

Cooper pair turbulence in atomic Fermi gases

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Abstract – We investigate under what conditions a uniform quench of a superfluid atomic Fermi gas leads to the emergence of spatial inhomogeneities. We demonstrate that, if the system is larger than the coherence length, the superfluid order parameter becomes spatially nonuniform. Spatial modulations develop through a parametric excitations of pairing modes with opposite momenta. Their growth is eventually suppressed by nonlinear effects resulting in a state characterized by a random superposition of wave packets of the superfluid order parameter. This state can be probed by measuring the molecular momentum distribution following a fast sweep to the BEC side of the Feshbach resonance.

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How does a homogeneous interacting many-body system develop spatial modulations as a result of a uniform quench? In general, this can happen due to an interplay between the extra energy introduced by the quench and an intrinsic coupling of the degrees of freedom at various length scales [1–4]. By the very nature of the problem, the energy distribution of these degrees of freedom is initially far from a state of thermodynamic equilibrium. This situation is common in a nonlinear medium and can be described by the term “wave turbulence” (see [5] and references therein). In the case when one of the parameters of the medium or an applied field is periodically varied in time, wave turbulence due to a parametric instability can develop. Examples of such a phenomenon include the decay of high-frequency electric field into Langmuir and ion-sound waves in plasma [6], the spin-wave instability in an rf-magnetic field in dielectric ferromagnets [7] and instability of coherent spin precession in superfluid ³He [8,9]. A natural question is then whether a quench can lead to a parametric excitation of spatial modes.

In this letter we address this issue for a system of fermionic atoms in a homogeneous superfluid state. The system is uniformly quenched by a change of the applied magnetic field. We demonstrate that spatial modulations develop in the course of the nonadiabatic dynamics triggered by the quench. These inhomogeneities correspond to a nonzero center of mass momentum of Cooper

pairs and can be experimentally probed by projecting the Cooper pair wave function onto molecular wave function by a fast sweep to the BEC side of the Feshbach resonance [10,11]. The subsequent measurements of the molecular momentum distribution should reveal two peaks: one at zero momentum and the other at the momentum of the spatial modulations (see below).

Previous analysis of the nonadiabatic pairing dynamics revealed two types of asymptotic states the fermionic condensate can reach depending on the strength of the initial perturbation [12,13]. The first type has a constant value $\Delta(t) = \Delta_s$, while the second one is characterized by periodic $\Delta(t)$. This analysis does not take into account the pair breaking processes and is thus valid only for $t < \tau_\varepsilon$, where τ_ε is the quasi-particle relaxation time. Moreover, the emerging asymptotic states are spatially uniform: order parameter evolution was obtained as a solution of an effectively zero-dimensional problem. Results of refs. [12,13] are thus valid when the system size is smaller than the coherence length, $L < \xi = v_F/\Delta_s$. The description of the order parameter evolution for $L > \xi$ requires an additional investigation.

In what follows we perform a stability analysis of the solutions for the wave function and order parameter obtained in refs. [12,13] with respect to spatial fluctuations. We find that the asymptotic states with constant order parameter remain stable, while the state with periodic $\Delta(t)$ does not. The physical origin of the instability lies in the possibility of parametric excitation

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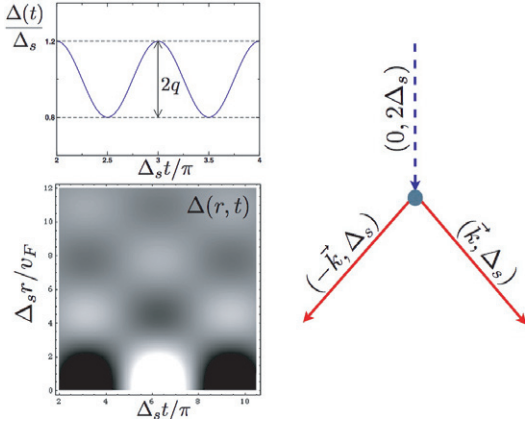


Fig. 1: Asymptotic state with the periodic order parameter $\Delta(t) = \Delta_s[1 + q\cos(2\Delta_s t)]$ (top left) is unstable with respect to the parametric excitation of two modes with opposite momenta (right panel). Frequency of each mode is half the oscillation frequency of the homogeneous order parameter. Initial exponential growth of the parametric instability is followed by a transient regime after which the condensate reaches a spatially inhomogeneous post-threshold state (bottom left). The value of the order parameter increases from white to black.

of the spatially modulated pairing modes, fig. 1. In a homogeneous medium, the periodic (in time) order parameter can be considered as an energy pump allowing a coupling between two spatial modes with opposite momenta, and, at the same time, providing enough energy to overcome the damping due to the scattering of Cooper pairs. Subsequent scattering effects of the resonant modes limit the initial exponential growth and result in a state with a *spatially inhomogeneous* order parameter. We demonstrate that as the process of the parametric instability develops, the energy of the homogeneous state is transferred into that of the pairing wave packets with the typical size of the order of the coherence length, ξ , signaling the onset of the Cooper pair turbulence.

Consider the state with a periodically varying order parameter. Analytically, $\Delta(t)$ is described by the Jacobi elliptic function dn [14]. Here we assume that the amplitude of the oscillations is small. This allows us to keep only the first two terms in the Fourier series for dn :

$$\Delta(t) = \Delta_s[1 + q\cos(2\Delta_s t)], \quad q \ll 1. \quad (1)$$

We note that the nondissipative dynamics within the BCS model is described by the Bogoliubov-de Gennes equations, which can be cast into the form of equations of motion for classical vector variables $\vec{s}_{\mathbf{p}}$ [15]: $\dot{\vec{s}}_{\mathbf{p}} = \vec{b}_{\mathbf{p}} \times \vec{s}_{\mathbf{p}}$. Here $\vec{b}_{\mathbf{p}} = 2(-\Delta(t), 0, \varepsilon_{\mathbf{p}})$ plays the role of an external magnetic field. The periodic external field makes it possible for the parametric instabilities to develop. Given that the asymptotic state (1) is robust against homogeneous perturbations [14,16], it seems natural to look for an instability with respect to spatial fluctuations. In particular, we will investigate the conditions of the

parametric decay of the homogeneous pairing mode (1) with an energy $2\Delta_s$ into two pairing modes with energies and momenta (ω_1, \mathbf{k}_1) and (ω_2, \mathbf{k}_2) , fig. 1. In a continuous medium the energy and momentum have to be conserved: $\mathbf{k}_1 + \mathbf{k}_2 = 0$ and $2\Delta_s = \omega_1 + \omega_2$. These type of instabilities appear in various physical systems, see, *e.g.*, ref. [5] for an extensive review.

To analyze the stability of the asymptotic state (1), we employ the Bogoliubov-de Gennes (BdG) equations¹:

$$\begin{aligned} i\dot{u}_{\mathbf{p}}(\mathbf{r}, t) &= \hat{\xi}u_{\mathbf{p}}(\mathbf{r}, t) + \Delta(\mathbf{r}, t)v_{\mathbf{p}}(\mathbf{r}, t), \\ i\dot{v}_{\mathbf{p}}(\mathbf{r}, t) &= -\hat{\xi}v_{\mathbf{p}}(\mathbf{r}, t) + \bar{\Delta}(\mathbf{r}, t)u_{\mathbf{p}}(\mathbf{r}, t). \end{aligned} \quad (2)$$

Here $\hat{\xi} = -\nabla^2/2m - \mu$, μ is the chemical potential and the order parameter $\Delta(\mathbf{r}, t)$ is

$$\Delta(\mathbf{r}, t) = g \sum_{\mathbf{p}} u_{\mathbf{p}}(\mathbf{r}, t)\bar{v}_{\mathbf{p}}(\mathbf{r}, t), \quad (3)$$

where g is the BCS coupling constant. We linearize eqs. (2) with respect to the deviations $\phi_{\mathbf{p}}(\mathbf{r}, t)$ and $\psi_{\mathbf{p}}(\mathbf{r}, t)$ from the homogeneous solution $U_{\mathbf{p}}(t)$ and $V_{\mathbf{p}}(t)$:

$$\begin{bmatrix} u_{\mathbf{p}}(\mathbf{r}, t) \\ v_{\mathbf{p}}(\mathbf{r}, t) \end{bmatrix} = \begin{bmatrix} U_{\mathbf{p}}(t) + \phi_{\mathbf{p}}(\mathbf{r}, t) \\ V_{\mathbf{p}}(t) + \psi_{\mathbf{p}}(\mathbf{r}, t) \end{bmatrix} e^{i\mathbf{p}\cdot\mathbf{r}}. \quad (4)$$

The order parameter (3) also contains a small inhomogeneous part $\Delta(\mathbf{r}, t) = \Delta(t) + \delta\Delta(\mathbf{r}, t)$. Plugging (4) into (2) and Fourier transforming with respect to the spatial coordinates, we find

$$\begin{aligned} i\partial_t \begin{bmatrix} \phi_{\mathbf{p}}(\mathbf{k}, t) \\ \psi_{\mathbf{p}}(\mathbf{k}, t) \end{bmatrix} &= \begin{pmatrix} \varepsilon_{\mathbf{p}+\mathbf{k}} & \Delta(t) \\ \Delta(t) & -\varepsilon_{\mathbf{p}+\mathbf{k}} \end{pmatrix} \begin{bmatrix} \phi_{\mathbf{p}}(\mathbf{k}, t) \\ \psi_{\mathbf{p}}(\mathbf{k}, t) \end{bmatrix} \\ &+ \begin{pmatrix} 0 & \delta\Delta(\mathbf{k}, t) \\ \delta\bar{\Delta}(-\mathbf{k}, t) & 0 \end{pmatrix} \begin{bmatrix} U_{\mathbf{p}}(t) \\ V_{\mathbf{p}}(t) \end{bmatrix}. \end{aligned} \quad (5)$$

The time dependence of $U_{\mathbf{p}}(t)$ and $V_{\mathbf{p}}(t)$ (see ref. [17]) suggests that for linear corrections (4) we write $\phi_{\mathbf{p}}(\mathbf{k}, t) = a_{\mathbf{p}}(\mathbf{k}, t)e^{i\xi_{\mathbf{p}}t} + b_{\mathbf{p}}(\mathbf{k}, t)e^{-i\xi_{\mathbf{p}}t}$ and $\psi_{\mathbf{p}}(\mathbf{k}, t) = \tilde{a}_{\mathbf{p}}(\mathbf{k}, t)e^{i\xi_{\mathbf{p}}t} + \tilde{b}_{\mathbf{p}}(\mathbf{k}, t)e^{-i\xi_{\mathbf{p}}t}$ with $\xi_{\mathbf{p}} = \sqrt{\varepsilon_{\mathbf{p}}^2 + \Delta_s^2} + O(q)$ and $\varepsilon_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m} - \mu$. We also write

$$\delta\Delta(\mathbf{k}, t) = C_{\mathbf{k}}(t)e^{i\Delta_s t} + \tilde{C}_{\mathbf{k}}(t)e^{-i\Delta_s t}, \quad (6)$$

where $C_{\mathbf{k}}(t) = c_{\mathbf{k}}e^{\nu(t-t_0)}$, $\tilde{C}_{\mathbf{k}}(t) = \tilde{c}_{\mathbf{k}}e^{\nu(t-t_0)}$, ν determines the growth rate, and t_0 is a time scale when (1) is reached (in what follows we set $t_0 = 0$). In the expression (6) we have neglected the higher harmonics $e^{\pm i3\Delta_s t}$, $e^{\pm i5\Delta_s t}$ etc. This is justified for $q \ll 1$, since their inclusion yields higher order in q corrections to the growth rate ν and to the order parameter amplitudes [18]. In the linear corrections to the Bogoliubov amplitudes (see above) we also keep only the lowest harmonics $\omega = \pm\Delta_s$, *i.e.* $a_{\mathbf{p}}(\mathbf{k}, t) \rightarrow (a_{1,\mathbf{p}}(\mathbf{k})e^{i\Delta_s t} + a_{-1,\mathbf{p}}(\mathbf{k})e^{-i\Delta_s t})e^{\nu t}$, $b_{\mathbf{p}}(\mathbf{k}, t) \rightarrow (b_{1,\mathbf{p}}(\mathbf{k})e^{i\Delta_s t} + b_{-1,\mathbf{p}}(\mathbf{k})e^{-i\Delta_s t})e^{\nu t}$ etc.

¹In this paper we consider only the case of a clean superfluid.

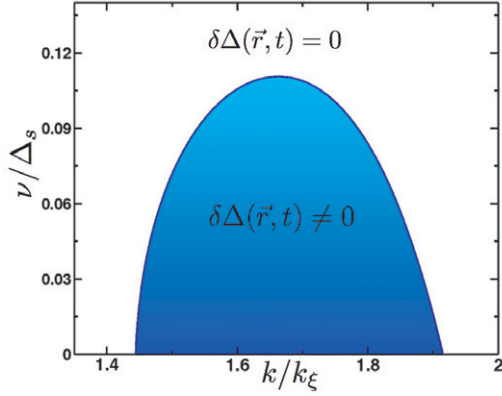


Fig. 2: Region of the parametric instability of the homogeneous $\Delta(t)$ (1) with respect to generation of the pairing modes with opposite momenta $(\mathbf{k}, -\mathbf{k})$. For a given momentum, the fastest growing modes are the ones on the boundary between the stable and unstable regions. Instability growth rate is plotted for $q=0.05$ in the units of Δ_s (see eq. (1)) and momentum is in the units of $k_\xi = \Delta_s/v_F$. The maximum rate is reached at $\nu_m \approx 2q\Delta_s$. For small q the shape of the instability curve is $\nu(k) \approx \nu_m - 2\Delta_s(k - 1.6k_\xi)^2/k_\xi^2$.

Next, we express the Bogoliubov amplitudes in terms of $c_{\mathbf{k}}$ and $\tilde{c}_{\mathbf{k}}$ by equating the coefficients in front of $e^{\pm i\Delta_s t}$ in eq. (5). The resulting amplitudes are substituted into the self-consistency eq. (3). One obtains a linear system for the variables $c_{\mathbf{k}}, \tilde{c}_{-\mathbf{k}}, c_{-\mathbf{k}}$ and $\tilde{c}_{\mathbf{k}}$. Expressing $\tilde{c}_{\mathbf{k}}, \tilde{c}_{-\mathbf{k}}$ in terms of $c_{\mathbf{k}}$ and $c_{-\mathbf{k}}$, we derive

$$(\omega_{\mathbf{k}} + i\gamma_{\mathbf{k}})c_{\mathbf{k}} + h_{\mathbf{k}}c_{-\mathbf{k}}^* = 0, \quad (\omega_{\mathbf{k}} - i\gamma_{\mathbf{k}})c_{-\mathbf{k}}^* + h_{\mathbf{k}}c_{\mathbf{k}} = 0, \quad (7)$$

where $\omega_{\mathbf{k}}, \gamma_{\mathbf{k}}$, and $h_{\mathbf{k}}$ are nonlinear functions of the growth rate ν . Equations (7) can be interpreted as the equations of motion for a classical field $c_{\mathbf{k}}$ [5]. Then, $\omega_{\mathbf{k}}$ has the meaning of the excitation spectrum of this field and $h_{\mathbf{k}} \sim O(q)$ stands for the pumping amplitude, which gives rise to a parametric instability. Finally, $\gamma_{\mathbf{k}}$ describes the damping of the parametric modes due to the intrinsic relaxation processes.

Nonzero solutions of eqs. (7) for $\nu(\mathbf{k})$ exist provided $\omega_{\mathbf{k}}^2(\nu) = h_{\mathbf{k}}^2(\nu) - \gamma_{\mathbf{k}}^2(\nu)$. Thus, the stability analysis is reduced to the solution of the nonlinear equation for $\nu(\mathbf{k})$. We have analyzed this equation numerically and present the results on fig. 2. We find that the instability region is centered around $k_m \approx 1.6k_\xi$ and has a width $\delta k \approx 1.2\sqrt{q}k_\xi$, where $k_\xi = 1/\xi$ is the coherence wave vector. From (7) it follows that for a fixed q the parametric growth will be suppressed as soon as the energy pumped into the system fully goes into dissipation. This condition determines the maximum growth rate ν_m , i.e. $\gamma_{\mathbf{k}}(\nu_m) = h_{\mathbf{k}}(\nu_m)$. Our estimate yields $\nu_m \approx 2q\Delta_s$. Lastly, we have also verified that asymptotic states with constant order parameter remain stable with respect to the spatial fluctuations of the above type.

The initial growth of the parametric instability (11) will be limited by nonlinear effects which lead to the

transient behavior with subsequent transition into a post-resonance state. The latter is defined as a state in which Fourier components of the order parameter (6) are time independent, $C_{\mathbf{k}}(t) = c_{\mathbf{k}}$ and $\tilde{C}_{\mathbf{k}}(t) = \tilde{c}_{\mathbf{k}}$. Below we focus on finding the resulting post-resonance state of the condensate. From the linear analysis we have seen that the fastest growing modes are the ones with a certain magnitude of the momentum. Thus, in the BdG eqs. (2) among the nonlinear in powers of $c_{\mathbf{k}}, \tilde{c}_{\mathbf{k}}$ terms we keep the resonant ones with frequencies $\omega = \pm\Delta_s$ and momenta $|\mathbf{k}| = |\mathbf{k}'| = k_s$, where k_s is a new post-resonance state momentum to be determined below. Next we repeat exactly the same procedure that lead us to eq. (7). The resulting set of nonlinear in $c_{\mathbf{k}}$ equations for the order parameter amplitudes can be written as eq. (7) with renormalized coefficients

$$\begin{aligned} \omega_{\mathbf{k}} &\rightarrow \Omega_{\mathbf{k}} = \omega_{\mathbf{k}}(0) + \sum_{|\mathbf{k}'|=k_s} T_{\mathbf{k}\mathbf{k}'} |c_{\mathbf{k}'}|^2, \\ h_{\mathbf{k}} &\rightarrow P_{\mathbf{k}} = h_{\mathbf{k}}(0) + \sum_{|\mathbf{k}'|=k_s} S_{\mathbf{k}\mathbf{k}'} c_{\mathbf{k}'} c_{-\mathbf{k}'}, \end{aligned} \quad (8)$$

where $T_{\mathbf{k}\mathbf{k}'}$ and $S_{\mathbf{k}\mathbf{k}'}$ are the scattering matrix elements. They vary slowly on a scale of k_ξ and are almost independent of the angle between \mathbf{k} and \mathbf{k}' . In what follows we neglect the k -dependence in the scattering matrix elements, $T_{\mathbf{k}\mathbf{k}'} = T$ and $S_{\mathbf{k}\mathbf{k}'} = S$. Note that each contribution in (8) is either phase independent or depends on a sum of the two phases of $c_{\mathbf{k}}$ and $c_{-\mathbf{k}}$. This can be interpreted as follows. There are two physical processes which limit the parametric excitations: one has to do with the reduction of the absolute value of the amplitudes, while the other is related to the phase decoherence of the two pairing modes with opposite momenta. In its spirit approximation (8) known as the ‘‘S-theory’’ is similar to the mean-field BCS model where only diagonal in momentum terms are kept in the interaction and was first considered by Zakharov, L’vov and Starobinets [19] to study the parametric excitation of spin waves in uniaxial ferromagnets. The inclusion of off-diagonal terms in eq. (8) is expected to cause a broadening of the post-resonance state momentum $\delta k \sim \sqrt{q}k_\xi$, see fig. 2.

To determine the parameters of our post-resonance state, we insert $c_{\mathbf{k}} = |c_{\mathbf{k}}|e^{i\alpha_{\mathbf{k}}}$, and eqs. (8) for $\Omega_{\mathbf{k}}$ and $P_{\mathbf{k}}$ into (7). The post-resonance state momentum k_s is determined by the condition that the magnitude of the pumping field $|P_{\mathbf{k}}|$ does not exceed the damping $\gamma_{\mathbf{k}}$ for any k . As a result we have $\Omega_{k_s} = 0$. Phase $\Psi_s = \alpha_{\mathbf{k}} + \alpha_{-\mathbf{k}}$ and amplitude $|c_{k_s}|$ are given by $\sin\Psi_s = \gamma_{k_s}/h_{k_s}$ and $|c_{k_s}|^2 = h_{k_s} \cos\Psi_s / |S|$ (the corresponding expressions for $|\tilde{c}_{k_s}|$ and $\tilde{\Psi}_s = \tilde{\alpha}_{\mathbf{k}} + \tilde{\alpha}_{-\mathbf{k}}$ can be derived similarly). We obtain

$$\begin{aligned} \Delta(\vec{r}, t) &= \Delta_s + \sqrt{q}\Delta_s c_s \sum_{|\mathbf{k}|=k_s} e^{i\mathbf{k}\cdot\vec{r}} \\ &\times \left[e^{i(\alpha_{\mathbf{k}} + \Delta_s t)} + w_s e^{i(\tilde{\alpha}_{\mathbf{k}} - \Delta_s t)} \right], \end{aligned} \quad (9)$$

where $k_s \approx 1.73k_\xi$, $c_s \approx 0.77$ and $w_s \approx 0.95$ for $\Delta_s = 0.1\mu$. Note that the post-resonance momentum $k_s > k_m \approx 1.6k_\xi$, *i.e.* the energy cascades to smaller length scales, as expected of turbulent behavior.

The individual phases $\alpha_{\mathbf{k}}$ and $\tilde{\alpha}_{\mathbf{k}}$ cannot be determined within the diagonal approximation (8). In a continuous medium, one can treat them as random variables. For the correlators we take $\langle e^{i\alpha_{\mathbf{k}}} \rangle = 0$, $\langle e^{i\alpha_{\mathbf{k}_1}} e^{i\tilde{\alpha}_{\mathbf{k}_2}} \rangle = 0$ and $\langle e^{i\alpha_{\mathbf{k}_1}} e^{i\alpha_{\mathbf{k}_2}} \rangle = \delta_{\mathbf{k}_1, -\mathbf{k}_2}^{(3)} e^{i\Psi_s}$, where $\langle \dots \rangle$ stands for averaging over the phase distribution.

To get further insight into the nature of the post-resonance state, consider the following choice for the phases $\alpha_{\mathbf{k}} = \Psi_s/2 - \mathbf{k} \cdot \mathbf{r}_0$ and $\tilde{\alpha}_{\mathbf{k}} = \tilde{\Psi}_s/2 - \mathbf{k} \cdot \mathbf{r}_0$. It leads to a spherically symmetric wave packet, a ‘‘bubble’’, with a periodic amplitude $A(t)$

$$\Delta(\vec{r}, t) = \Delta_s + \frac{\sqrt{q}\Delta_s c_s \sin(k_s|\vec{r} - \vec{r}_0|)}{k_s|\vec{r} - \vec{r}_0|} A(t), \quad (10)$$

$$A(t) = e^{i(\frac{\Psi_s}{2} + \Delta_s t)} + w_s e^{i(\frac{\tilde{\Psi}_s}{2} - \Delta_s t)}.$$

In general we obtain a linear combination of these bubbles (10) by writing $e^{i\alpha_{\mathbf{k}}} = \int f(\mathbf{r}_0) e^{-i\mathbf{k} \cdot \mathbf{r}_0} d^3\mathbf{r}_0$ and similarly for $e^{i\tilde{\alpha}_{\mathbf{k}}}$. Then, eq. (9) can be viewed as a superposition of wave packets of the form (10) centered at different \vec{r}_0 with random amplitudes $A(\vec{r}_0, t)$. This suggests that the parametric instability results in a random distribution of the wave packets.

It is instructive to compare (10) with $\delta\Delta(\vec{r}, t)$ at the linear stage of the parametric instability, which can be derived using our result for $\nu(k)$ (see fig. 2) and eqs. (6), (7). Taking $c_{\mathbf{k}} = C e^{-i(\mathbf{k} \cdot \mathbf{r}_0 + \Delta_s \tau_{\mathbf{k}})}$ and $\tilde{c}_{\mathbf{k}} = c_{-\mathbf{k}}^*$, we obtain

$$\delta\Delta(\vec{r}, t) \approx \frac{C e^{\nu_m t} \cos[\Delta_s(t - \tau)] \sin(k_m R) e^{-R^2/l^2(t)}}{\sqrt{\Delta_s t} k_m R}, \quad (11)$$

where $l(t) \approx \xi \sqrt{\Delta_s t}$, $R = |\vec{r} - \vec{r}_0|$, C is a constant, and $e^{i\Delta_s \tau_{\mathbf{k}}} = (i\gamma_k - \omega_k)/h_k$. In deriving eq. (11), we also assumed $k_m R \gg \Delta_s t$ and replaced a slowly varying function $\tau_{\mathbf{k}} \rightarrow \tau$. Expression (11) describes the initial formation of a wave packet (10). Note that on a time scale $(q\Delta_s)^{-1}$ at which the order parameter deviation is of the order $\sqrt{q}\Delta_s$, the width of the packet is $l_p \approx \xi/\sqrt{q}$.

Above observations help to identify features of the post-resonance state (9) relevant for the experimental verification of our theory. To be specific, let us compute $|\Delta(\mathbf{p}, t)|^2$. This quantity determines the momentum distribution $N_{\mathbf{p}}$ of the condensed molecules after a fast sweep to the BEC side of the Feshbach resonance [20,21]. Equation (9) implies

$$N_{\mathbf{p}} \propto \Delta_s^2 (\delta_{p,0} + qc_s^2 [1 + w_s^2 + 2w_s \mathcal{K}_{\mathbf{p}}(t)] \delta_{p,k_s}), \quad (12)$$

where $\mathcal{K}_{\mathbf{p}}(t) = \cos(2\Delta_s t + \tilde{\alpha}_{\mathbf{p}} - \alpha_{\mathbf{p}})$ is essentially random. We note from eq. (12) that the molecular momentum distribution acquires an additional peak at $p = k_s$ thus signaling the presence of spatial modulations in the post-resonance state eq. (9).

Formation of an isolated wave packet (10) induces an oscillating supercurrent $\vec{j}_s \propto \vec{\nabla} \Phi(\vec{r}, t)$, where $\Phi(\vec{r}, t)$ is the phase of the order parameter. Setting $r_0 = 0$ we find that only the radial component of the current is nonzero. To the lowest order in q , $\vec{j}_s(\vec{r}, t) \propto \hat{e}_r \sqrt{q} \cos(\Delta_s t) [k_s r \cos(k_s r) - \sin(k_s r)] / (k_s r)^2$. This implies a spatial re-distribution of Cooper pairs similar to the Friedel oscillations in the density of a degenerate Fermi gas induced by a weak scattering potential.

In our discussion so far we treated the pairing mode (1) giving rise to the parametric instability as an independent external field. Inclusion of the feedback on this mode as weak turbulence ($q \ll 1$) develops may modify the post-resonance state (9). We leave a detailed analysis of possible feedback effects for future studies.

In the post-resonance state (9) the Fourier components of the order parameter are $c_{\mathbf{k},\omega} \sim \delta(k - k_s) \delta(\omega - \Delta_s)$. Inelastic scattering or thermal effects generally leads to a broadening in the momentum and frequency distributions of $c_{\mathbf{k},\omega}$ [7]. The latter might cause a damping of the temporal oscillations in eq. (9). On a time scale $t > \tau_\varepsilon$ dissipation due to quasi-particle scattering processes ultimately forces the system to reach an equilibrium state. Finally, we comment that in the transient regime leading to an asymptotic state with constant $\Delta(t) = \Delta_s$ the order parameter is $(\Delta(t) - \Delta_s) \propto \cos(2\Delta_s t) / \sqrt{t}$ (*cf.* (1)) [16]. Oscillatory behavior suggests that this asymptotic state might never be attained owing to the development of the parametric instability of the type considered above.

In conclusion, we have investigated the stability of the nonequilibrium asymptotic states of a fermionic superfluid, which can be generated, *e.g.*, by a uniform quench of the pairing strength. We have demonstrated that in a system of size L larger than the coherence length ξ the asymptotic state (1) with periodic in time order parameter is unstable with respect to spatial fluctuations. The instability is due to the parametric excitation of two pairing modes with opposite momenta. The initial exponential growth of deviations from the homogeneous state (11) is suppressed by nonlinear effects eventually leading to a spatially nonuniform post-resonance state described by eq. (9). This state can be interpreted as a superposition of bubbles of the superfluid order parameter (10) with random amplitudes. The parametric instability of the uniform oscillations can be experimentally probed by either measuring the local order parameter [22] or by performing the measurements of the condensate fraction using the technique of the fast sweep across the Feshbach resonance [10,11]. One of the experimental signatures of our post-resonance state is the additional peak in the condensate fraction at a nonzero momentum $p = k_s$, see eq. (12).

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REFERENCES

- [1] RUUTU V. M. H. *et al.*, *Nature (London)*, **382** (1995) 334.
 [2] WARNER G. L. and LEGGETT A. J., *Phys. Rev. B*, **71** (2005) 134514.
 [3] CASADO S., GONZÁLEZ-VINAS W. and MANCINI H., *Phys. Rev. E*, **74** (2006) 047101.
 [4] SADLER L. E. *et al.*, *Nature (London)*, **443** (2006) 312.
 [5] ZAKHAROV V. E., L'VOV V. S. and STAROBINETS S. S., *Sov. Phys. Usp.*, **17** (1975) 896.
 [6] L'VOV V. S. and RUBENCHIK A. M., *Sov. Phys. JETP*, **37** (1973) 263.
 [7] L'VOV V. S., *Sov. Phys. JETP*, **42** (1976) 1057.
 [8] BUNKOV YU. M., L'VOV V. S. and VOLOVIK G. E., *JETP Lett.*, **83** (2006) 530; **84** (2006) 289.
 [9] VOLOVIK G. E., arXiv:cond-mat/0701180.
 [10] REGAL C. A., GREINER M. and JIN D. S., *Phys. Rev. Lett.*, **92** (2004) 040403.
 [11] ZWIERLEIN M. *et al.*, *Phys. Rev. Lett.*, **92** (2004) 120403.
 [12] BARANKOV R. A. and LEVITOV L. S., *Phys. Rev. Lett.*, **96** (2006) 230403.
 [13] YUZBASHYAN E. A. and DZERO M., *Phys. Rev. Lett.*, **96** (2006) 230404.
 [14] BARANKOV R. A., LEVITOV L. S. and SPIVAK B. Z., *Phys. Rev. Lett.*, **93** (2004) 160401.
 [15] ANDERSON P. W., *Phys. Rev.*, **112** (1958) 1900.
 [16] YUZBASHYAN E. A., TSYPLYATYEV O. and ALTSHULER B. L., *Phys. Rev. Lett.*, **96** (2006) 097005.
 [17] DZERO M., YUZBASHYAN E. A., ALTSHULER B. L. and COLEMAN P., *Phys. Rev. Lett.*, **99** (2007) 160402.
 [18] LANDAU L. D. and LIFSHITZ E. M., *Classical Mechanics* (Pergamon Press, London) 1975.
 [19] ZAKHAROV V. E., L'VOV V. S. and STAROBINETS S. S., *Sov. Phys. JETP*, **32** (1971) 656; see also: L'VOV V. S., *Wave Turbulence under Parametric Excitations. Application to Magnetics* (Springer Verlag) 1994.
 [20] DIENER R. B. and TIN-LUN HO, preprint, arXiv:cond-mat/0404517.
 [21] ALTMAN E. and VISHWANATH A., *Phys. Rev. Lett.*, **95** (2005) 110404.
 [22] SHIN Y., SCHUNCK C. H., SCHIROTZEK A. and KETTERLE W., *Phys. Rev. Lett.*, **99** (2007) 090403.