. The string worldsheet is a supersymmetric 2d-QFT containing additional 2d degrees of freedom besides $X(z)$. The 2d coupling constants become, in addition to the space-time metric $g_{\mu\nu}(X)$, a collection of non-abelian gauge fields, scalar fields, fermion fields, etc. on space-time. The supersymmetric string worldsheet resolved two of the problems with the basic 2d-NLM. The 2d supersymmetry of the string worldsheet eliminates the unstable directions at the fixed points (tachyons in the S-matrix). The worldsheet $\beta = 0$ equation generalizing $R_{\mu\nu} = 0$ is a semi-realistic supersymmetric classical field equation that includes GR and potentially the SM.

In the mid-1980s, several assumptions and mathematical idealizations about string theory were adopted by the community more or less as truisms:

1. The string S-matrix was taken to be an asymptotic S-matrix without IR cutoff — a "theory of everything".
2. The string backgrounds were taken to be the conformally invariant worldsheet 2d-QFTs from which such asymptotic string S-matrices were derived: the exact worldsheet solutions of $\beta = 0$ given by Calabi-Yau manifolds ($R_{\mu\nu} = 0$) and generalizations.
3. It was *assumed* that the low momentum physics of string theory is the (supersymmetric) QFT that happens to have the same low momentum scattering amplitudes as the asymptotic string S-matrix. The string backgrounds were conflated with those supersymmetric QFTs.
4. It was assumed that there must exist a microscopic QFT or some other kind of fundamental microscopic quantum mechanical system from which the string S-matrix could be derived.

Questions (1987)

Circa 1987, the key questions seemed to me to be:

1. How does the 2d-RG act in string theory as a *mechanism*? The fixed point equation $\beta = 0$ is a only consistency condition for the string S-matrix recipe.
2. Where does *quantum* field theory come from in string theory? What produces a functional integral over space-time fields?
3. What is the *quantum* string background? It should be a quantum state of a QFT.

The λ-model (1988-2002)

The attempt to answer these questions was a long-drawn-out process. One seed was the idea that the string background is encoded in the local 2d physics of the worldsheet. Another seed was the vague notion that nonperturbative effects in string theory might come from infinite genus worldsheets. Eventually, these were combined in the idea that a froth of small handles would contribute to the local 2d physics of the worldsheet and thus to the string background. This motivated the calculation of the log divergence in the contribution of a single small handle (as in section 3.5 above), which took the form of a bi-local insertion in the worldsheet. This was a strong signal. The infrared log divergence of the scalar field 2-point function plays a fundamental role in 2d-QFT. Thus the idea of setting the 2d coupling constants λ^i fluctuating as 2d scalar fields $\lambda^i(z)$ with the 2d-IR log divergences of the λ-fluctuations cancelling the 2d-UV log divergences of the small handles.

The essential role of a 2d distance scale Λ^{-1} as a 2d UV cutoff in the string S-matrix calculation and as the 2d IR cutoff on the λ-fluctuations required abandoning the idealized asymptotic string S-matrix "of everything" for an effective string S-matrix with IR cutoff L (as in section 3.3 above). Integrating out the froth of small handles became the S-matrix RG.

Recognizing that the *a priori* measure $\rho(\lambda)d\lambda$ on the λ-fluctuations would govern the local worldsheet physics led to identifying $\rho(\lambda)d\lambda$ as the functional integral of the effective space-time QFT that is the quantum string background (as in sections 3.4 and 3.6 above).

At this point the task became to identify semi-classical 2d effects in the λ-model that might lead to checkable predictions.

A.2 Pragmatism and the S-matrix philosophy

The S-matrix was proposed as an alternative formal structure for fundamental physics at several points in the history of Quantum Mechanics when QFT seemed to have hit a wall. A version of the history is recounted in [5]. Heisenberg first proposed in the 1940s using the S-matrix as a fundamental formalism in response to the divergences of perturbative QED and the difficulty of accounting for cosmic ray showers. The S-matrix bootstrap program was proposed in the 1960s in response to the plethora of mesons and baryons and their strong couplings. In the end, QFT overcame its difficulties on both occasions and the S-matrix proposals lapsed. The third occasion was the string S-matrix proposal of the early 1970s which attracted interest at least in part because of the incompatibility between GR and QFT when extrapolated down to the Planck length.

Heisenberg's explicit rationale for using the S-matrix was the principle that fundamental physics should be expressed in terms of what is actually observed. This principle has had notable successes in fundamental theoretical physics. For example, the route of Bohr and Heisenberg to Matrix Mechanics was guided by focussing on observable transitions. But the principle was not followed literally. Matrix Mechanics in the end described the world by quantum states and transition amplitudes which are not themselves observable. Only their absolute squares are observable. The strategy of focussing on what is observable led to a formalism, Matrix Mechanics, that reliably produces observable quantities. A pragmatic version of the strategic principle might be *use the minimal formal machinery that is useful to produce the observable quantities of physics.* Quantum Mechanics in the form of quantum field theory is so successful at producing observable quantities that it can be considered "what is observable" at distances greater than about $(10^3\mathrm{GeV})^{-1}$.

The S-matrix philosophy proposed replacing Quantum Mechanics and QFT with an asymptotic S-matrix. But the asymptotic S-matrix is an extreme idealization of what is observable. All the useful work of physics at distances larger than the elementary particle scale uses Quantum Mechanics or its effective approximation Classical Mechanics. Is it feasible, for example, to describe the behavior of a galaxy in terms of an S-matrix? If fundamental physics were formulated as an S-matrix, what would effective QFT or effective Quantum Mechanics or effective Classical Mechanics be derived from?

On the other hand, a pragmatic version of the S-matrix philosophy does seem reasonable. Scattering amplitudes describe what is observable at short distances where 'short' is relative to the size of the observer. An effective S-matrix with an IR cutoff L is a practical formulation of what we can actually observe at distances smaller than the limit of our best hamiltonian quantum mechanical model.

From this point of view, string theory is interesting not because it offers an S-matrix "theory of everything", but rather because it is a way to construct a self-consistent S-matrix for short distance physics without requiring a short distance QFT. A short distance S-matrix that does not require a short distance QFT is entirely suitable in a pragmatic version of the S-matrix philosophy. Before string theory, the only successful way to construct an S-matrix was by deriving it from a QFT. But that does not mean that every S-matrix must derive from a microscopic QFT.

The pragmatist philospher C. S. Peirce (a contemporary of Ernst Mach) proposed that the symbolic tools of science take their significance from the work that they do. (This might well be a selective, idiosyncratic reading of Peirce.) A pragmatic strategy is to shape the formalisms of fundamental physics for the work they need to perform. The pragmatic view argues against pursuit of mathematical beauty, against pursuit of beautiful fundamental principles, against attempting to extrapolate to an absolutely fundamental theory based on absolutely fundamental principles. Eventually, a successful fundamental theory may be based on beautiful principles and may be formulated in beautiful mathematics. But there is no telling how far away that is or in what direction. There is no telling in advance which mathematically beautiful forms will prove useful for fundamental physics. Meanwhile, a practical strategy is to seek incremental changes in the formalisms of fundamental physics, incremental improvements that can actually do useful work in describing the fundamental physics of the real world.

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