A pragmatic approach to formal fundamental physics

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Abstract

A minimal practical formal structure for a fundamental theory is suggested. A mechanism that produces such a structure is reviewed. The proposed mechanism has possibilities of producing non-canonical phenomena in SU(2) and SU(3) quantum gauge theories. These might provide testable conditional predictions. One possibility is a vacuum condensate of SU(2) gauge fields derived from certain trajectories of the SU(2) Yang-Mills flow.

Contents

1 Formal fundamental physics

For 45 years, the most fundamental theory of physics has been the Standard Model combined with General Relativity. It describes almost everything known at distances larger than about $(10^3\text{GeV})^{-1}$. Dark matter, neutrino mixing, and some CP violation are the only observed phenomena left unexplained. The project of formal fundamental physics is to hypothesize a more comprehensive formal machinery that includes the SM+GR and makes predictions that can be checked against experiment. It seems overreaching to attempt to predict the SM+GR itself. Sufficient would be conditional predictions of the form *if the formalism produces the SM+GR then it must also produce such and such testable phenomena beyond the SM+GR*. An example is proton decay conditionally predicted by Grand Unification. If a conditional prediction were to check successfully against experiment, then the proposed formalism would become a serious candidate for a more fundamental theory.

SM+GR is an effective quantum field theory with short distance cutoff on the order of $(10^3\text{GeV})^{-1} = 10^{16}\ell_P$ where ℓ_P is the Planck distance. GR can be treated as an effective QFT because quantum effects are negligible at distances $\gg \ell_P$. The effective QFT with short distance cutoff $10^{16}\ell_P$ is indistinguishable from classical GR. A more comprehensive formalism should be capable of producing an effective QFT such as the SM+GR and should make definite conditional predictions of observable phenomena beyond the SM+GR.

In the 45 years since the SM was verified, none of the attempts at formal fundamental physics have worked (in the strict theoretical physics sense of 'worked' — making predictions that check successfully against experiment). One reaction is to give up on the project, perhaps hoping that experiment will eventually provide more guidance. Alternatively, it might be useful to reexamine the assumptions that have guided the formal fundamental physics enterprise and reconsider paths not taken. SM+GR already encodes very much experimental evidence. New high energy discoveries will most likely lead to a new effective QFT that improves incrementally on the SM. The question for formal fundamental physics is how to use a specific effective QFT such as the SM+GR as guidance towards a formalism more comprehensive and more predictive than effective QFT in general.

1.1 Against Quantum Gravity

It might be useful to question the truism that General Relativity and Quantum Mechanics have to be reconciled in a theory of Quantum Gravity. On the contrary, there is no appreciable conflict at distances $\gg \ell_P$. The smallest distance presently accessible to experiment is roughly $L_{\text{exp}} = (10^3\text{GeV})^{-1} = 10^{16}\ell_P$. To suppose a conflict is to extrapolate the validity of both Quantum Mechanics and General Relativity over 16 orders of magnitude from $10^{16}\ell_P$ down to ℓ_P. Nothing is known about physics at such small distances.

energy (GeV)	10^{18}	10^{15}	10^{12}	10^{9}	10^{6}	10^{3}	10^{0}	10^{-3}	10^{-6}	10^{-9}	10^{-12}
distance (ℓ_P)	10^{1}	10^{4}	10^{7}	10^{10}	10^{13}	10^{16}	10^{19}	10^{22}	10^{25}	10^{28}	10^{31}

ℓ_P $\qquad$ L_{exp} $\qquad$ 1mm

???? ??? ?? ? $\qquad$ Quantum Mechanics $\longrightarrow$

Such a presumptuous extrapolation beyond the physical evidence would be justifiable if it yielded a testable prediction, as for example the extrapolation of Grand Unification gave the conditional prediction of proton decay. But it is implausible that any proposed theory of Quantum Gravity can be checked experimentally given that the smallest distance presently accessible to experiment is $10^{16}\ell_P$. There is no practical possibility of checking whether any proposed theory of Quantum Gravity actually describes the real world. Without the possibility of an experimental test, except in fantasy, any such extreme extrapolation beyond the experimental evidence is unlikely to be useful for fundamental physics.

1.2 Against mathematical idealizations

Formal structures are used in physics for practical purposes, not as ideal mathematical forms. A quantum field theory is used as an *effective* theory describing physics at distances greater than some UV cutoff at the short distance limit of the evidence. An effective QFT says nothing about distances smaller than the UV cutoff. It does not even suppose the existence of a space-time continuum. Continuum QFT is a mathematical idealization which extrapolates far beyond the practical use of the formalism.

Likewise, an S-matrix is used as an *effective* theory that describes physics at distances smaller than the scattering region. The asymptotic S-matrix is a mathematical idealization which supposes ingoing scattering states produced infinitely early in time and infinitely far from the scattering region and outgoing scattering states detected infinitely later in time and infinitely far away. Actual scattering experiments take place within a finite region of space over a finite period of time. Again, the idealized asymptotic S-matrix extrapolates far beyond the practical use of the formalism.

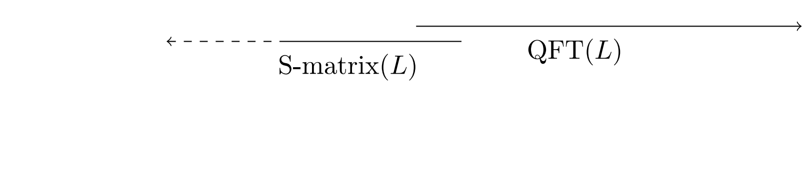
These mathematical idealizations serve mathematical purposes. But the practical limits of physical knowledge are encoded in the *effective* QFT with a UV cutoff and the *effective* S-matrix with an IR cutoff.

2 A minimal practical formal structure

2.1 An effective QFT for distances $>L$ and an effective S-matrix for distances $<L$, for observers at every distance scale $L \gg \ell_P$

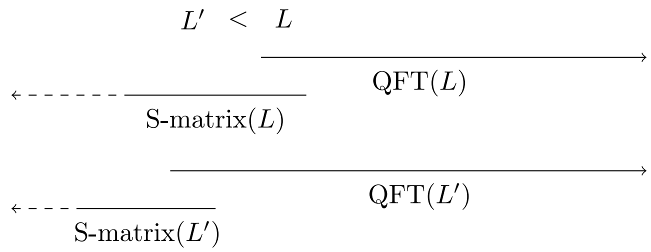
A leading edge high energy experiment of size L probes for new physics at distances $<L$. The observer has in hand an effective QFT for distances $\gtrsim L$. Short distance physics is probed by sending things in and measuring what comes out. Measurements are expressed as scattering amplitudes between states of the effective QFT. There is only an effective S-matrix with IR cutoff L for short distance physics.

Prudence and practicality suggest a formalism that corresponds with what is observable. For every $L \gg \ell_P$ there should be an effective QFT(L) with UV cutoff L and an effective S-matrix(L) with IR cutoff L. The meaning of "short distance physics" depends on the scale L of the observer. L is a sliding distance scale. The condition $L \gg \ell_P$ expresses the impracticality of experimenting anywhere near the Planck scale.



2.2 QFT renormalization group operates from smaller distance L to larger; S-matrix renormalization group operates from larger L to smaller

The descriptions of physics must be consistent as progress pushes to shorter distance L'.



C1 QFT(L) must derive from QFT(L') by the QFT renormalization group.

C2 S-matrix(L) must agree with the scattering amplitudes derived from QFT(L') at intermediate distances between L' and L.

C3 S-matrix(L') must derive from S-matrix(L) by the "S-matrix renormalization group".

The S-matrix RG is the operation on effective S-matrices that takes an effective S-matrix with IR cutoff distance L to an effective S-matrix with smaller IR cutoff distance L' by using the scattering states at scale L to make the scattering states at the smaller scale L'. The S-matrix RG and the QFT RG operate in opposite directions on the distance scale. The ideas of the effective S-matrix and the S-matrix RG are illustrated by the constructions in section 3.3 and section 3.5 below. Technical definitions remain to be formulated.

2.3 An S-matrix does not imply a hamiltonian

The formal structure QFT(L) + S-matrix(L) is local in L. An observer at scale L makes only a modest extrapolation by supposing there will be a somewhat more fundamental effective QFT(L') at somewhat smaller distance L'. There is no presumption of QFT or any quantum mechanical hamiltonian all the way down to ℓ_P. An S-matrix does not necessarily come from a microscopic hamiltonian. An S-matrix can be derived from a microscopic hamiltonian, but not vice versa.

Nor does an S-matrix imply an effective QFT in the IR, even if the IR limit of the S-matrix matches the scattering amplitudes of the effective QFT. Such a coincidence only means that the effective QFT is consistent with the S-matrix. There is still need for a mechanism that actually *produces* the effective QFT. One possible mechanism is the RG acting on a microscopic QFT or other microscopic hamiltonian system. But there could be mechanisms for producing effective QFT that do not depend on such an extrapolation.

3 A mechanism that produces such a formal structure

A search for a mechanism that would produce a realistic QFT began with [1, 2]. A mechanism was finally proposed in [3]. The mechanism produces a formal structure such as described above. The line of thought is sketched in the Appendix.

3.1 Summary

1. String theory provides a way to construct a self-consistent S-matrix for short distance physics without using a microscopic QFT.

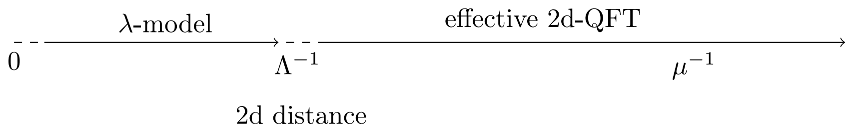
2. When the string worldsheet is an effective 2d-QFT with 2d UV cutoff distance Λ^{-1}, the string S-matrix is an effective S-matrix(L) with IR cutoff distance L given in dimensionless units by $L^2 = \ln(\Lambda/\mu)$ where $\mu^2|dz|^2$ is the worldsheet metric. The condition $L \gg 1$ is the requirement $\Lambda^{-1} \ll \mu^{-1}$, i.e., the requirement that the 2d cutoff distance Λ^{-1} be insignificant at the 2d distance scale μ^{-1} of the S-matrix calculation.

3. The string background is encoded in the local worldsheet physics at 2d distance Λ^{-1}.

4. The S-matrix RG acts on S-matrix(L) by integrating out the froth of small handles in the worldsheet, increasing the 2d UV cutoff Λ^{-1}, decreasing the IR cutoff L.

5. The effects of the froth of small handles are replicated by a certain 2d nonlinear model (2d-NLM) called the *λ-model*. The λ-model is mathematically natural. The target manifold is the space of effective 2d-QFTs of the worldsheet, parametrized by the effective 2d coupling constants $\lambda^i(\Lambda)$ at 2d scale Λ^{-1}. These $\lambda^i(\Lambda)$ are the modes of the classical background space-time fields with UV cutoff L. The target manifold of the λ-model is thus the space of classical space-time fields with UV cutoff L.

6. The froth of small handles is replaced by the λ-fluctuations at 2d distances $< \Lambda^{-1}$, which produce an effective worldsheet QFT with 2d UV cutoff Λ^{-1}.



7. Integrating out the λ-fluctuations has the same effect as integrating out the froth of small handles so the 2d-RG of the λ-model implements the S-matrix RG.

8. The 2d-RG of the λ-model also produces a measure on the target manifold. This is the *a priori* measure of the 2d-NLM. A measure on the target manifold of the λ-model is a functional integral over the space-time fields with UV cutoff L, i.e., an effective quantum field theory QFT(L).

9. The quantum states of QFT(L) are the quantum string backgrounds.

10. QFT(L) and S-matrix(L) automatically satisfy the consistency conditions **C1**, **C2**, **C3**. The λ-model produces a consistent realization of the minimal practical formal structure described above.

11. The effective QFT(L) is produced by a 2d mechanism that does not necessarily correspond to canonical quantization (except perturbatively). There are concrete possibilities of nonperturbative semi-classical 2d effects which could produce non-canonical degrees of freedom and non-canonical interactions in QFT(L). These 2d effects are the 2d winding modes and 2d instantons coming from nontrivial homotopy groups π_1 and π_2 of the space of space-time fields which is the target manifold of the λ-model. These homotopy groups are nontrivial when the space-time fields include SU(2) or SU(3) gauge fields in four space-time dimensions.

The last point is the main reason for investigating the λ-model as a formalism for fundamental physics. There are possibilities of conditional predictions of the form *if QFT(L) contains SM+GR, then it predicts certain specific non-canonical degrees of freedom and interactions beyond those of the canonically quantized effective quantum field theory.*

3.2 2d-QFT of the string worldsheet

In the general renormalizable 2d nonlinear model

$$\int e^{-\int d^2z\, g_{\mu\nu}(X)\partial X^\mu \bar{\partial} X^\nu} \mathcal{D}X \qquad X(z) \in M \tag{1}$$

the field $X(z)$ takes values in a target manifold M. The 2d coupling constants are given by a Riemannian metric $g_{\mu\nu}(X)$ on M. The manifold M is taken compact and $g_{\mu\nu}(X)$ euclidean signature so that the 2d-QFT will be well defined. The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda} g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2) \tag{2}$$

drives the 2d-NLM to a solution of $R_{\mu\nu} = 0$.

The 2d-QFT of the string worldsheet is an elaboration of the general 2d-NLM in which the target manifold M is space-time and the 2d coupling constants consist of the space-time metric $g_{\mu\nu}(X)$ and also some non-abelian gauge fields, scalar fields, fermion fields, etc. on the space-time M. The equation $\beta = 0$ generalizing $R_{\mu\nu} = 0$ is a semi-realistic supersymmetric classical field equation which includes GR and potentially the SM.

In abstract language,

$$\begin{aligned}
\lambda^i &= \text{the 2d coupling constants,} \\
\phi_i(z) &= \text{the corresponding spin-0 scaling fields of the 2d-QFT,} \\
|\phi_i\rangle &= \text{the corresponding radial quantization states on the unit circle in 2d,} \\
G_{ij} &= \text{the natural metric } \langle \phi_i | \phi_j \rangle.
\end{aligned}$$

The index i labels the modes of the space-time fields. For example, the modes of the space-time metric and the corresponding 2d fields are

$$\delta_i g_{\mu\nu}(X) = e^{ip_\mu(i)X^\mu} h_{\mu\nu}(i) \qquad \phi_i(z) = e^{ip_\mu(i)X^\mu(z)} h_{\mu\nu}(i)\, \partial X^\mu \bar{\partial} X^\nu \tag{3}$$

The λ^i form a system of local coordinates on the space of 2d-QFTs. The nearby 2d-QFTs are given by inserting in the worldsheet

$$e^{\int d^2z\, \lambda^i \phi_i(z)} \tag{4}$$

The 2d scaling-dimensions are

$$\dim(\phi_i) = 2 + \delta(i) \qquad \dim(\lambda^i) = -\delta(i) \qquad \delta(i) = p(i)^2 \tag{5}$$

The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda} \lambda^i = \beta^i(\lambda) \qquad \beta^i(\lambda) = -\delta(i)\lambda^i + O(\lambda^2) \tag{6}$$

drives the worldsheet 2d-QFT towards the $\beta = 0$ submanifold which is parametrized by the marginal coupling constants

$$\dim(\lambda^i) = -\delta(i) = -p(i)^2 = 0 \tag{7}$$

which correspond to the zero-modes of the space-time fields. There are no relevant operators, no λ^i with $\delta(i) < 0$. (There are no tachyons in the string S-matrix.) The $\beta = 0$ submanifold is stable under the 2d-RG. There are no unstable directions.

3.3 Effective string S-matrix with IR cutoff L

Let $\mu^2|dz|^2$ be the worldsheet metric. Impose a 2d UV cutoff $\Lambda^{-1} \ll \mu^{-1}$. The cutoff string propagator (the cutoff integral over 2d-cylinders) is

$$\int_0^{\ln(\Lambda/\mu)} \left(\sum_{i,j} |\phi_i\rangle\, G^{ij} e^{-\tau\delta(i)} \langle\phi_j| \right) d\tau = \sum_{i,j} |\phi_i\rangle \, \frac{1-e^{-L^2\delta(i)}}{\delta(i)} \, G^{ij} \, \langle\phi_j| \tag{8}$$

where

$$L^2 = \ln(\Lambda/\mu) \qquad e^{-L^2\delta(i)} = (\Lambda/\mu)^{-\delta(i)} \tag{9}$$

The only modes that propagate are those that satisfy

$$\delta(i) > L^{-2} \qquad \text{which is} \qquad p(i)^2 > L^{-2} \tag{10}$$

so the 2d UV cutoff Λ^{-1} puts an IR cutoff L on the string S-matrix. An effective 2d-QFT of the worldsheet gives an effective string S-matrix(L) with L given by $L^2 = \ln(\Lambda/\mu)$.

3.4 Effective 2d coupling constants $\lambda^i(\Lambda)$

The effects of the 2d coupling constants $\lambda^i(\Lambda)$ at 2d scale Λ^{-1} are suppressed by the 2d-RG running from Λ^{-1} up to μ^{-1}

$$\lambda^i(\mu) = (\Lambda/\mu)^{-\delta(i)}\, \lambda^i(\Lambda) = e^{-L^2\delta(i)} \lambda^i(\Lambda) \tag{11}$$

If $L^2\delta(i) > 1$ then $\lambda^i(\Lambda)$ is effectively irrelevant; its effects on the worldsheet are negligible. The only $\lambda^i(\Lambda)$ that matter are the effectively marginal couplings

$$\delta(i) < L^{-2} \qquad \text{which is} \qquad p(i)^2 < L^{-2} \tag{12}$$

so there is a UV cutoff distance L on the modes of the space-time fields that are the coupling constants of the effective 2d-QFT of the worldsheet.

The 2d UV cutoff Λ^{-1} separates the 2d coupling constants λ^i into two subsets. The λ^i with $\delta(i) > L^{-2}$ are effectively irrelevant. The corresponding $\phi_i(z)$ are the vertex operators for the propagating modes in the effective string S-matrix. The λ^i with $\delta(i) < L^{-2}$ are the effectively marginal coupling constants. These are not exact solutions of $\beta = 0$. They are the off-shell classical string backgrounds at distances $> L$. The off-shell classical backgrounds are prerequisites for quantum backgrounds at scales $> L$.

3.5 Implement the S-matrix renormalization group

Consider the effect of a small handle in the worldsheet. A small handle is made by identifying the boundaries of two holes of radius r around two points z_1 and z_2 which are close together in the worldsheet. The identification is

$$z \leftrightarrow z' \qquad (z - z_1)(z' - z_2) = r^2 e^{i\theta} \tag{13}$$

Insert a sum over radial quantization states on each boundary circle. Integrate over the moduli z_1, z_2, r, θ. The integral over θ projects on the spin-0 states. The effect of the small handle becomes the bi-local insertion

$$\frac{1}{2} \sum_{i_1,i_2} \int d^2z_1\, \phi_{i_1}(z_1) \int d^2z_2\, \phi_{i_2}(z_2) \int_{\Lambda^{-1}}^{\frac{1}{2}|z_1-z_2|} dr\; r^{-1-\delta(i_1)-\delta(i_2)}\; g_{\mathrm{str}}^2 G^{i_1 i_2} \tag{14}$$

where $G_{i_1 i_2}$ is the natural metric on the space of 2d-QFTs and g_{str} is the string coupling constant. The integration region of interest here is $\Lambda^{-1} < |z_1 - z_2| \ll \mu^{-1}$. This is where the small handle contributes to the local worldsheet physics.

The cutoff dependent contribution comes from the effectively marginal fields

$$\frac{1}{2} \sum_{\delta(i_1)\sim 0} \sum_{\delta(i_2)\sim 0} \int d^2 z_1 \, \phi_{i_1}(z_1) \int d^2 z_2 \, \phi_{i_2}(z_2) \; g_{\mathrm{str}}^2 G^{i_1 i_2} \ln \left(\Lambda |z_1 - z_2|\right) \tag{15}$$

The cutoff dependence of the small handle expressed by (15) can be canceled by letting the effectively marginal 2d coupling constants λ^i fluctuate locally on the worldsheet. Make the λ^i into sources $\lambda^i(z)$ so the worldsheet insertion becomes

$$e^{\int d^2 z \, \lambda^i(z) \phi_i(z)} \tag{16}$$

Then set the $\lambda^i(z)$ fluctuating with 2-point correlation function

$$\langle \, \lambda^{i_1}(z_1) \; \lambda^{i_2}(z_2) \, \rangle = -g_{\mathrm{str}}^2 G^{i_1 i_2} \ln \left(\Lambda |z_1 - z_2|\right) \tag{17}$$

The cancellation of the single small handle (15) by the gaussian λ-fluctuations (17) holds in every background 2d-QFT. Therefore the cutoff dependence of the entire nongaussian froth of small handles is canceled by λ-fluctuations governed by the 2d-NLM

$$\int e^{-\int d^2 z \; g_{\mathrm{str}}^{-2} G_{ij}(\lambda) \partial \lambda^i \bar{\partial} \lambda^j} \; e^{\int d^2 z \, \lambda^i(z) \phi_i(z)} \; \mathcal{D}\lambda \qquad \lambda(z) \in \mathcal{M} \tag{18}$$

This 2d-NLM is the λ-model. The target manifold is

$$\begin{aligned} \mathcal{M} &= \text{the manifold of effective worldsheet 2d-QFTs with 2d cutoff } \Lambda^{-1} \\ &= \text{the space of classical space-time fields with UV cutoff } L \text{ given by } L^2 = \ln(\Lambda/\mu) \end{aligned}$$

The sum over handles does not depend on the arbitrary choice of Λ^{-1} so the sum over λ-fluctuations at 2d distances $< \Lambda^{-1}$ has the same effect as the sum over small handles, since either of these sums cancels the Λ^{-1} dependence of the sum over handles at 2d distances $> \Lambda^{-1}$. Therefore integrating out the λ-fluctuations at 2d distances $< \Lambda^{-1}$ is equivalent to integrating out the froth of small handles. Integrating out the λ-fluctuations at 2d distances $< \Lambda^{-1}$ produces an effective 2d-QFT with UV cutoff Λ^{-1} which gives an effective string S-matrix(L) with IR cutoff L.

$$0 \xrightarrow{\lambda\text{-model}} \Lambda^{-1} \xrightarrow{\text{effective 2d-QFT}} \mu^{-1}$$

2d distance

The 2d-RG of the λ-model operates from smaller 2d distance Λ'^{-1} up to larger 2d distance Λ^{-1} by integrating out the λ-fluctuations at 2d distances between Λ'^{-1} and Λ^{-1}. This is equivalent to integrating out the small handles, taking the effective S-matrix(L') with larger IR cutoff L' to the effective S-matrix(L) with smaller IR cutoff L. Thus the 2d-RG of the λ-model implements the S-matrix RG.

The "froth of small handles" has only a perturbative meaning in string theory. The λ-model replicates the perturbative froth and is a nonperturbatively well defined 2d-NLM. So the λ-model gives nonperturbative meaning to the froth of small handles.

3.6 Production of an effective QFT with UV cutoff L

Like any 2d-NLM, the λ-model is specified by two pieces of data

- the metric $g_{\mathrm{str}}^{-2} G_{ij}(\lambda)$ on the target manifold $\mathcal{M}$
- the *a priori* measure $\rho(\lambda)d\lambda$ on the target manifold $\mathcal{M}$ from which comes the functional volume element in the functional integral (18)

$$\mathcal{D}\lambda = \prod_z \rho(\lambda(z))\, d\lambda(z) \tag{19}$$

A point z in the effective worldsheet represents a 2d block of dimensions $\Lambda^{-1} \times \Lambda^{-1}$. The measure $\rho(\lambda(z))\, d\lambda(z)$ summarizes the λ-fluctuations inside the block that have been integrated out, hence *a priori* meaning *from the earlier* or *from what has gone before.*

The *a priori* measure $\rho(\lambda)d\lambda$ evolves under the 2d-RG. It diffuses in $\mathcal{M}$ because of the λ-fluctuations. At the same time the λ^i are flowing along $\beta^i(\lambda)$ towards the $\beta = 0$ submanifold. So $\rho(\lambda)d\lambda$ evolves under a driven diffusion process. Taking $d\lambda$ to be the metric volume element, so $\rho(\lambda)$ is a function on $\mathcal{M}$, the driven diffusion equation is

$$\Lambda \frac{\partial}{\partial \Lambda}\, \rho(\lambda) = \nabla_i \left(g_{\mathrm{str}}^2 G^{ij} \partial_j + \beta^i \right) \rho(\lambda) \tag{20}$$

Integrating out the λ-fluctuations up to Λ^{-1} drives $\rho(\lambda)d\lambda$ to the equilibrium measure

$$\rho(\lambda)d\lambda \to e^{-g_{\mathrm{str}}^{-2} S(\lambda)} d\lambda \qquad \text{where} \quad \beta^i = G^{ij}\partial_j S$$

The λ^i are the space-time field modes with UV cutoff L so the *a priori* measure $\rho(\lambda)d\lambda$ is the functional integral of an effective QFT(L) with classical action $g_{\mathrm{str}}^{-2} S(\lambda)$.

The λ-model produces an effective 2d-QFT and an effective *a priori* measure $\rho(\lambda)d\lambda$ at every 2d distance $\Lambda^{-1} \ll \mu^{-1}$. Thus the λ-model produces an effective S-matrix(L) and an effective QFT(L) at every distance $L \gg 1$. The consistency conditions **C1, C2, C3** are automatically satisfied. The S-matrix RG acts on S-matrix(L) by design. The QFT RG acts on QFT(L) because of the decoupling of the effectively irrelevant $\lambda^i(\Lambda)$ in the 2d-RG. The agreement between S-matrix(L) and QFT(L) on scattering amplitudes at scales $\sim L$ is guaranteed because the scattering amplitudes of S-matrix(L) near the IR cutoff L are given by the 2d correlation functions of vertex operators near the 2d UV cutoff Λ^{-1} which are determined by the *a priori* measure $\rho(\lambda)d\lambda$ which is QFT(L).

3.7 Possible non-canonical degrees of freedom and couplings in SU(2) and SU(3) quantum gauge theory

The λ-model is a nonperturbative 2d-NLM with possibilities of nonperturbative semi-classical 2d effects [4]. These would be 2d winding modes associated to $\pi_1(\mathcal{M})$ and 2d instantons associated to $\pi_2(\mathcal{M})$ where the target manifold $\mathcal{M}$ is the manifold of space-time fields. Suppose the λ-model produces an effective QFT(L) with SU(N) gauge fields in four macroscopic space-time dimensions. Then the mathematical results

$$\begin{gathered} \pi_k \text{ of the manifold of } \mathrm{SU}(N) \text{ gauge fields on } \mathbb{R}^4 \cup \{\infty\} = \pi_{k+3}(\mathrm{SU}(N)) \\ \pi_4(\mathrm{SU}(2)) = \mathbb{Z}_2 \qquad \pi_5(\mathrm{SU}(2)) = \mathbb{Z}_2 \qquad \pi_4(\mathrm{SU}(3)) = 0 \qquad \pi_5(\mathrm{SU}(3)) = \mathbb{Z} \end{gathered} \tag{21}$$

imply that there are 2d winding modes when the gauge group is SU(2) and 2d-instantons when the gauge group is SU(2) or SU(3). These nonperturbative semi-classical effects in the λ-model offer possibilities of conditional predictions of the form *if the λ-model produces SM+GR then it also produces non-canonical degrees of freedom from the $\mathbb{Z}_2$ winding mode in the SU(2) gauge fields and non-canonical interactions from the 2d instantons in the SU(2) and SU(3) gauge fields.*

3.8 2d winding modes and 2d instantons

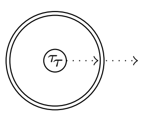
Let the gauge field $A_{+-}(u,0)$ be an SU(2) instanton-anti-instanton configuration on $\mathbb{R}^4$ in the limit where one or both of the instanton sizes goes to zero. The parameter u is the relative orientation. It lies in the adjoint representation $\mathrm{SU}(2)/\mathbb{Z}_2$. The remaining moduli of the instanton-anti-instanton pair are left implicit. $A_{+-}(u,0)$ is a solution of the Yang-Mills equation (which is the 2d-RG equation $\beta = 0$). The nontrivial element of $\pi_1 = \mathbb{Z}_2$ is the nontrivial closed loop in $\mathrm{SU}(2)/\mathbb{Z}_2$. Blowing up the zero-size instanton to a small size ρ gives a four-parameter family of gauge fields $A_{+-}(u,\rho)$ which is an $\mathbb{R}^4/\mathbb{Z}_2$ orbifold. The 2d winding mode is the $\mathbb{Z}_2$ twist field of this orbifold. The moduli of the $\mathbb{Z}_2$ twist field are the remaining moduli of the instanton-anti-instanton pair (including their fermionic zero-modes).

The 2d instantons in the SU(2) and SU(3) gauge fields on $\mathbb{R}^4$ are also found in zero-size instanton-anti-instanton configurations. For SU(3) the relative orientation of a zero-size instanton-anti-instanton pair is parametrized by $\mathrm{SU}(3)/\mathrm{U}(1)$ which has a topologically nontrivial 2-sphere. For SU(2) the minimal nontrivial 2-sphere is in the zero-size configurations of two instantons and two anti-instantons.

3.9 Vacuum condensate of SU(2) Yang-Mills flow defects

Suppose the target manifold includes SU(2) gauge fields on $\mathbb{R}^4$. Suppose a portion of the *a priori* measure falls in the $\mathbb{Z}_2$-odd sector. Then the λ-model will contain a gas of $\mathbb{Z}_2$ twist fields. Each $\mathbb{Z}_2$ twist field will pin the worldsheet to an orbifold point, a zero-size instanton-anti-instanton pair. But any $\mathbb{Z}_2$-even cluster of twist fields will be unstable, perched high up at twice the instanton action without topological protection against being pushed by the 2d-RG down to the flat SU(2) gauge field. The 2d-RG acts on the SU(2) gauge fields as the Yang-Mills flow — the gradient flow of the Yang-Mills action. For each orbifold fixed point $A_{+-}(\mathbf{1},0)$ in which the instanton and anti-instanton are aligned, there is a unique downward trajectory $A_{\mathrm{YMF}}(t)$ that starts from $A_{+-}(\mathbf{1},0)$ at $t=-\infty$ and ends at $t=+\infty$ at the flat gauge field. The instanton and anti-instanton grow together and annihilate along the downward trajectory.

A $\mathbb{Z}_2$-even cluster of twist fields will appear in the worldsheet as a Yang-Mills flow defect operator



The core is the $\mathbb{Z}_2$-even cluster of twist fields. The inner dotted arrow is the slow flow in the $\beta \approx 0$ region near the orbifold fixed point. The fast part of the flow is between the solid circles. The outer dotted arrow is the slow flow near the flat gauge field. The 2d gas of these defects will produce a vacuum condensate in the effective $\mathrm{QFT}(L)$. The downward Yang-Mills trajectory will determine the properties of the condensate. The initial and final stages of the trajectory can be controlled by perturbing around $\beta = 0$. The intermediate stage can at least be calculated numerically.

4 To do

Most urgent is to determine if the λ-model does in fact make conditional predictions of observable non-canonical effects in SU(2) and SU(3) gauge theory in 4 dimensions. This requires figuring out how to calculate semi-classical corrections to the *a priori* measure of the λ-model coming from the 2d winding modes and instantons. Most promising would seem to be the vacuum condensate of $\mathbb{Z}_2$ winding modes for SU(2) gauge fields sketched in section 3.9 above. If such conditional predictions can be made and checked, then there will be compelling motivation for further investigation of the λ-model. There are many basic questions to investigate. Most of the technical foundation remains to be built. But the effort might not be worthwhile unless and until there is a successful conditional prediction.

The λ-model operates at 2d distances up to Λ^{-1}. The space-time distance scale L is given by $L^2 = \ln(\Lambda/\mu)$. So the λ-model builds QFT(L) from the largest distances *down* to L (nevertheless ensuring that the QFT RG is satisfied). This top-down construction of effective QFT might have useful consequences concerning naturalness or its lack.

The *a priori* measure $\rho(\lambda)d\lambda$ is a measure on the target manifold which is the space of effective 2d-QFTs of the string worldsheet. In the extreme limit $L \to \infty$, $\Lambda^{-1} \to 0$, only the exactly marginal 2d couplings λ^i fluctuate. The target manifold at $L = \infty$, $\Lambda^{-1} = 0$ is the space of worldsheet 2d conformal field theories which give idealized asymptotic S-matrices. The λ-model dynamically produces a measure $\rho(\lambda)d\lambda$ on this space of idealized classical backgrounds. Only if some portion of the measure concentrates near a background with large space-time dimensions will $\rho(\lambda)d\lambda$ take the form of the functional integral of an effective QFT(L) at $L < \infty$. Exploring the dynamics of such concentration and the variety of places the measure might concentrate is an intimidating task. More practical is to suppose that some part of the measure does concentrate near a large 4d space-time with space-time fields that include those of the SM+GR, then try to make predictions conditional on this assumption. If these predictions can be checked against experiment, then it might be worth asking if the λ-model does in fact cause some portion of $\rho(\lambda)d\lambda$ to concentrate at backgrounds with macroscopic 4d space-times.

Wick rotation is an after-thought. Space-time euclidean signature is assumed so that the 2d-NLM of the worldsheet will be a well defined effective 2d-QFT. Then only finitely many modes $\lambda^i(\Lambda)$ of the space-time fields fluctuate and their fluctuations are governed by a positive definite metric $G_{ij}(\lambda)$. So the λ-model is an effective mechanism. Wick rotation is to be carried out *ad hoc* in the effective QFT(L) and in the effective S-matrix(L), if and when the *a priori* measure concentrates at a macroscopic space-time. There is no hint of an explanation of Wick rotation in this machinery.

Cosmology might be done by relating the distance scale L to the time scale of the cosmological observer, the early universe being described by an outgoing scattering state in S-matrix(L) and the later universe by a state in QFT(L).

Appendix. Notes on the line of thought

A.1 Search for a mechanism that produces QFT

The ideas expressed in this note were developed mainly during the period 1977–2002 in the process of searching for and eventually formulating a mechanism that would produce quantum field theory.

The 2d-RG as a mechanism for space-time physics (1977–79)

The line of thought began with the renormalization of the general 2d nonlinear model

$$\int e^{-\int d^2z\, g_{\mu\nu}(X)\partial X^\mu \bar{\partial} X^\nu} \mathcal{D}X \qquad X(z) \in M \tag{22}$$

where $g_{\mu\nu}(X)$ is a Riemannian metric on a manifold M. The 2d-RG

$$\Lambda \frac{\partial}{\partial \Lambda} g_{\mu\nu}(X) = -R_{\mu\nu}(X) + O(R^2) \tag{23}$$

drives the 2d-NLM to a solution of $R_{\mu\nu} = 0$. This was very exciting (at least for me). The 2d-RG appeared as a *mechanism* that *produces* solutions of a GR-like space-time field equation $R_{\mu\nu} = 0$. This suggested the possibility that a mechanism — the 2d-RG — might actually be the answer to questions like *where does space-time field theory come from?* or even *where do the laws of physics come from?* The goal became a mechanism that actually produces the laws of physics (instead of the goal of fundamental principles such as symmetry to constrain the laws of physics).

It had become clear by the late 1970s that there are far too many effective QFTs. A mechanism was needed that would produce effective QFT more selectively than the QFT renormalization group. The 2d-RG seemed promising for the purpose since it at least produced solutions of classical field theory. The goal became a mechanism that produces *quantum* field theory and that has the 2d-RG as its classical limit.

The general 2d-NLM had two other shortcomings as a mechanism for producing space-time physics. First, the solutions of $\beta = 0$, the 2d-RG fixed points, have unstable directions along which the RG flow diverges from the fixed point rather than converges. Second, the $\beta = 0$ equation $R_{\mu\nu} = 0$ is not quite Einstein's equation.

The 2d-RG incorporated into string theory (1981–85)

In the early 1980s it was realized that the 2d-RG fixed point equation $\beta = 0$ (i.e., 2d scale invariance) is a consistency condition for calculating the string S-matrix from a worldsheet 2d-QFT. The string worldsheet is a supersymmetric 2d-QFT containing additional 2d degrees of freedom besides $X(z)$. The 2d coupling constants are, in addition to the space-time metric $g_{\mu\nu}(X)$, a collection of non-abelian gauge fields, scalar fields, fermion fields, etc. on space-time. The string worldsheet resolved the two shortcomings of the basic 2d-NLM. First, the 2d supersymmetry of the string worldsheet eliminates the unstable directions at the fixed points (the tachyons in the S-matrix). Second, the worldsheet $\beta = 0$ equation generalizing $R_{\mu\nu} = 0$ is a semi-realistic supersymmetric classical field equation that includes GR and potentially the SM.

In the mid-1980s, several assumptions and mathematical idealizations became truisms:

1. The string S-matrix was taken to be an asymptotic S-matrix without IR cutoff — a "theory of everything".

2. The string backgrounds were taken to be the conformally invariant worldsheet 2d-QFTs from which such asymptotic string S-matrices are derived — the exact worldsheet solutions of $\beta = 0$ given by Calabi-Yau manifolds ($R_{\mu\nu} = 0$) and generalizations.

3. It was *assumed* that the IR physics of string theory is the supersymmetric QFT that happens to have the same low momentum scattering amplitudes as the asymptotic string S-matrix. The string backgrounds were conflated with those supersymmetric QFTs.

4. It was assumed that there must exist a microscopic QFT or some other kind of microscopic mechanical hamiltonian system from which the string S-matrix is derived.

Questions (1987)

Circa 1987, the key questions seemed to me to be:

1. How does the 2d-RG act in string theory as a *mechanism*? The fixed point equation $\beta = 0$ is only a consistency condition for the string S-matrix recipe.

2. Where does *quantum* field theory come from in string theory? What produces a functional integral over space-time fields?

3. What is the *quantum* string background? It should be a quantum state in a QFT.

The λ-model (1988-2002)

The attempt to answer these questions was a long-drawn-out process. One seed was the idea that the string background is encoded in the local 2d physics of the worldsheet. Another seed was the vague notion that nonperturbative effects in string theory might come from infinite genus worldsheets. Eventually, these were combined in the idea that a froth of small handles would contribute to the local 2d physics of the worldsheet and thus to the string background. This motivated the calculation of the log divergence in the contribution of a single small handle which took the form of a bi-local insertion in the worldsheet (as in section 3.5 above). This log divergence was a strong signal. The infrared log divergence of the scalar field 2-point function plays a fundamental role in 2d-QFT. Thus the idea of setting the 2d coupling constants fluctuating as 2d scalar fields $\lambda^i(z)$ with the 2d-IR log divergence of the scalar field fluctuations cancelling the 2d UV log divergences of the small handles.

The essential role of a 2d distance scale Λ^{-1} as 2d UV cutoff in the string worldsheet and as 2d IR cutoff on the λ-fluctuations required abandoning the idealized asymptotic string S-matrix "of everything" for an effective string S-matrix with IR cutoff L (as in section 3.3 above). Integrating out the froth of small handles became the S-matrix RG.

Recognizing that the *a priori* measure $\rho(\lambda)d\lambda$ of the 2d-NLM would govern the local worldsheet physics led to identifying $\rho(\lambda)d\lambda$ as the functional integral of the effective space-time QFT that is the quantum string background (as in sections 3.4 and 3.6 above).

At this point the task became to identify semi-classical 2d effects in the λ-model that might lead to checkable predictions.

A.2 Pragmatism and the S-matrix philosophy

The S-matrix has been proposed as a formal structure for fundamental physics at several points in history when QFT has seemed to hit a wall. The history is recounted in [5]. Heisenberg first proposed using the S-matrix as a fundamental formalism in the 1940s in response to the divergences of perturbative QED and the difficulty of accounting for cosmic ray showers. In the 1960s the S-matrix bootstrap program was proposed in response to the plethora of mesons and baryons and their strong couplings. On both occasions QFT overcame its difficulties and the S-matrix proposals lapsed. The third occasion was the string S-matrix proposal of the early 1970s which attracted interest at least in part because of the incompatibility between GR and QFT when extrapolated down to the Planck length.

Heisenberg's explicit rationale for using the S-matrix was the principle that fundamental physics should be expressed in terms of what is actually observed. This principle has had notable successes in fundamental theoretical physics. For example, the route of Bohr and Heisenberg to Matrix Mechanics was guided by focussing on observable transitions. But the principle was not followed literally. Matrix Mechanics in the end described the world by quantum states and transition amplitudes which are not themselves observable. Only their absolute squares are observable. The strategy of focussing on what is observable led to a formalism, Matrix Mechanics, that reliably produces observable quantities. A pragmatic version of the principle might be *use the minimal formal machinery that is useful to produce the observable quantities of physics.* Quantum Mechanics in the form of QFT is so successful at producing observable quantities that it can be considered to be "what is observable" at distances greater than about $(10^3\mathrm{GeV})^{-1}$.

The S-matrix philosophy proposed replacing Quantum Mechanics and QFT with an asymptotic S-matrix. But the asymptotic S-matrix is an extreme idealization of what is observable. All the useful work of physics at distances larger than the elementary particle frontier uses Quantum Mechanics or its effective approximation Classical Mechanics. Is it feasible, for example, to describe the behavior of a galaxy in terms of an S-matrix? If fundamental physics is formulated as an S-matrix, what produces effective QFT and effective Quantum Mechanics and effective Classical Mechanics?

On the other hand, a pragmatic version of the S-matrix philosophy does seem reasonable. Scattering amplitudes describe what is observable at short distances where 'short' is relative to the size of the observer. An effective S-matrix with an IR cutoff L is a practical formulation of what we can actually observe at distances smaller than the limit of our best hamiltonian quantum mechanical model.

From this point of view string theory is interesting not because it offers an S-matrix "theory of everything" but rather because it is a way to construct a self-consistent S-matrix for short distance physics without requiring a short distance QFT. A short distance S-matrix that does not require a short distance QFT is entirely suitable in a pragmatic version of the S-matrix philosophy. The only way to construct an S-matrix before string theory was to derive it from a QFT. But that does not mean that every S-matrix and in particular the string S-matrix must be derived from a microscopic QFT.

The pragmatist philospher C. S. Peirce (a contemporary of Ernst Mach) proposed that the symbolic tools of science take their significance from the work that they do. (This might well be a selective, idiosyncratic reading of Peirce.) A pragmatic strategy is to shape the formalism of fundamental physics for the work it needs to perform. The pragmatic view argues against pursuit of mathematical beauty, against pursuit of beautiful fundamental principles, against attempting to extrapolate to an absolutely fundamental theory based on absolutely fundamental principles. A successful fundamental theory may eventually be based on beautiful principles and formulated in beautiful mathematics. But there is no telling how far away that is or in what direction. There is no telling in advance which mathematically beautiful forms will prove useful for fundamental physics. Meanwhile, a practical strategy is to try to make incremental improvements in the formalism of fundamental physics that can actually do useful work in describing the fundamental physics of the real world.

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References

[1] D. Friedan, "Nonlinear Models in $2 + \epsilon$ Dimensions," *Phys. Rev. Lett.* **45** (1980) 1057.

[2] D. Friedan, *Nonlinear Models in* $2 + \epsilon$ *Dimensions.* PhD thesis, U.C. Berkeley, 1980. Published as Lawrence Berkeley Laboratory Report LBL-11517 (1980) and *Annals Phys.* **163** (1985) 318.

[3] D. Friedan, "A tentative theory of large distance physics," *JHEP* **10** (2003) 063, `arXiv:hep-th/0204131`.

[4] D. Friedan, "A loop of $SU(2)$ gauge fields stable under the Yang-Mills flow," in *Perspectives in mathematics and physics: Essays dedicated to Isadore Singer's 85th birthday*, T. Mrowka and S.-T. Yau, eds., vol. 15 of *Surveys in Differential Geometry*, pp. 131–204. International Press of Boston, 2010. `arXiv:1008.1189 [hep-th]`.

[5] J. T. Cushing, *Theory Construction and Selection in Modern Physics: The S Matrix.* Cambridge University Press, 1990.