New superfluid states of fermionic matter in and out of (far from) equilibrium

Emil Yuzbashyan

Rutgers University

New superfluid states of fermionic matter in and out of (far from) equilibrium

Collaborators:

E.Y., arXiv:0807.3181 (2008).

G. Catelani, E.Y., *Phys. Rev. A* (2008).

M. Dzero, E.Y., B. Altshuler, arXiv:0805.2798 (2008).

E.Y., O. Tsyplyatyev, arXiv:0712.4280 (2007).

M. Dzero, E.Y., B. Altshuler, P. Coleman, *Phys. Rev. Lett.* (2007).

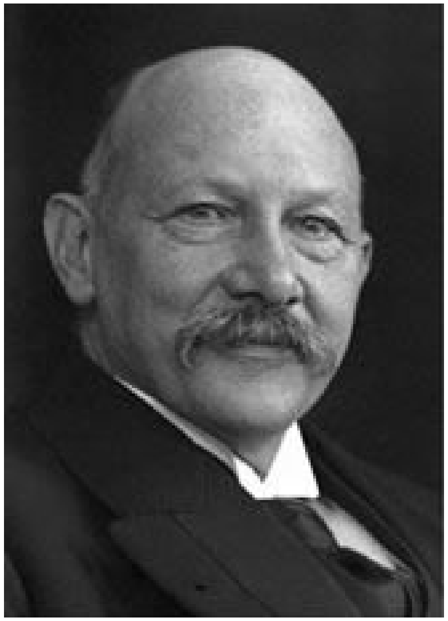
E.Y., M. Dzero, *Phys. Rev. Lett.* (2006).

E. Y., O. Tsyplyatyev, B. Altshuler, *Phys. Rev. Lett.* (2006).

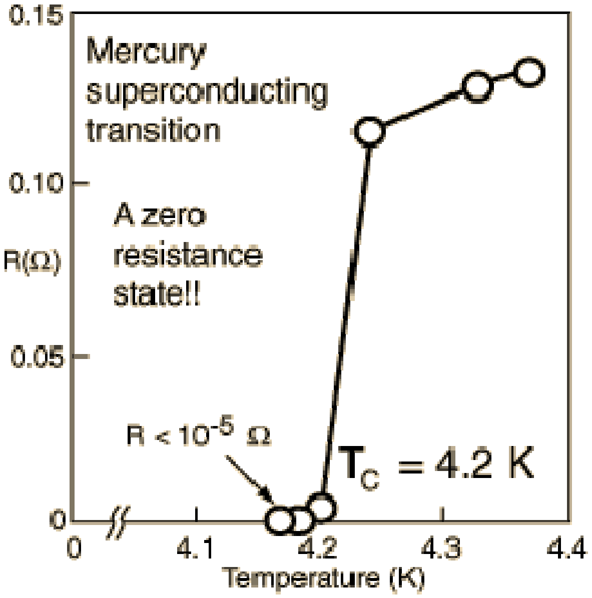
E.Y., B. Altshuler, V. Kuznetsov, V. Enolskii, *Phys. Rev. B* (2005).

Superconductivity = fermionic superfluidity

1911: the discovery



Heike Kammerlingh Onnes
(1853-1926)



H. K. Onnes, Commun. Phys. Lab.12,120, (1911)

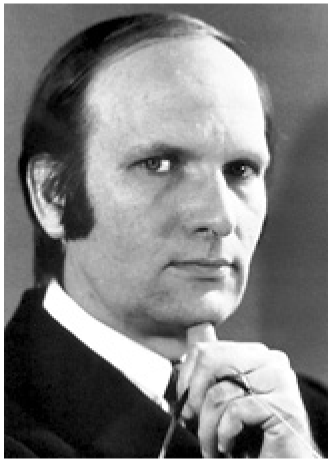
"Mercury has passed into a new state, which on account of its extraordinary electrical properties may be called the superconductive state."

(courtesy of Joerg Schmalian)

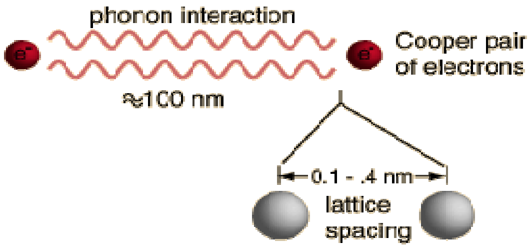
1957: the explanation (equilibrium superconductivity)

J. Bardeen

L. Cooper



J. R. Schrieffer



(courtesy of J. Schmalian)

At low energies the effective interaction between electrons is attractive.

(Most) energy is gained by pairing up electrons with opposite spin and momentum.

$$\boxed{\langle b_k \rangle = \langle c_{-k\downarrow} c_{k\uparrow} \rangle \neq 0}$$

two electron bound state
(Cooper pair)

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois

(Received July 8, 1957)

$$H_{\text{red}} = 2\sum_{k>k_F} \epsilon_{\mathbf{k}} b_{\mathbf{k}}^* b_{\mathbf{k}} + 2\sum_{k<k_F} |\epsilon_{\mathbf{k}}| b_{\mathbf{k}} b_{\mathbf{k}}^* - \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} b_{\mathbf{k}'}^* b_{\mathbf{k}}. \quad (2.14)$$

$$b_{\mathbf{k}} = c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow},$$

$$b_{\mathbf{k}}^* = c_{\mathbf{k}\uparrow}^* c_{-\mathbf{k}\downarrow}^*$$

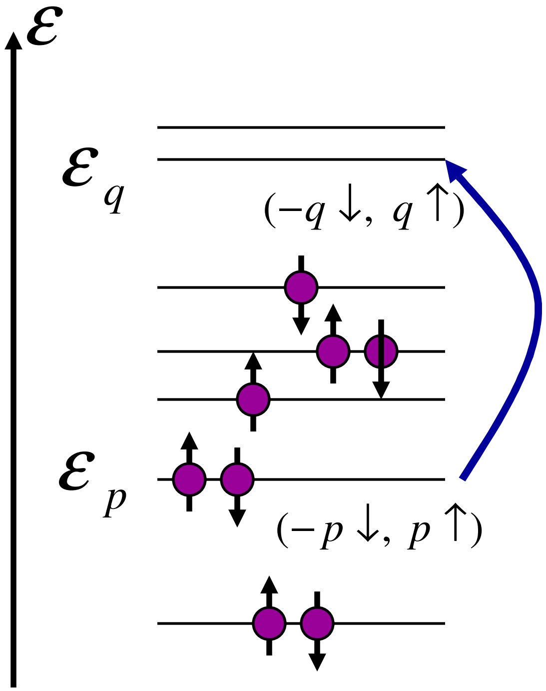
One-particle energy

BCS interaction

bozons ?

BCS Hamiltonian

energy levels:



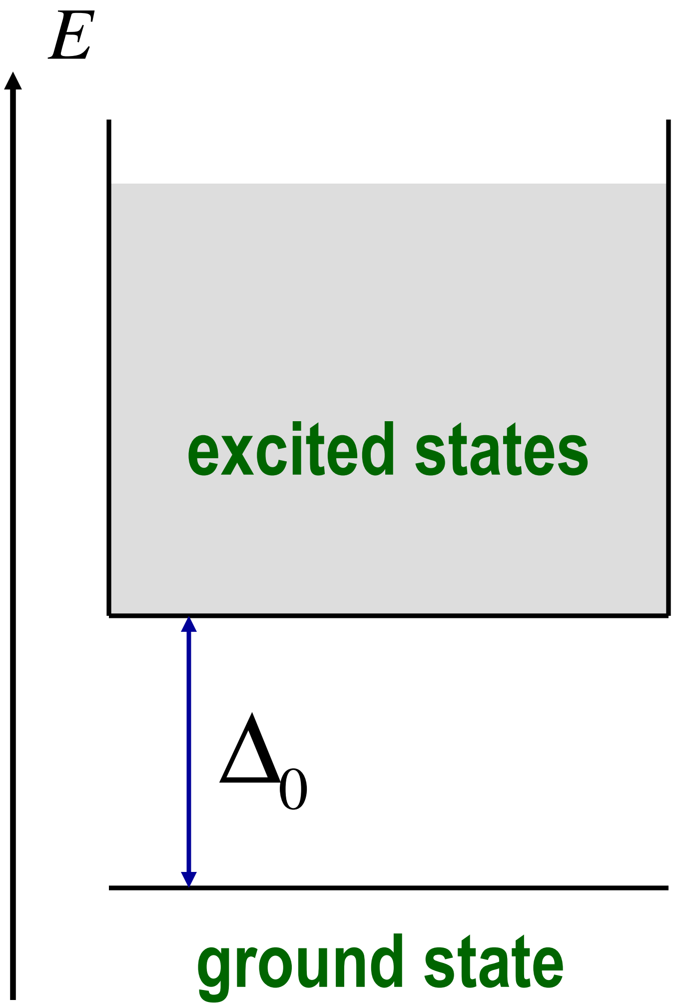
$$\boxed{H_{BCS} = \sum_{p} \varepsilon_p n_p - g \sum_{p,q} c^{+}_{q\uparrow} c^{+}_{-q\downarrow} c_{-p\downarrow} c_{p\uparrow}}$$

BCS coupling constant

superconducting order parameter = gap function = anomalous average

$$\boxed{\Delta = g \sum_{p} \left\langle c_{-p\downarrow} c_{p\uparrow} \right\rangle}$$

BCS theory (equilibrium superconductivity)



single-particle excitations:

$$\boxed{E(p) = \sqrt{\varepsilon_p^2 + \Delta_0^2}}$$

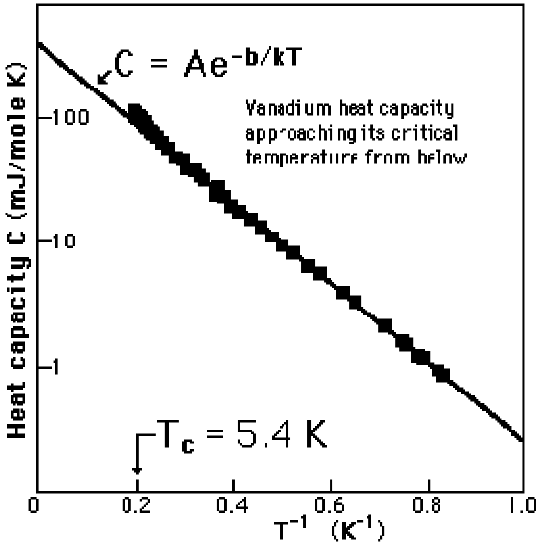
energy gap (value of the order parameter in ground state):

$$\Delta_0 = D\, e^{-1/g\nu_F}$$

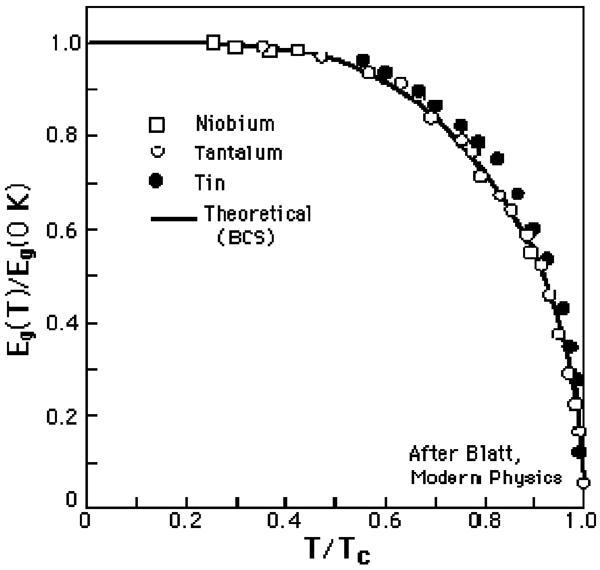
BCS theory (equilibrium superconductivity)

1957: the success

low temperature heat capacity



temperature dependence of the gap



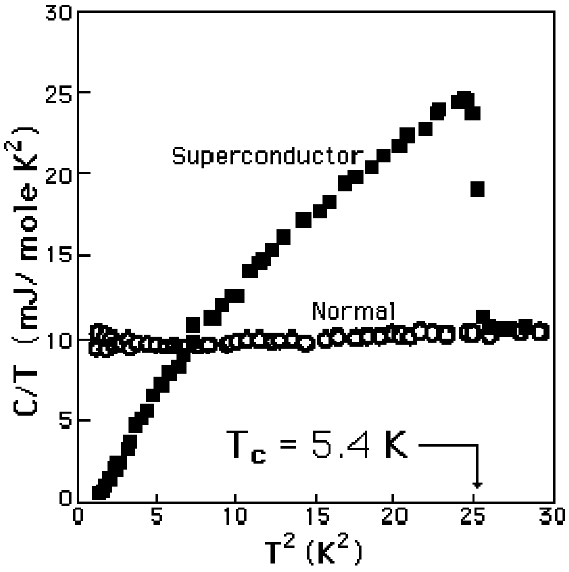
(courtesy of Joerg Schmalian)

BCS theory (equilibrium superconductivity)

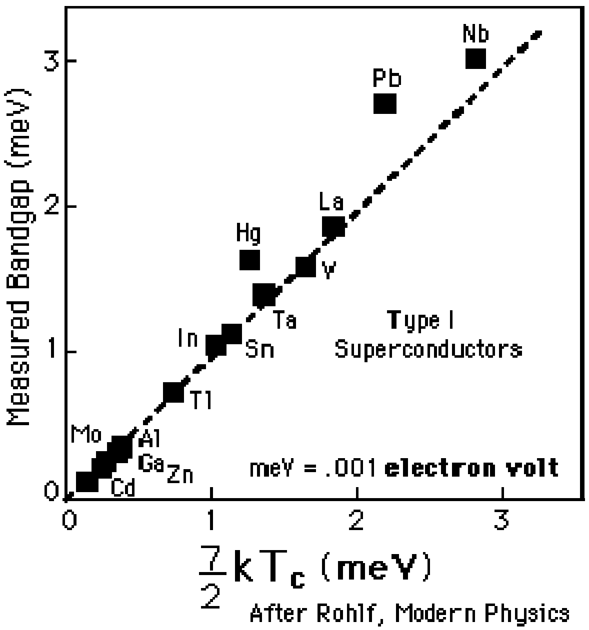
1957: the success

$$\Delta c(T_c)/c_n(T_c) \approx 1.43$$

$$E_g / k_B T_c = 2\pi e^{-\gamma_E} \approx 3.53$$



heat capacity of vanadium



After Rohlf, Modern Physics

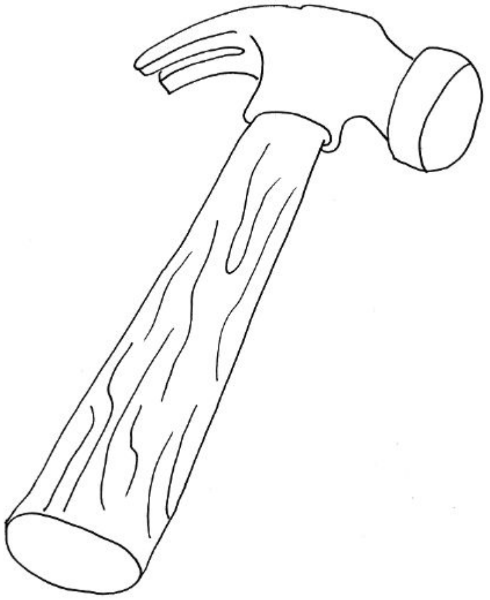
(courtesy of Joerg Schmalian)

BCS theory = equilibrium superconductivity

What about non- (far from) equilibrium superconductivity?

(where concepts based on excitation spectrum & energy gap do not apply)

Far from equilibrium superconductibity?



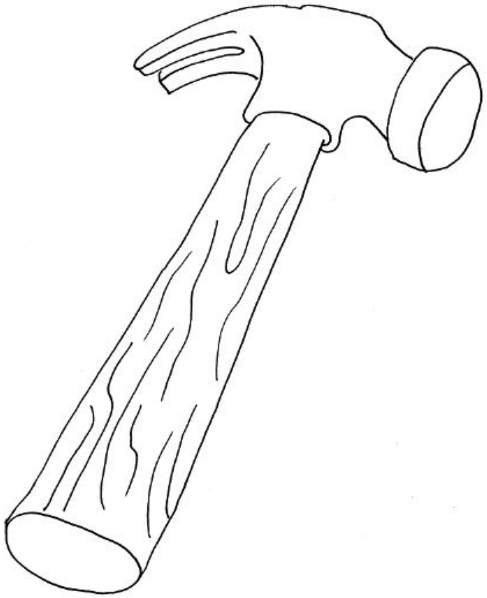
?

$$\Delta(t) = ?$$

Time scales: 1) energy relaxation time τ_ε

2) Cooper instability time $\hbar/\Delta_0$

How to describe **nonadiabatic** dynamics at times of the order of $1/\Delta$?



Superconductor

Δ_0

?

$$\Delta(t) = ?$$

Perturbation time shorter than energy relaxation time τ_ε and comparable with Cooper instability time Δ_0^{-1}

(nonadiabatic regime)

Nonequilibrium Superconductivity – History

Time-Dependent Ginzburg-Landau equation

E. Abrahams & T. Tsuneto, 1966
A. Schmid, 1966
L.P. Gorkov & G.M. Eliashberg, 1968

only special cases like gapless superconductivity

Boltzmann eqn + selfconsistency eqn for the order parameter

A. I. Larkin & Yu. N. Ovchinnikov, 1968
O. Betbeder-Matibed & P. Nozieres, 1969
A.G. Aronov et al, 1981
⋮
N.B. Kopnin "Theory of Nonequilibrium Superconductivity", 2001

only at times longer than Cooper instability time

How to describe nonadiabatic dynamics at times of the order of $1/\Delta$?

Difficulty – far from equilibrium

How to describe nonadiabatic dynamics at times of the order of $1/\Delta$?

P.W. Anderson, 1958
V.P. Galaiko, 1972
A. F. Volkov & Sh.M. Kogan, 1974
Yu.M. Galperin, V.I. Kozub & B.Z. Spivak, 1981
V.S. Shumeiko, Doctoral Thesis, 1990

Conventional superconductors are in adiabatic regime

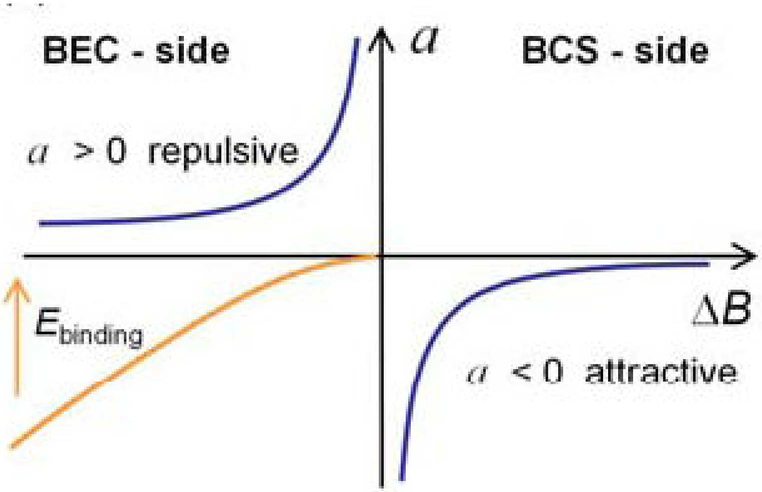
$$\Delta_0^{-1} \approx 100\ \text{ps (Al)}$$

Physical realizations

1) Ultra-cold fermions (^{40}K, ^{6}Li).

Abrupt change of the BCS coupling constant

$$g \propto \text{scatt. length}, \quad g_i \to g_f$$

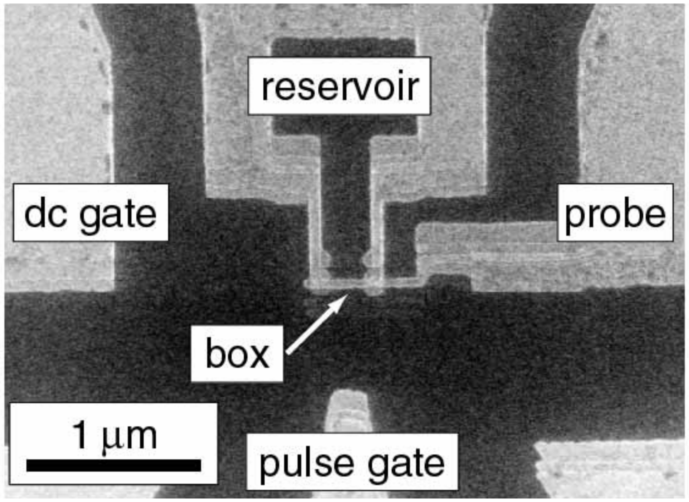


Greiner, Regal &Jin (JILA, ^{40}K)

pert. time ~ 10 μs, $\Delta_{eq}^{-1} \sim 1$ ms, $\tau_\varepsilon \sim 100$ ms

2) Superconducting qubits.

Nonequilibrium conditions can be generated by fast voltage pulses



Nakamura, Pashkin &Tsai

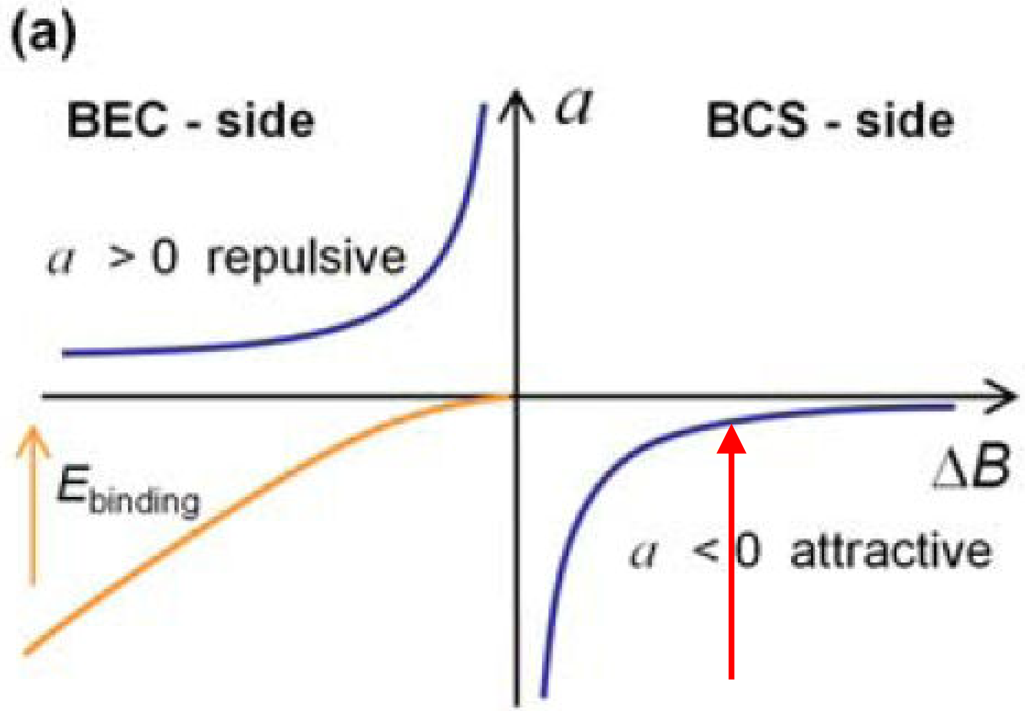
pert. time ~ $\Delta_{eq}^{-1} \sim 100$ ps, $\tau_\varepsilon \sim 1\ \mu$s

How to describe nonadiabatic dynamics at times of the order of $1/\Delta$?

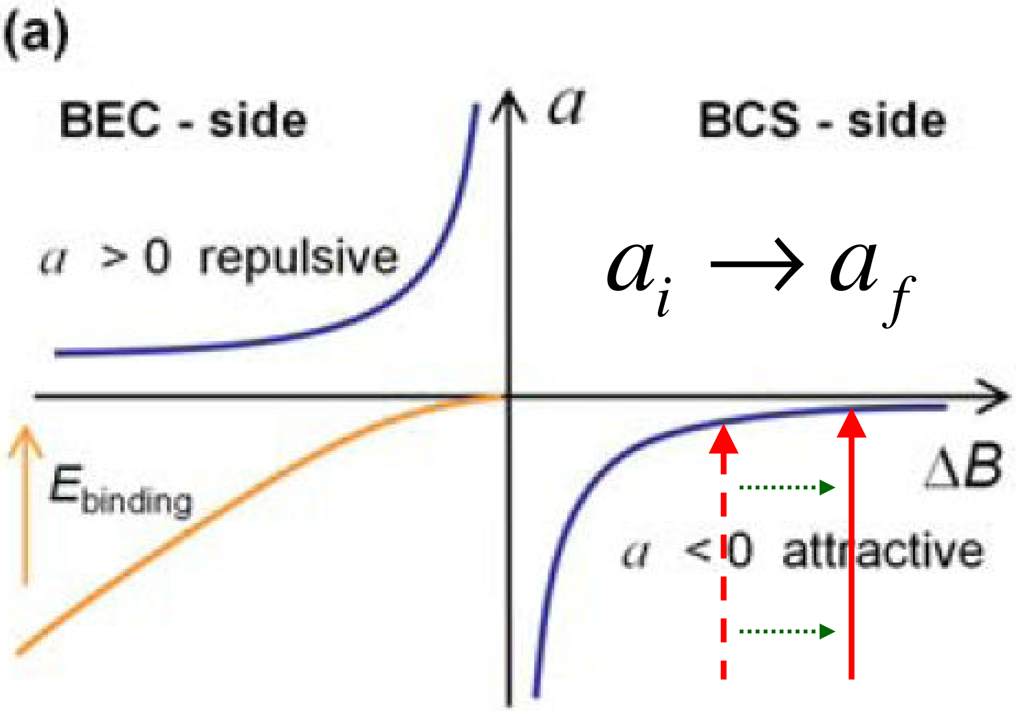
R.A. Barankov, L.S. Levitov & B.Z. Spivak, 2004
A.V. Andreev, V. Gurarie, L. Radzihovsky, 2004
M.H.C. Amin, E.V. Bezuglyi, A.S. Kijko, A.N. Omelyanchouk, 2004
E.Y., B. Altshuler, V. Kuznetsov, V. Enolskii, 2004
M.H. Szumanska, B.D. Simons & K. Burnett, 2004
I. Tikhonenkov, A. Vardi, 2004
T. Miyakawa, P. Meystre, 2004
G.L. Warner & A.J. Leggett, 2005
W. Yi and L.-M. Duan, 2005
T. Domanski, 2005
Michael W. Jack and Han Pu, 2005

Cold gases are naturally in the non-adiabatic regime!

Problem

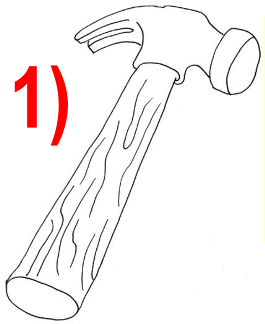

(a)
BEC - side
a
BCS - side
a > 0 repulsive
E_{binding}
ΔB
a < 0 attractive

Problem



$$a_i \rightarrow a_f$$

$$g = k_F\,|a|/\pi v_F$$



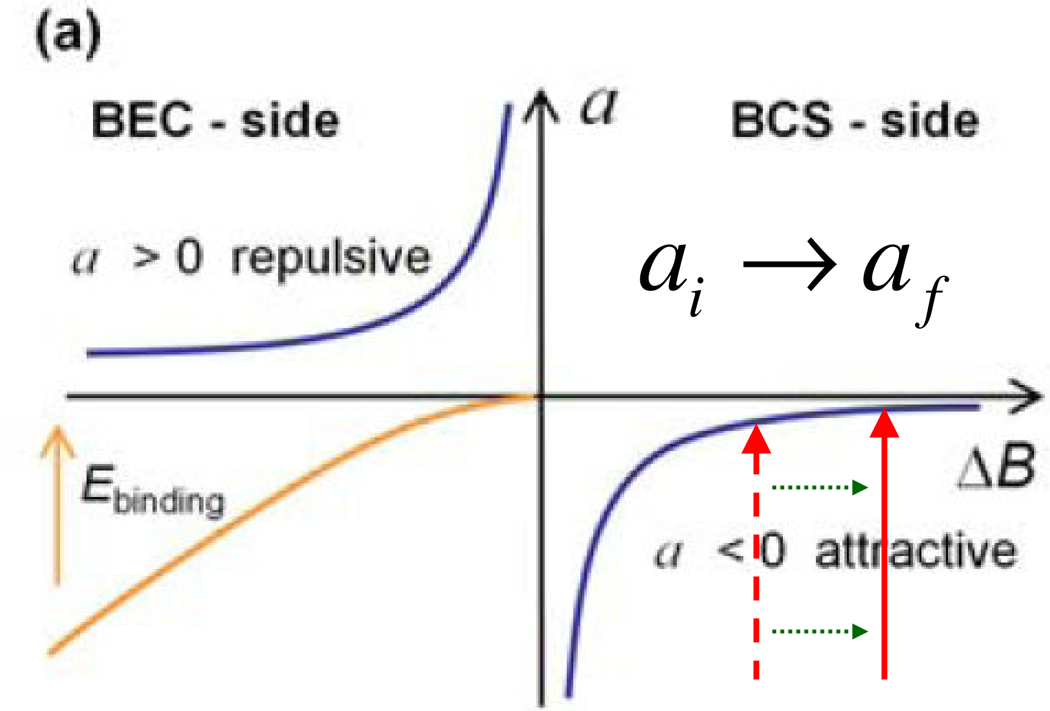
1) $$g_i \rightarrow g_f$$

2) $$\Psi_{cond}(t=0) = \left|\text{Ground state with wrong coupling}\right\rangle$$

3) $$\Psi_{cond}(t>0) = ?$$

$t \sim \Delta^{-1}$, weak coupling

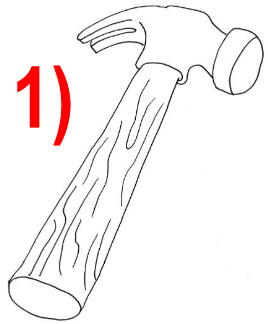
Problem



$$a_i \to a_f$$

$$g = 2k_F\,|a|/\pi\nu_F$$

$$\Delta_{eq} = 2E_F e^{-1/g\nu_F}$$



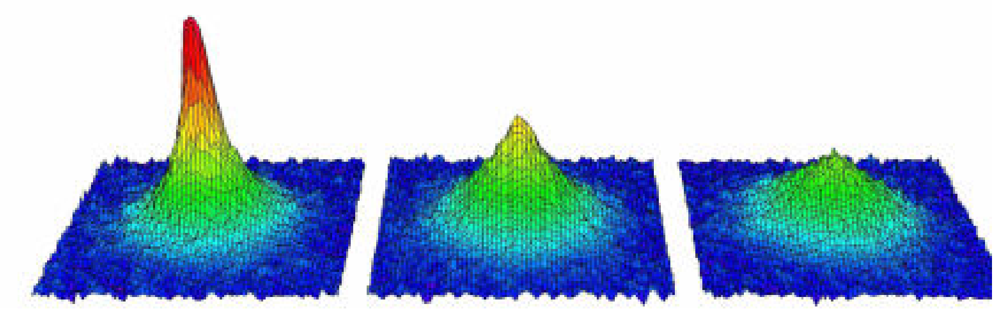
1) $\Delta_i \to \Delta_f$

2) $\Psi_{cond}(t=0) = \left|\text{Ground state with wrong coupling}\right\rangle$

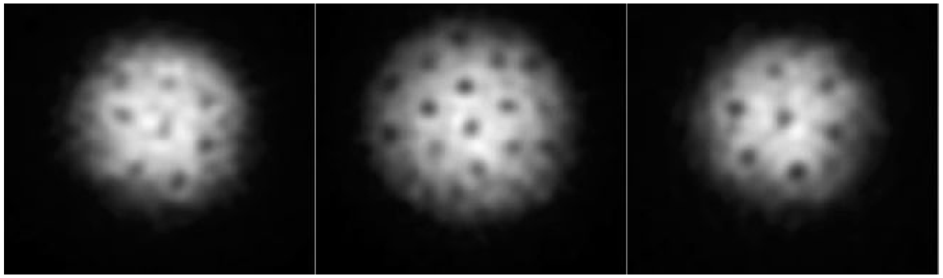
3) $\Psi_{cond}(t>0) = ?$

$t \sim \Delta^{-1}$, weak coupling

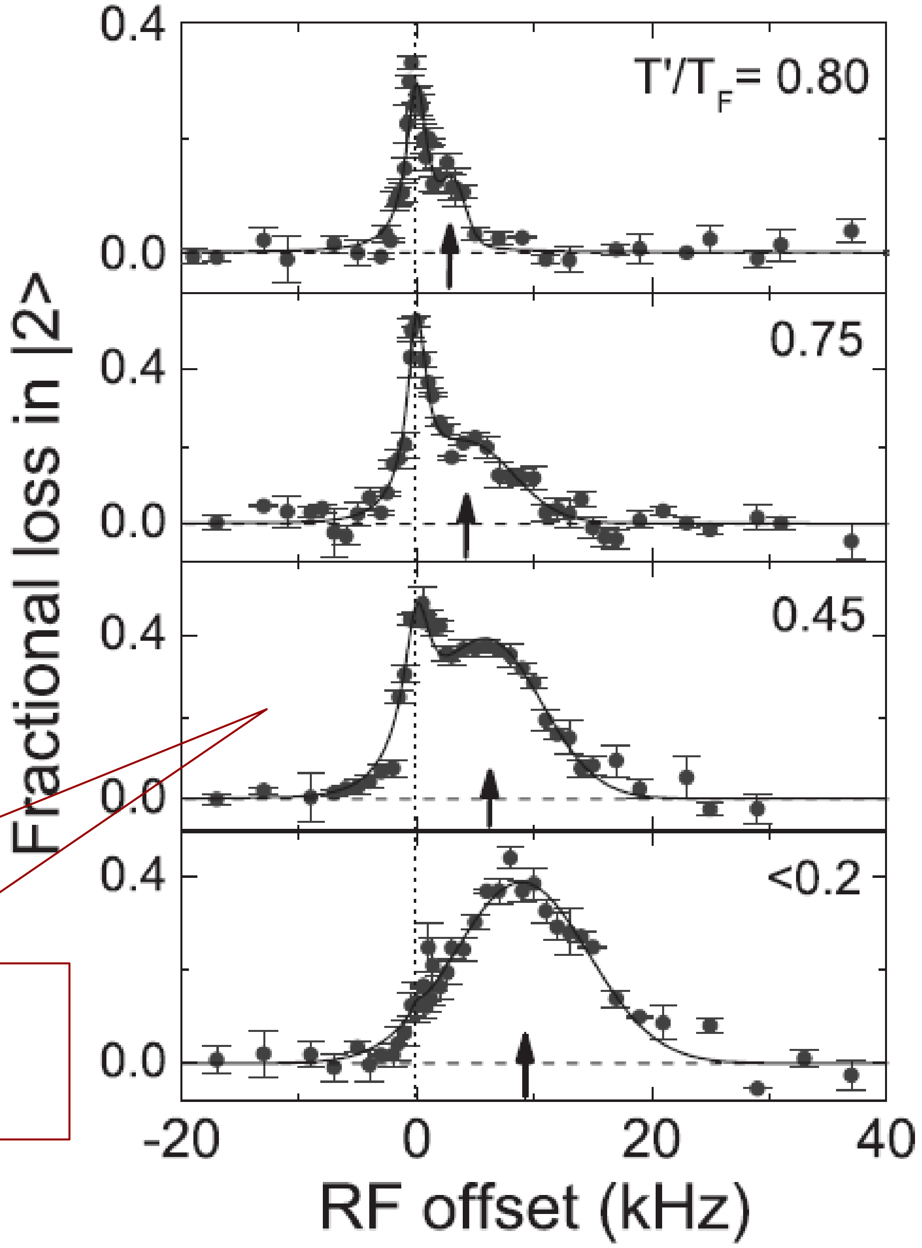
Cooper pairing in cold atomic fermions



Optical images of condensate. Regal, Greiner & Jin ’04.



Vortex lattice. Zwierlein et. al. ‘05



Direct measurement of Δ on BCS side. Chin et. al. ’04.

Anderson pseudospins

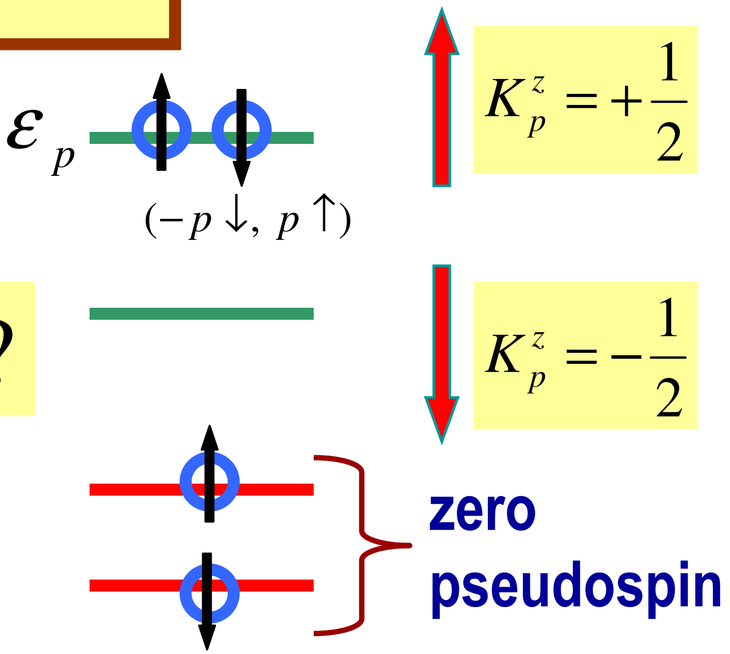
$$H_{BCS} = \sum_p \varepsilon_p n_p - g \sum_{p,q} c^+_{q\uparrow} c^+_{-q\downarrow} c_{-p\downarrow} c_{p\uparrow}$$

P. W. Anderson (1958)

$$K_p^z = - \begin{matrix} r\, c^+_{p\uparrow} c^+_{-p\downarrow} \\ c_{-p\downarrow} c_{p\uparrow} \end{matrix}$$

bozons ?

$$K_p^+ = c^+_{p\uparrow} c^+_{-p\downarrow}, \qquad K_p^- = c_{-p\downarrow} c_{p\uparrow}$$



$K_p^z = +\frac{1}{2}$

$K_p^z = -\frac{1}{2}$

zero pseudospin

$$H_{BCS} = \sum_p 2\varepsilon_p K_p^z - g \sum_{p,q} K_q^+ K_p^-$$

Ground state

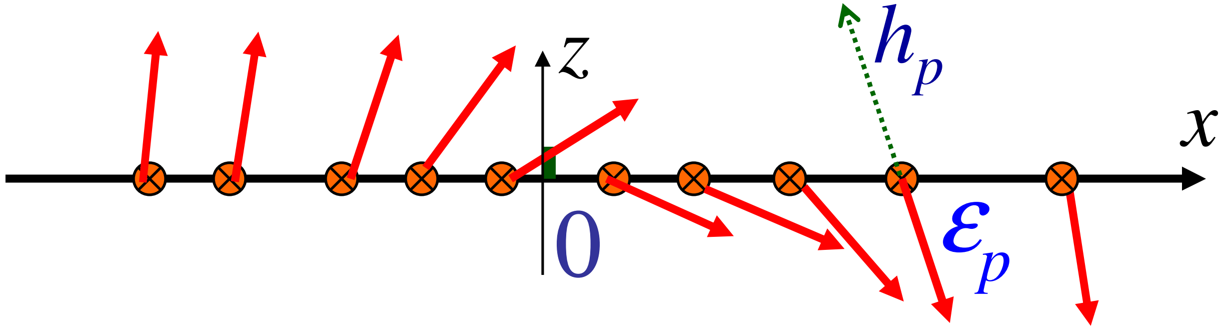
$$H_{BCS} = \sum_p 2\varepsilon_p K_p^z - g\sum_{p,q} K_q^+ K_p^-$$

mean-field $$H_{BCS} = \sum_p \vec{h}_p \vec{K}_p \qquad \vec{h}_p = (-2\Delta,\ 0,\ 2\varepsilon_p)$$

$$\Delta = g\sum_p \left\langle K_p^x \right\rangle = g\sum_p \left\langle c_{-p\downarrow} c_{p\uparrow} \right\rangle$$

$$|\vec{h}_p| = 2\sqrt{\varepsilon_p^2 + \Delta^2}$$

Paired ground state



BCS (1957)

Excitations

$$H_{BCS} = \sum_p 2\varepsilon_p K_p^z - g\sum_{p,q} K_q^+ K_p^-$$

mean-field $H_{BCS} = \sum_p \vec{h}_p \vec{K}_p \qquad \vec{h}_p = (-2\Delta,\ 0,\ 2\varepsilon_p)$

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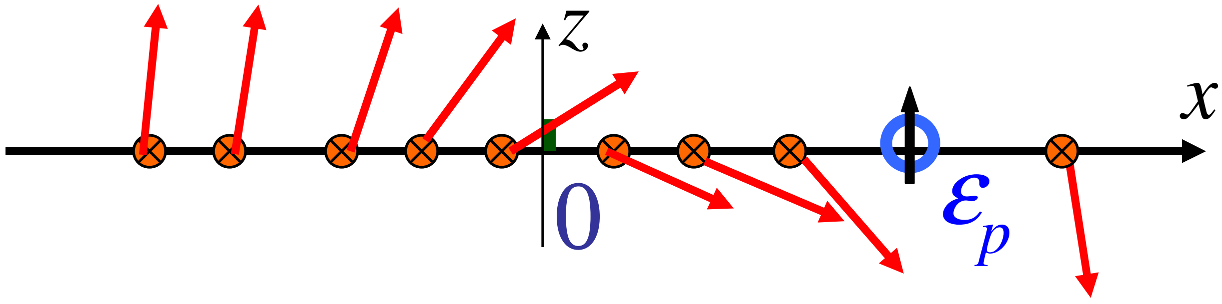
$$|\vec{h}_p| = 2\sqrt{\varepsilon_p^2 + \Delta^2}$$

1. single-particle excitations

BCS (1957)



$$E_{\mathrm{ex}} = \sqrt{\varepsilon_p^2 + \Delta^2} \qquad S = 1/2 \qquad Q = e$$

Excitations

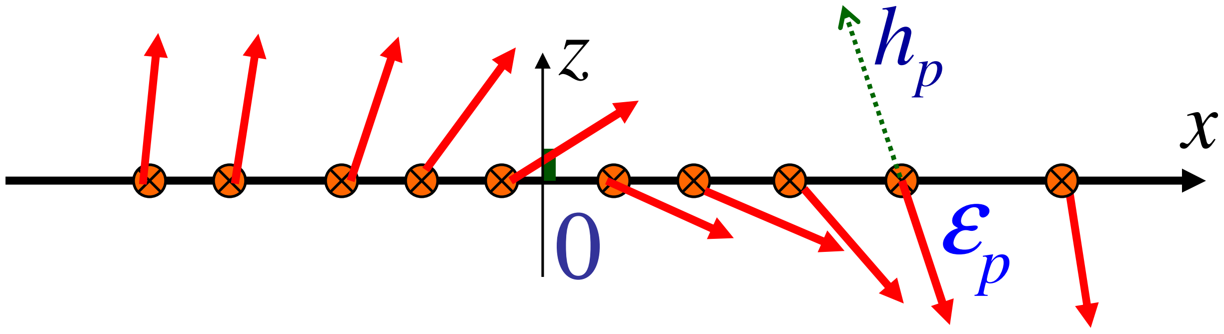
$$H_{BCS} = \sum_p 2\varepsilon_p K_p^z - g \sum_{p,q} K_q^+ K_p^-$$

mean-field $$H_{BCS} = \sum_p \vec{h}_p \vec{K}_p \qquad \vec{h}_p = (-2\Delta,\ 0,\ 2\varepsilon_p)$$

$$\Delta = g \sum_p \langle K_p^x \rangle = g \sum_p \langle c_{-p\downarrow} c_{p\uparrow} \rangle$$

$$|\vec{h}_p| = 2\sqrt{\varepsilon_p^2 + \Delta^2}$$

2. Excited pairs (pseudospin flips)



BCS (1957)

Excitations

$$H_{BCS} = \sum_p 2\varepsilon_p K_p^z - g \sum_{p,q} K_q^+ K_p^-$$

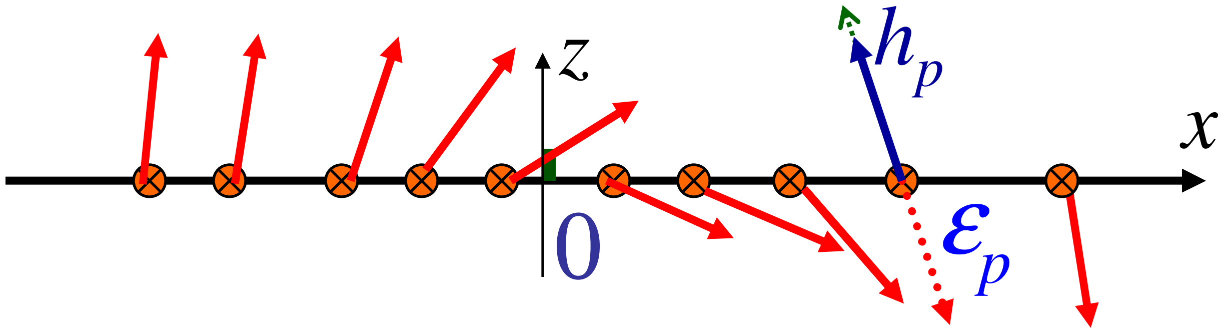
mean-field $H_{BCS} = \sum_p \vec{h}_p \vec{K}_p \qquad \vec{h}_p = (-2\Delta,\ 0,\ 2\varepsilon_p)$

$$\Delta = g \sum_p \left\langle K_p^x \right\rangle = g \sum_p \left\langle c_{-p\downarrow} c_{p\uparrow} \right\rangle$$

$$|\vec{h}_p| = 2\sqrt{\varepsilon_p^2 + \Delta^2}$$

2. Excited pairs (pseudospin flips)

BCS (1957)



$$E_{\mathrm{ex}} = 2\sqrt{\varepsilon_p^2 + \Delta^2} \qquad S = 0 \qquad Q = 0$$

How to detect!?

How to describe nonadiabatic dynamics at times of the order of $1/\Delta$?

$$H_{BCS} = \sum_{p} 2\varepsilon_{p} K_{p}^{z} - g \sum_{p,q} K_{q}^{+} K_{p}^{-}$$

$$\Delta = g \sum_{p} \left\langle K_{p}^{x} \right\rangle$$

$$\frac{d\vec{K}_{p}}{dt} = i\left[H_{BCS},\ \vec{K}_{p} \right] = \left(-2g\sum_{p} K_{p}^{x},\ -2g\sum_{p} K_{p}^{y},\ 2\varepsilon_{p} \right) \times \vec{K}_{p}$$

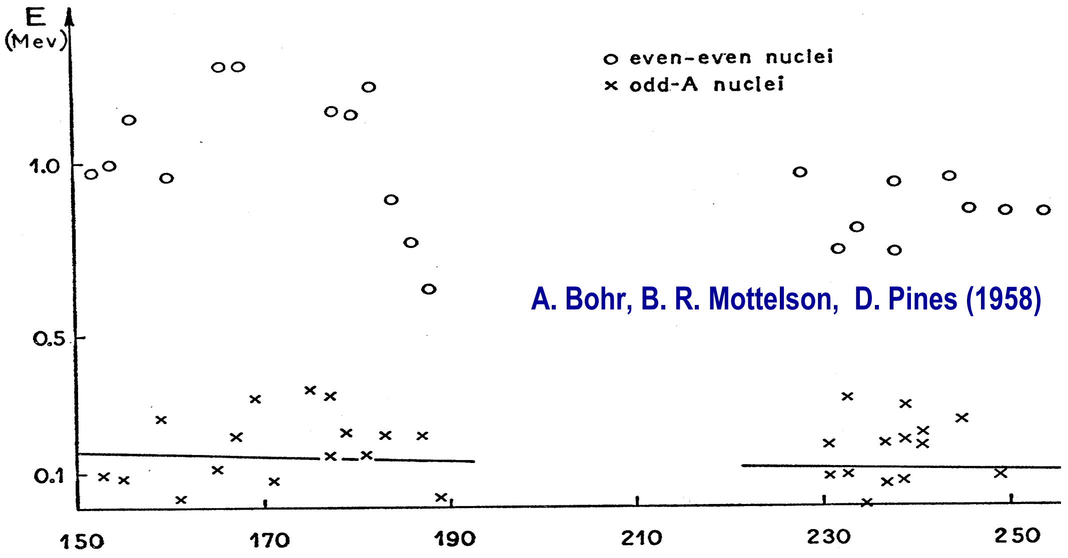
Nonlinear, many-body, far from equilibrium – normally would be intractable analytically

H_{BCS} is integrable – infinitely many integrals of motion!

1. Nuclear superconductivity

Richardson &Sherman (1964)

“Exact eigenstates of the pairing-force Hamiltonian”



A. Bohr, B. R. Mottelson, D. Pines (1958)

Central spin models

$$\boxed{H_p = \vec{K}_p \bullet \sum_q \gamma_q \vec{K}_q + \vec{B} \bullet \vec{K}_p}$$ – integrals of motion for H_{BCS}

Sachdev & Bhatt (1987) *”Spin dynamics across the metal-insulator transition”*

Prokof’ev & Stamp (1993) *”Giant spins and topological decoherence…”*

Khaetskii, Loss, Glazman (2002) *”Electron spin evolution induced by interaction with nuclei in a quantum dot”*

⋮

Taylor et. al. (2008) *”Relaxation, dephasing, and quantum control of electron spins in double quantum dots”*

(new paper every week)

Secret life of BCS integrals

Gaudin magnets

$$\boxed{H_p = \vec{K}_p \bullet \sum_q \gamma_q \vec{K}_q + \vec{B} \bullet \vec{K}_p}$$ – integrals of motion for H_{BCS}

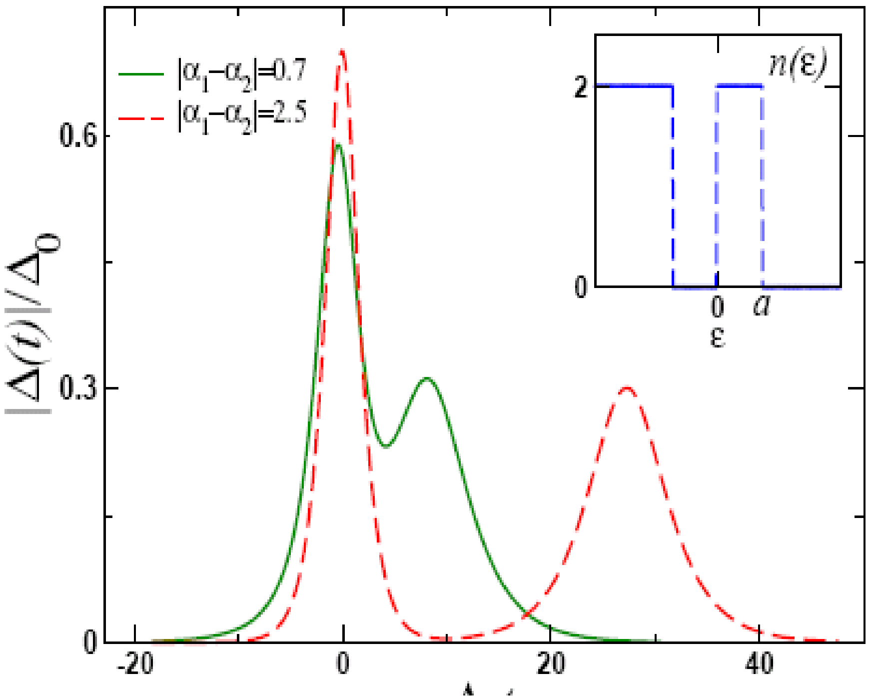
Gaudin (1972 – 1976) *"Diagonalisation d'une classe d'hamiltoniens de spin"*
Sklyanin (1987) *"Separation of variables in the Gaudin model"*
Kuznetsov (1992) *"Quadrics on real Riemannian spaces of constant curvature: … connection with Gaudin magnet"*
⋮

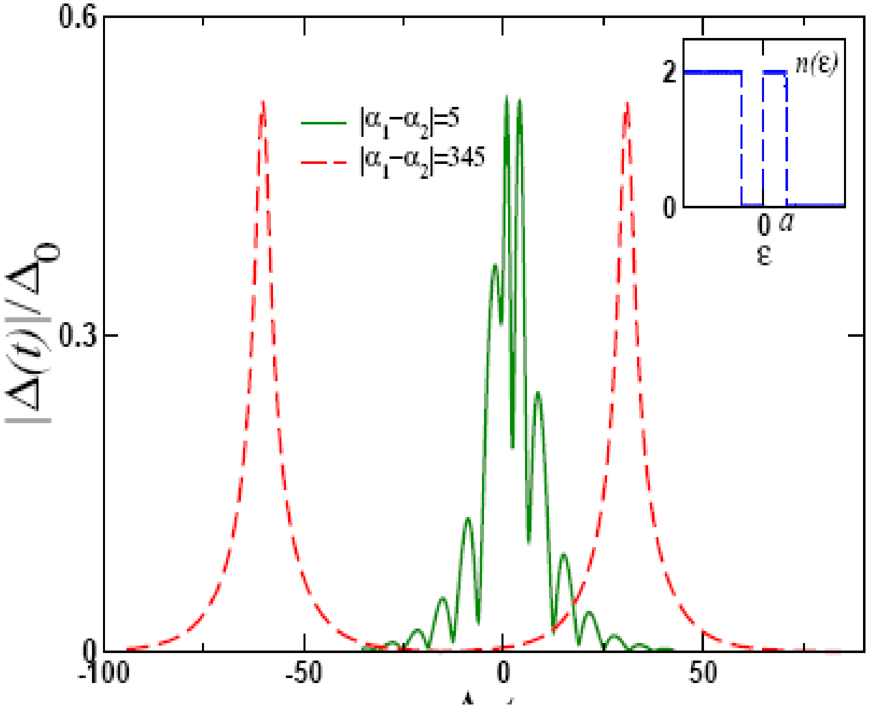
Exact solution for dynamics of H_{BCS}

E.Y., Altshuler, Kuznetsov, Enolskii, *PRB* (2005)

(nonlinear integrable dynamics *cf.* Korteweg–de Vries, nonlinear Schrodinger, Landau-Lifshitz, sine-Gordon etc.)

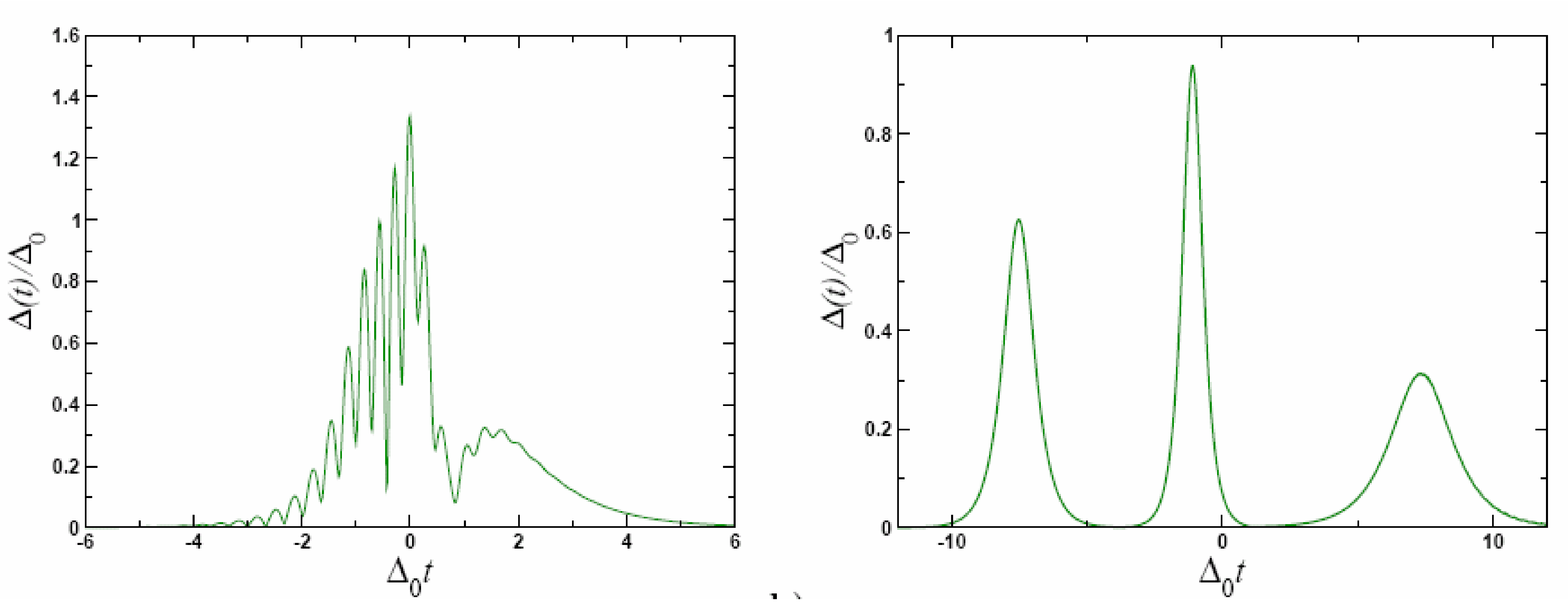
Normal and anomalous multi-solitons





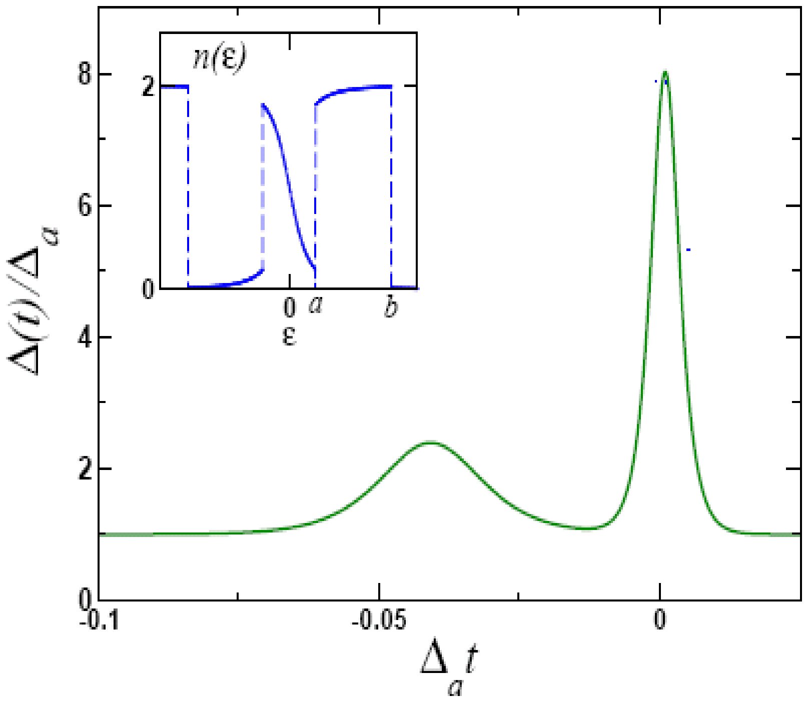
E.Y., arXiv:0807.3181 (2008)

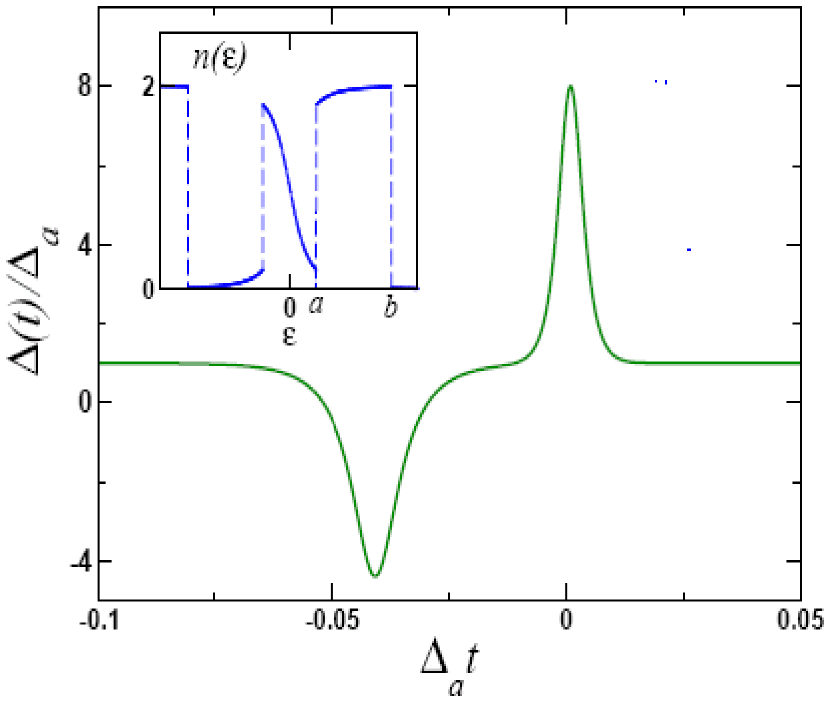
Normal and anomalous multi-solitons



E.Y., arXiv:0807.3181 (2008)

Normal and anomalous multi-solitons





E.Y., arXiv:0807.3181 (2008)

How to describe nonadiabatic dynamics at times of the order of $1/\Delta$?

$$H_{BCS} = \sum_{p} 2\varepsilon_p K_p^z - g \sum_{p,q} K_q^+ K_p^-$$

$$\Delta = g \sum_p \langle K_p^x \rangle$$

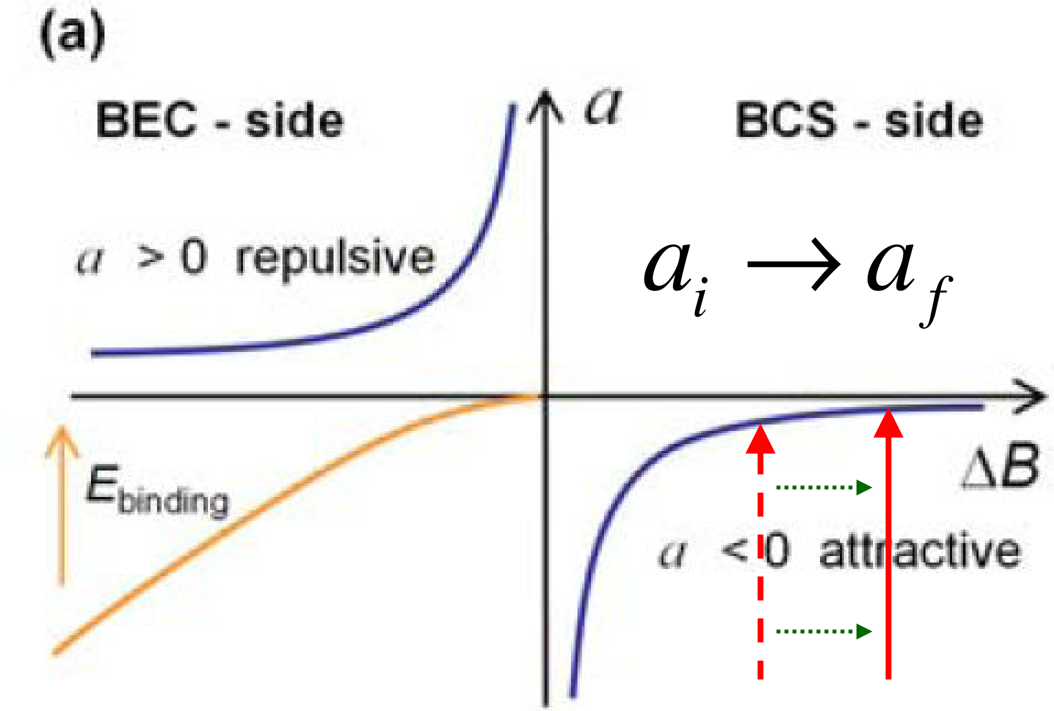
$$\frac{d\vec{K}_p}{dt} = i\left[H_{BCS},\ \vec{K}_p\right] = \left(-2g\sum_p K_p^x,\ -2g\sum_p K_p^y,\ 2\varepsilon_p\right) \times \vec{K}_p$$

Nonlocal & no translational invariance – new approach needed

E.Y., M. Dzero, *Phys. Rev. Lett.* (2006).

E. Y., O. Tsyplyatyev, B. Altshuler, *Phys. Rev. Lett.* (2006).

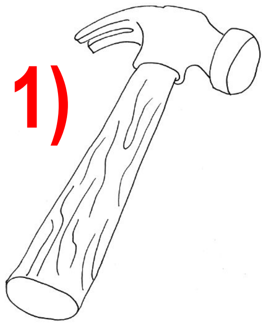
Problem



$$a_i \to a_f$$

$$g = 2k_F \,|a|\, / \pi \nu_F$$

$$\Delta_{eq} = 2E_F e^{-1/g\nu_F}$$



1) $\Delta_i \to \Delta_f$

2) $\Psi_{cond}(t=0) = \left|\text{Ground state with wrong coupling}\right\rangle$

3) $\Psi_{cond}(t>0) = ?$

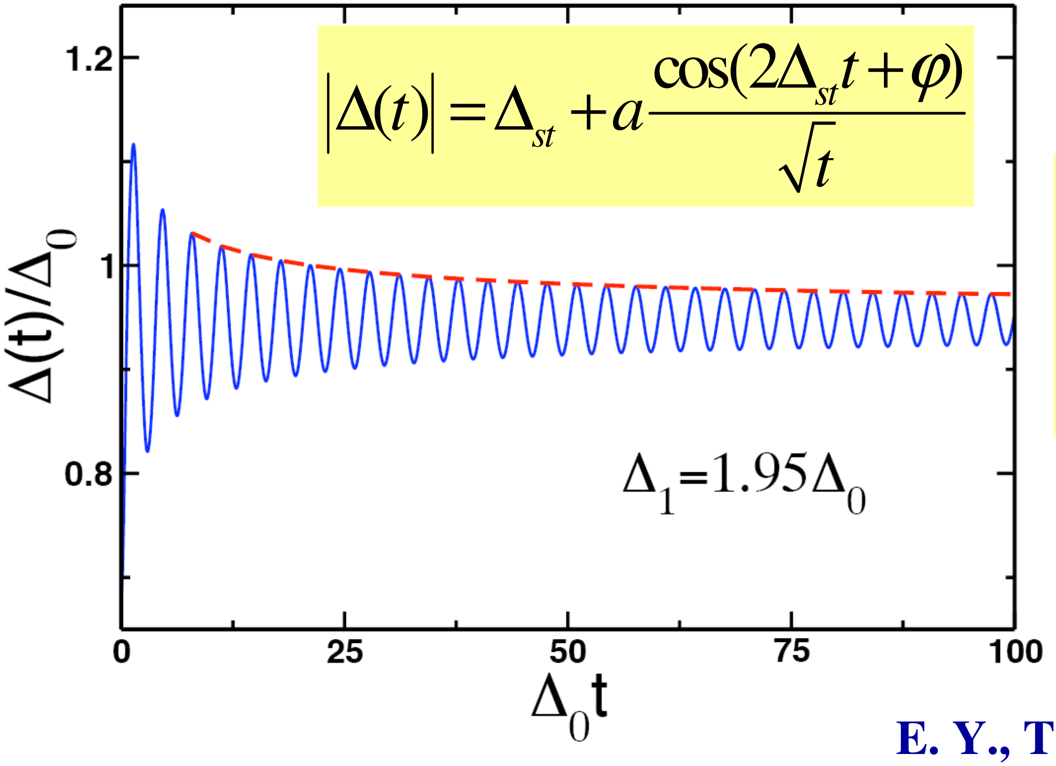
$t \sim \Delta^{-1}$, weak coupling

Gaped regime

$g_i = g_f$ → $|\Delta(t)| = \Delta_i = \Delta_f$

$\Delta_i = 2E_F e^{-1/g_i \nu_F}$ $\Delta_f = 2E_F e^{-1/g_f \nu_F}$

This type of behavior persists for some range of $\dfrac{\Delta_f}{\Delta_i}$



$$|\Delta(t)| = \Delta_{st} + a\frac{\cos(2\Delta_{st}t + \varphi)}{\sqrt{t}}$$

$$\boxed{|\Delta(t)| \to \Delta_{st}}$$

$$\Delta_{st} = \Delta_i \cos\eta \quad < \Delta_f$$

$$\exp\left[-\eta\tan(\eta/2)\right] = \frac{\Delta_f}{\Delta_i}$$

$$|\eta| \le \pi/2$$

E. Y., Tsyplyatyev, Altshuler *PRL* (2006)

Gaped regime

$|\Delta(t)| \to \Delta_{st}$

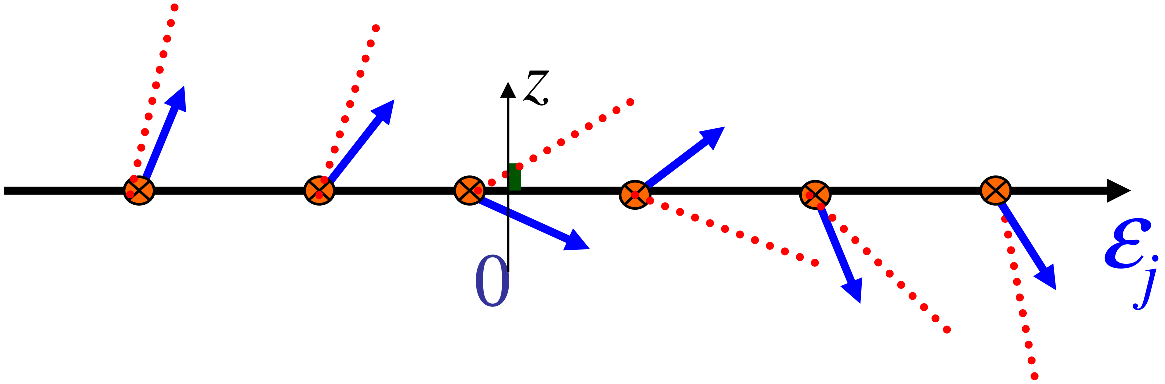
What is the nature of the steady state?

Each spin rotates in a constant magnetic field

$$\frac{d\vec{K}_p}{dt} = (-2\Delta_{st}, 0, 2\varepsilon_p) \times \vec{K}_p$$

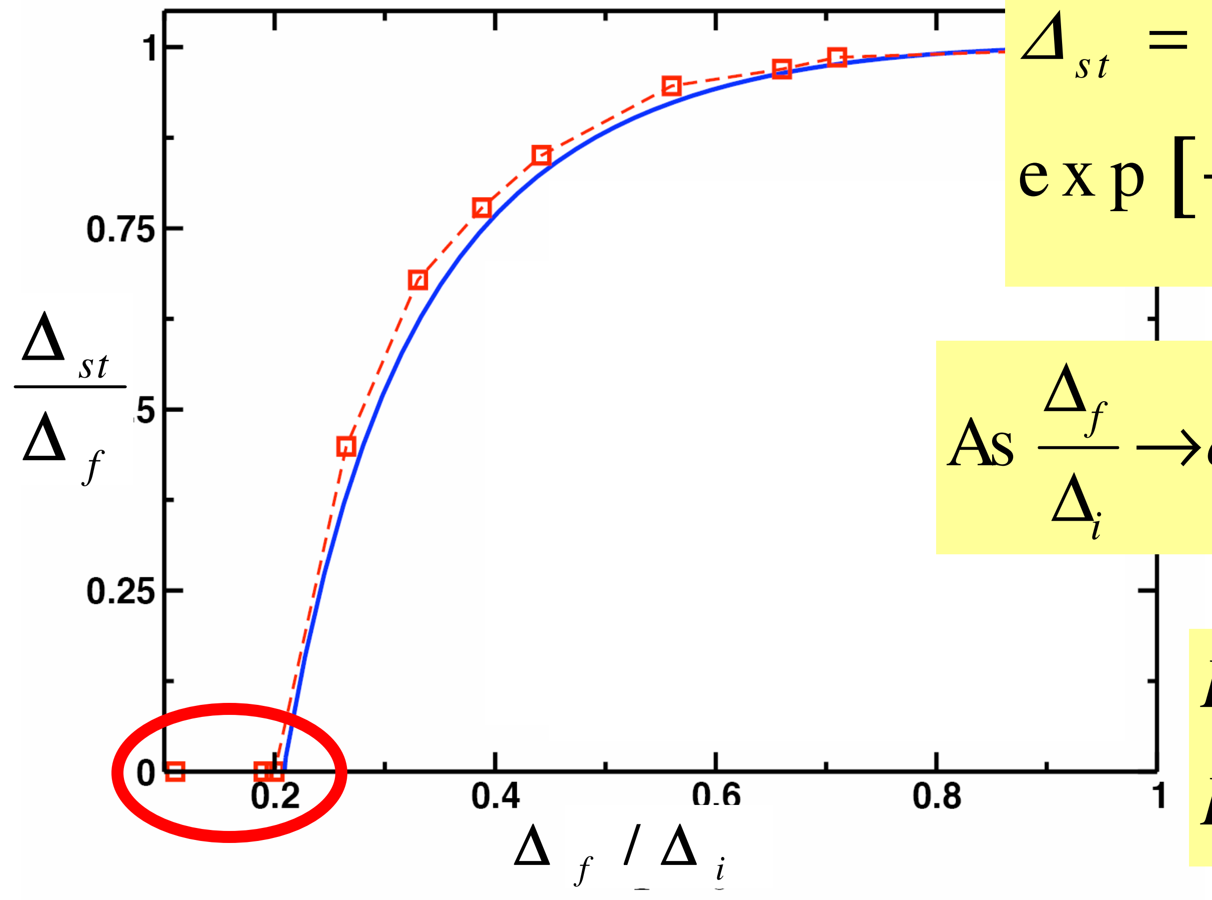
$$\omega(\varepsilon) = 2\sqrt{\varepsilon^2 + \Delta_{st}^{\ 2}}$$

$$\Delta(t) = g\sum_p \langle K_p^x \rangle \quad = \text{const} + \int_{-\infty}^{\infty} a(\omega)\cos \omega t \; d\varepsilon \to \text{const}$$



Dephasing similar to inhomogeneous line broadening in NMR

Dynamical "phase transition" at $\left(\frac{\Delta_f}{\Delta_i}\right)_c = e^{-\pi/2} \approx 0.21$



$$\Delta_{st} = \Delta_i \cos\eta \quad < \Delta_f$$

$$\exp\left[-\eta \tan(\eta/2)\right] = \frac{\Delta_f}{\Delta_i}$$

$$\text{As } \frac{\Delta_f}{\Delta_i} \to e^{-\pi/2} \quad \eta \to \frac{\pi}{2} \text{ and } \Delta_{st} \to 0!!$$

$E_n - E_{sf} \propto \Delta_f^2$ decreases

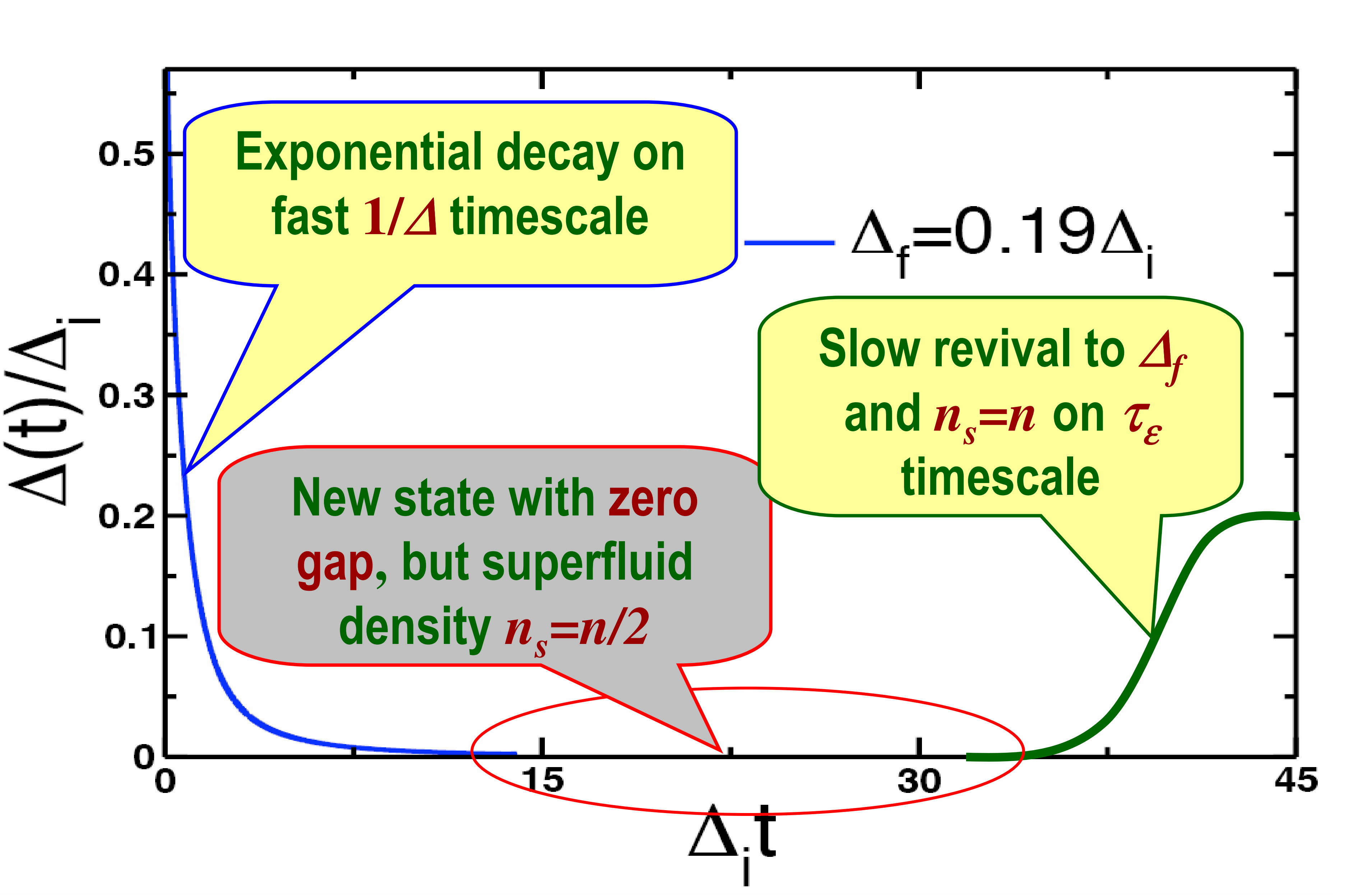
$E_i - E_{sf}$ increases

$$\text{For } \frac{\Delta_f}{\Delta_i} \le e^{-\pi/2} \approx 1/5, \quad \Delta_{st} = 0!!$$

Barankov & Levitov *PRL* (2006)

E. Y. & Dzero: *PRL* (2006)

II. Gapless regime $\Delta_f / \Delta_i < e^{-\pi/2} \approx 1/5$



Exponential decay on fast $1/\Delta$ timescale

New state with zero gap, but superfluid density $n_s = n/2$

Slow revival to Δ_f and $n_s = n$ on τ_ε timescale

What happens when Δ_f / Δ_i is increased?

$\Delta_f / \Delta_i = \infty$, i.e $\Delta_i = 0$

$$\frac{d\mathbf{K}_j}{dt} = \left(-2g\sum_j K_j^x,\ -2g\sum_j K_j^y,\ 2\varepsilon_j \right) \times \mathbf{K}_\mathbf{j}$$

Linear analysis around the normal state

normal modes $\omega_j = 2\varepsilon_j$ **unstable mode** $\omega_0 = i\Delta_f$

$$|\Delta(t)| = \int_{-E_F}^{E_F} A(\varepsilon)\cos(2\varepsilon t + \varphi)\, d\varepsilon + A_0 e^{\Delta_f t} \approx A_0 e^{\Delta_f t}$$

$$\Delta_i = 2E_F e^{-1/g_i \nu_F} \quad \Delta_f = 2E_F e^{-1/g_f \nu_F}$$

What happens when Δ_f / Δ_i is increased?

$$\Delta_f \,/\, \Delta_i > e^{\pi/2}$$

$$\frac{d\mathbf{K}_j}{dt} = \left(-2g\sum_j K_j^x,\; -2g\sum_j K_j^y,\; 2\varepsilon_j \right) \times \mathbf{K}_\mathbf{j}$$

Nonlinear analysis

E.Y., Tsyplyatyev, Altshuler: (2006).

$$|\Delta(t)| = \int_{-E_F}^{E_F} A(\omega)\cos(\omega t + \varphi)\,d\omega + \sum_{n=1}^{\infty} B_n \cos(n\omega_0 t + \varphi_n)$$

vanishes for $t \gg 1/\Delta_f$

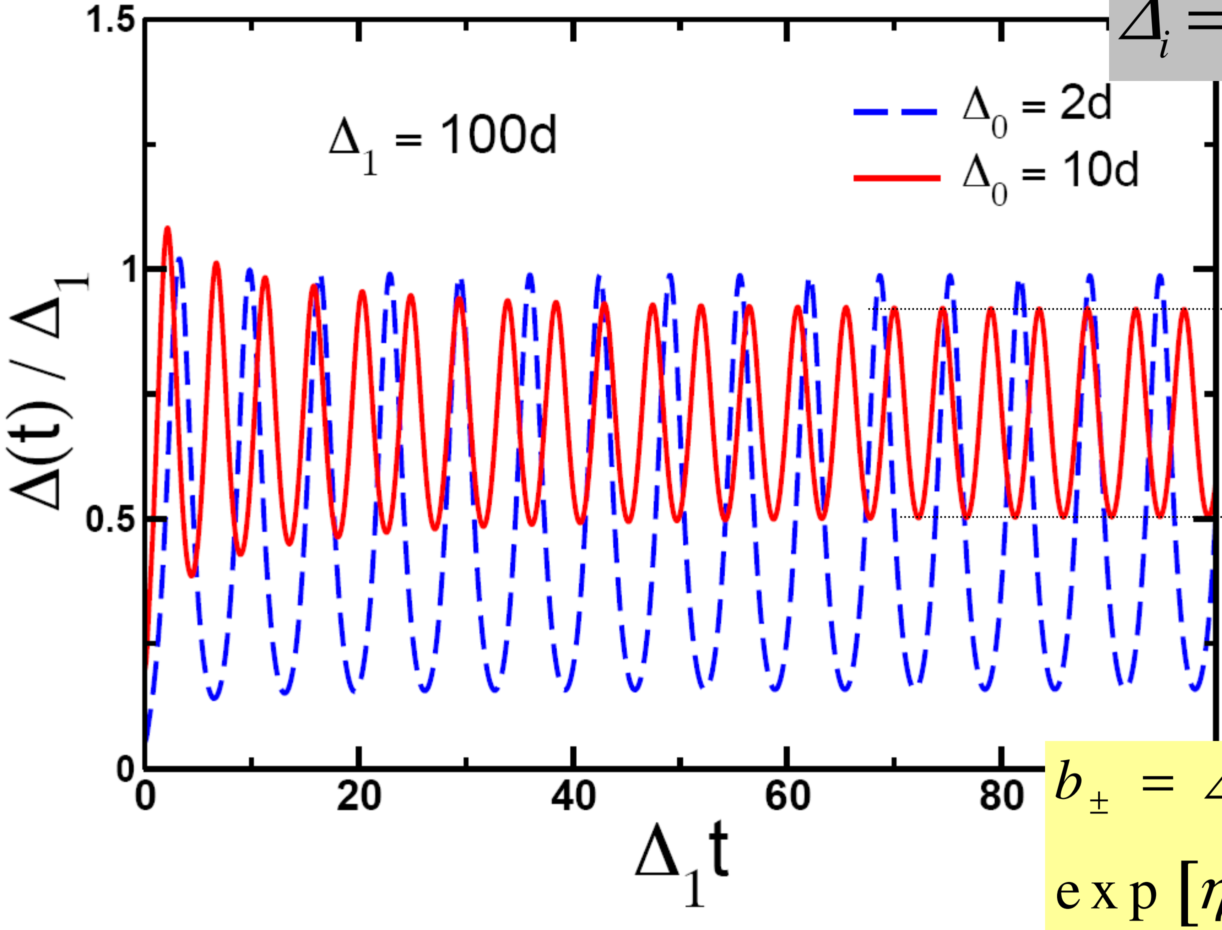
periodic!

$$|\Delta(t)| \to \text{ periodic oscillations for } \Delta_f / \Delta_i > e^{\pi/2}$$

Barankov & Levitov: *PRL* (2006)

Dynamical transition to oscillating order parameter

$$\Delta_i = 2E_F e^{-1/g_i \nu_F} \quad \Delta_f = 2E_F e^{-1/g_f \nu_F}$$



$$\Delta_+ = \frac{b_+ + b_-}{2}$$

$$\Delta_- = \frac{b_+ - b_-}{2}$$

$$|\Delta(t)| \to \Delta_+ \mathrm{dn}[\Delta_+(t - t_0), k] \quad k = \Delta_+^2 / \Delta_-^2$$

Barankov & Levitov: *PRL* (2006)

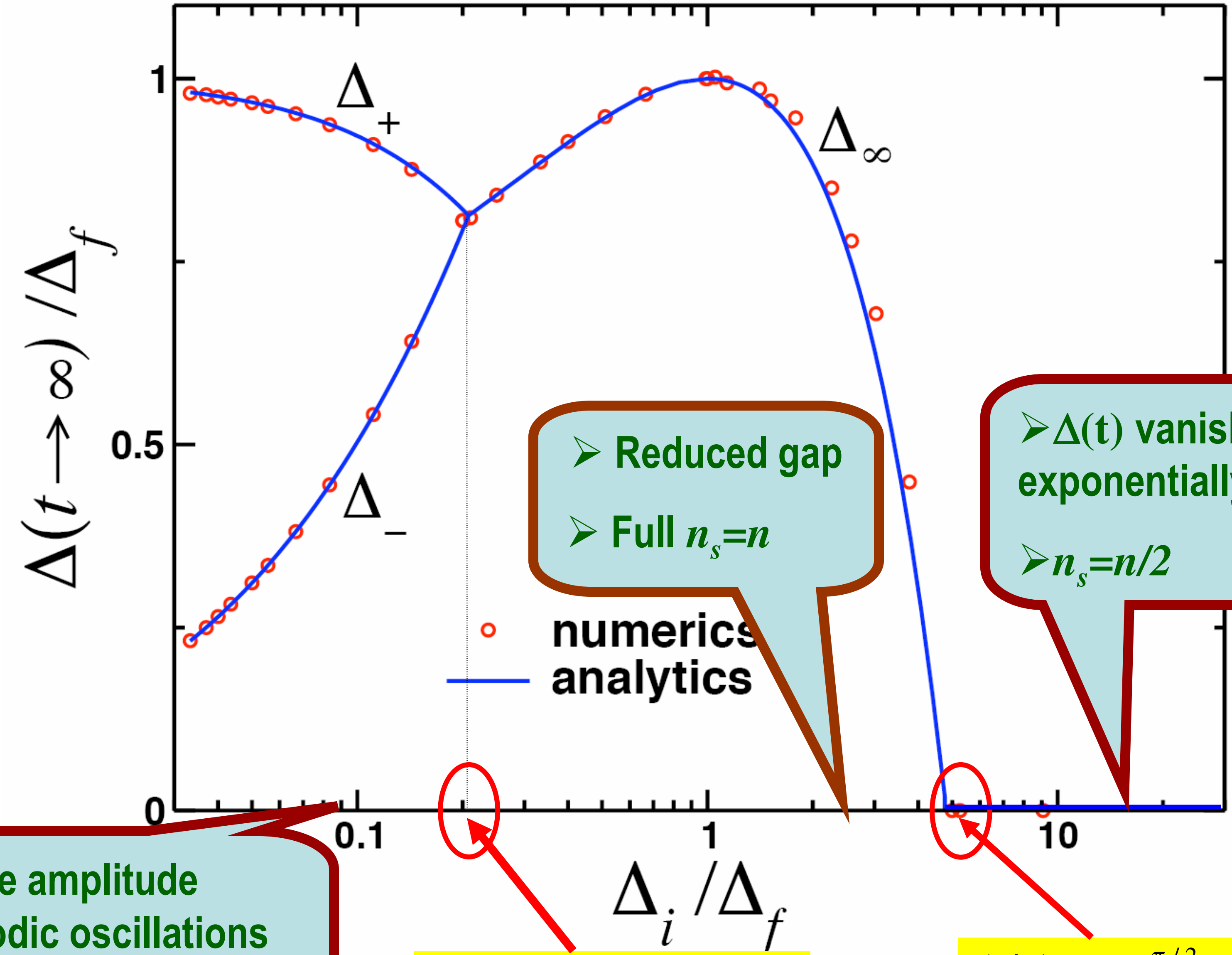
$$b_\pm = \Delta_i \cosh \eta_\pm \quad < \Delta_f$$

$$\exp\left[\eta_+ \tanh(\eta_+/2)\right] = \frac{\Delta_f}{\Delta_i}$$

$$\exp\left[\eta_- \coth(\eta_-/2)\right] = \frac{\Delta_f}{\Delta_i}$$

E.Y., unpublished

Time evolution in non-adiabatic regime



- Reduced gap
- Full $n_s=n$

- Δ(t) vanishes exponentially
- $n_s=n/2$

Large amplitude periodic oscillations of Δ(t)

$\Delta_i/\Delta_f = e^{-\pi/2} \approx 1/5$

$\Delta_i/\Delta_f = e^{\pi/2} \approx 5$

Barankov & Levitov *PRL* (2006); E. Y. & Dzero: *PRL* (2006)

Many body systems driven far from equilibrium can acquire dynamical phases (steady states) with properties quite distinct from equilibrium phases

The Luttinger model following a sudden interaction switch-on
Cazalilla, M. A. In *Physical Review Letters 97 156403 (2006)*

Quench dynamics and non equilibrium phase diagram of the Bose-Hubbard model
Kollath, Corinna; Laeuchli, Andreas; Altman, Ehud In *Physical Review Letters 98 180601 (2007)*

Hard-core bosons on optical superlattices: Dynamics and relaxation in the superfluid and insulating regimes Rigol, Marcos; Muramatsu, Alejandro; Olshanii, Maxim (2006-12-15) In *Physical Review A 74 053616 (2006)*

Strongly correlated fermions after a quantum quench
Manmana, S. R.; Wessel, S.; Noack, R. M. et al In *Physical Review Letters 98 210405 (2007)*

Analysis of quench dynamics of coupled one dimensional condensates using quantum sine Gordon model
Gritsev, Vladimir; Demler, Eugene; Lukin, Mikhail et al arXiv.org:cond-mat/0702343

Quenching, relaxation, and a central limit theorem for quantum lattice systems
Cramer, M.; Dawson, C. M.; Eisert, J. et al arXiv.org:cond-mat/0703314

Time-dependent evolution of two coupled Luttinger liquids
Perfetto, E. In *Physical Review B 74 205123 (2006)*

Spontaneous quantum condensation in an optically-pumped microcavity far from equilibrium Eastham, P. R.; Phillips, R. T. arXiv.org:0708.2009

Quantum Quenches in Extended Systems
Calabrese, Pasquale; Cardy, John In *Journal of Statistical Mechanics Theory and Experiment 0706 008 (2007)*

Thermalization and its mechanism for generic isolated quantum systems
Rigol, Marcos; Dunjko, Vanja; Olshanii, Maxim (2007-08-09) oai:arXiv.org:0708.1324

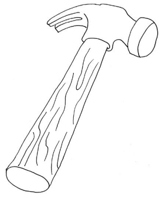
Non-thermal steady states after an interaction quench in the Falicov-Kimball model
Eckstein, Martin; Kollar, Marcus arXiv.org:0707.2789

Nonequilibrium pairing instability in ultracold Fermi gases with population imbalance
Tomadin, Andrea; Polini, Marco; Tosi, M. P. arXiv.org:0711.2384

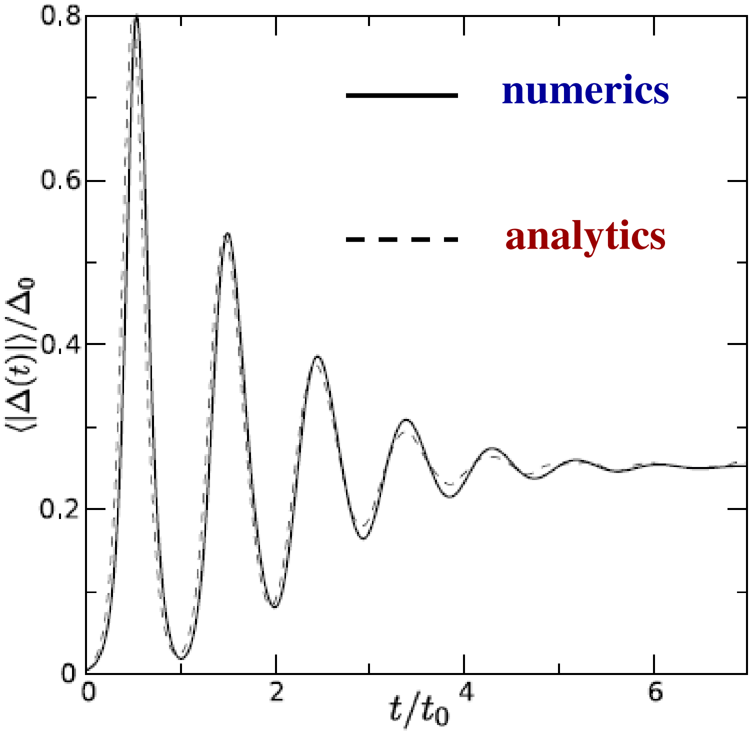
Dephasing due to thermal fluctuations

E. Y., Tsyplyatyev: arXiv:0712.4280

1. $T > T_c(g_i)$ at t=0 – normal phase, then



$g_i \rightarrow g_f$



fast exponential decay

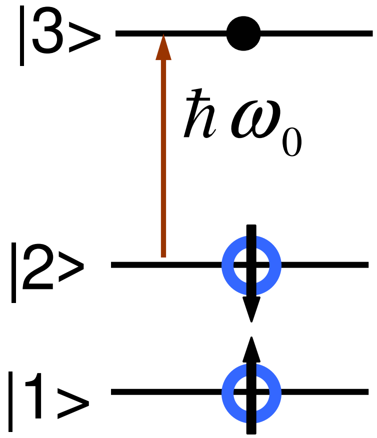
$$\frac{t_0}{\langle \tau \rangle} = \frac{1}{\pi^2} \ln \left(\frac{4\Delta_f^2}{T\delta} \right)$$

average period

level spacing

2. $T < T_c(g_i)$ effectively no decay

RF Spectroscopy

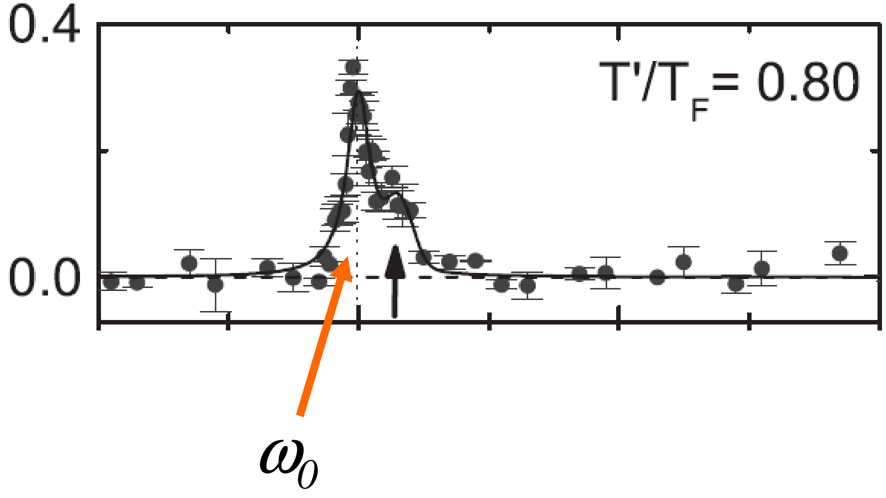


$$^6Li \quad |1\rangle = |F = 1/2, m_F = 1/2\rangle \quad |2\rangle = |F = 1/2, m_F = -1/2\rangle$$
$$|3\rangle = |F = 3/2, m_F = -3/2\rangle$$

Measure loss of atoms in state |2>

$$I = -\frac{dN_2}{dt}$$

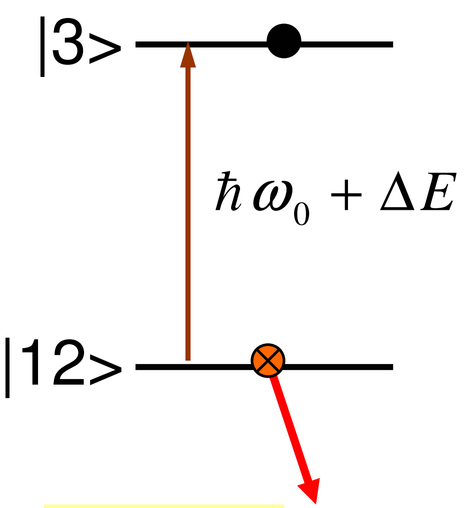
Unpaired state



sharp peak at $\omega_0 = \dfrac{E_3 - E_2}{\hbar}$

RF Spectroscopy

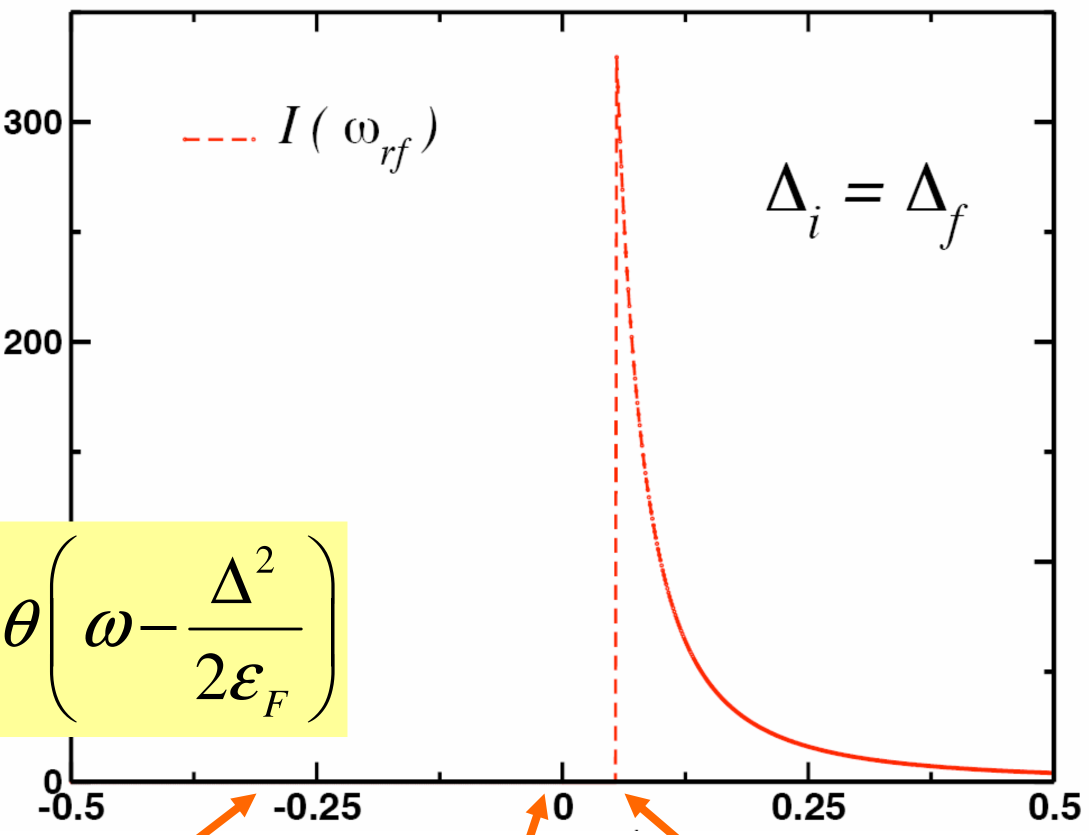
Paired ground state



$$I = \frac{dN_2}{dt}$$

$$I_{eq}(\omega) = 2\pi |T|^2 \frac{\Delta^2}{\omega^2}\theta\left(\omega - \frac{\Delta^2}{2\varepsilon_F}\right)$$

Torma & Zoller *PRL* (2000)



no loss at negative ω

ω_0

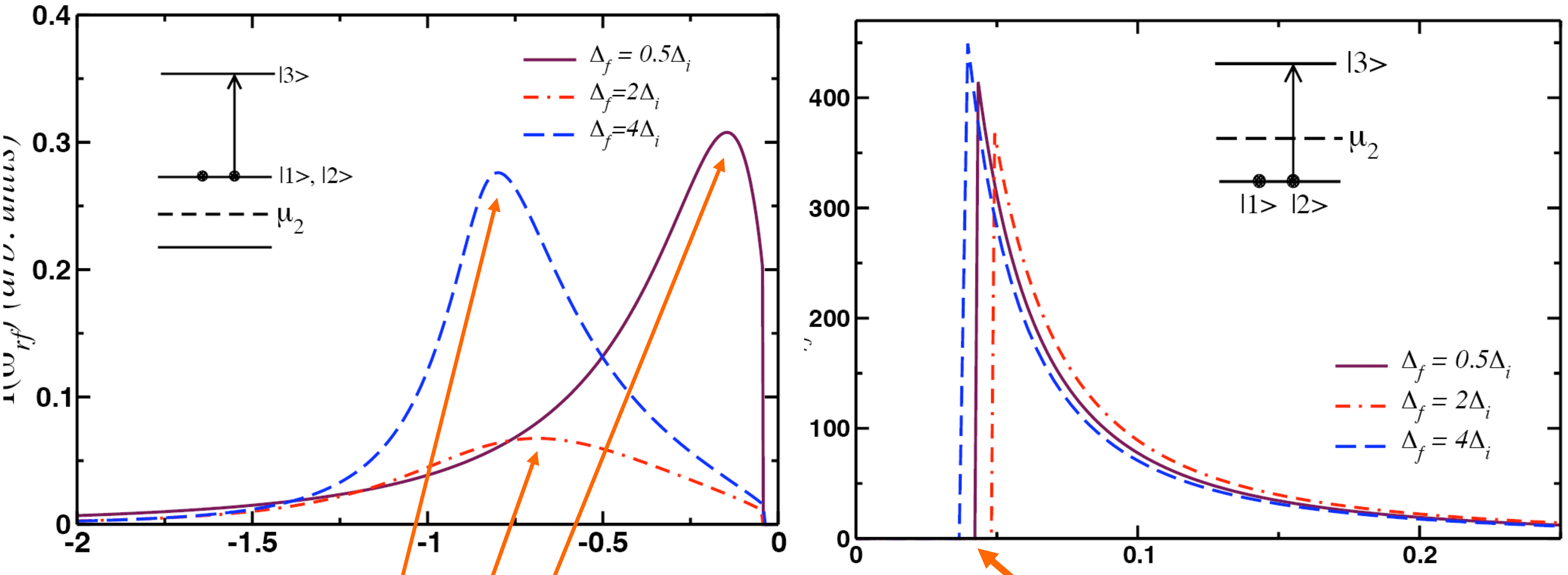
$$(\Delta E)_{\min} \approx \Delta^2 / 2\varepsilon_F$$

Gaped regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

$$\boxed{|\Delta(t)| \rightarrow \Delta_{st}}$$

$$I(\omega) = 2\pi |T|^2 \frac{\Delta^2}{\omega^2}\left[\sin^2 \chi(\omega)\ \theta\left(\omega - \Delta^2/2\varepsilon_F\right) + \cos^2 \chi(\omega)\theta\left(-\omega - \Delta^2/2\varepsilon_F\right)\right]$$

M. Dzero, E.Y., B. Altshuler, P. Coleman, ***Phys. Rev. Lett.*** **(2007).**

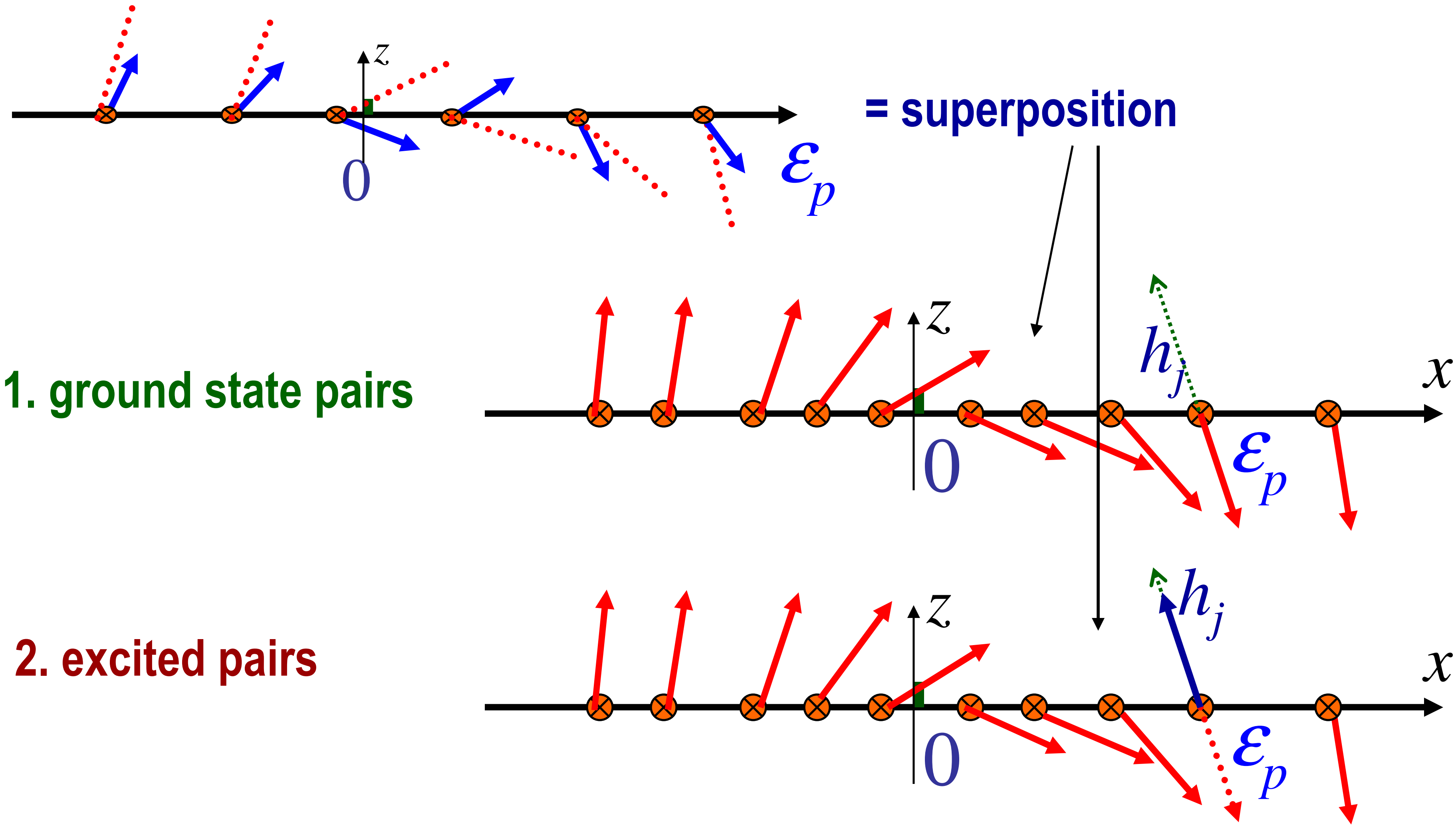


$$\hbar\tilde{\omega} \approx -\Delta_{st}$$

$$\hbar\omega \approx \Delta_{st}^2/\varepsilon_F$$

Gaped regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

Each spin rotates in a constant magnetic field



= superposition

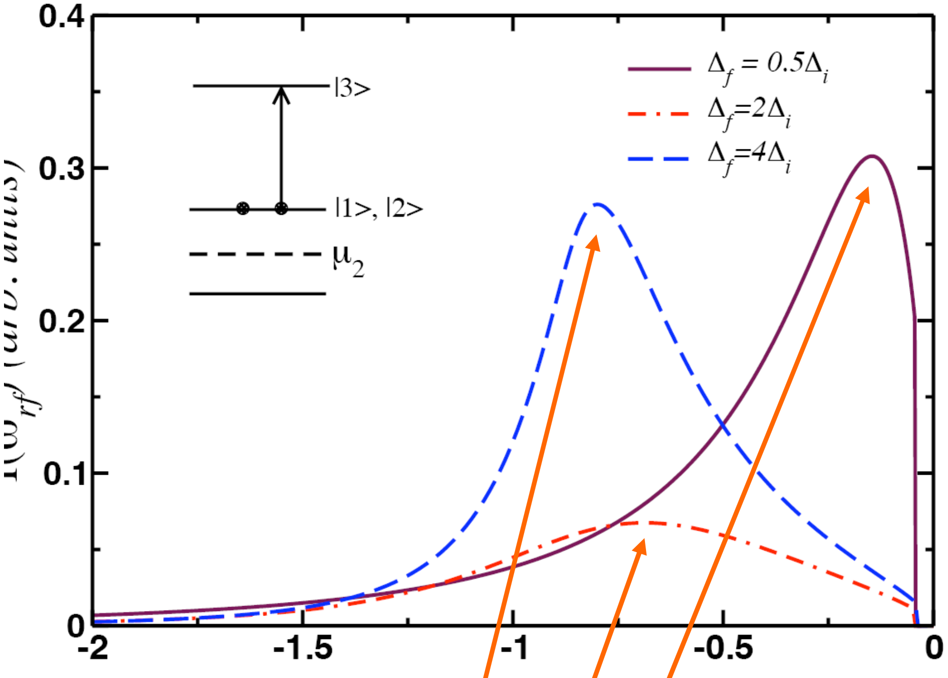
1. ground state pairs

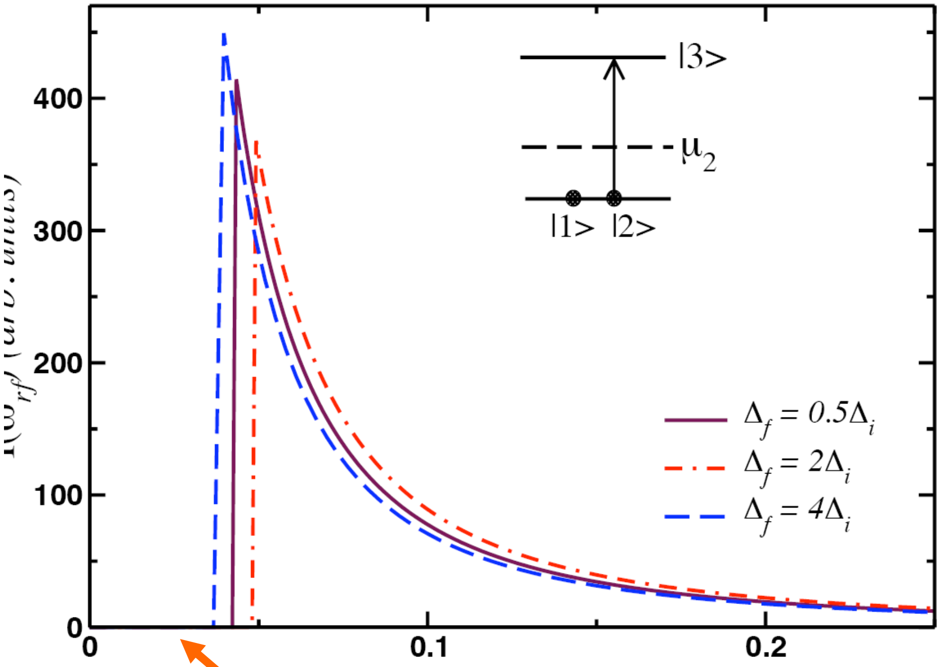
2. excited pairs

Gaped regime $e^{-\pi/2} < \frac{\Delta_f}{\Delta_i} < e^{\pi/2}$

$$\boxed{|\Delta(t)| \to \Delta_{st}}$$

M. Dzero, E.Y., B. Altshuler, P. Coleman, *Phys. Rev. Lett.* (2007)





$$\boxed{\hbar\tilde{\omega} \approx -\Delta_{st}}$$

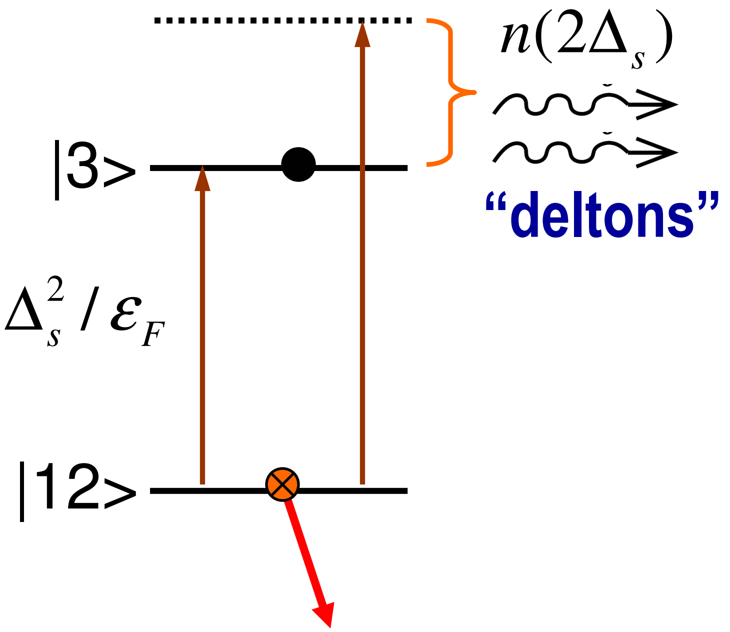
$$\boxed{\hbar\omega \approx \Delta_{st}^2 / 2\varepsilon_F}$$

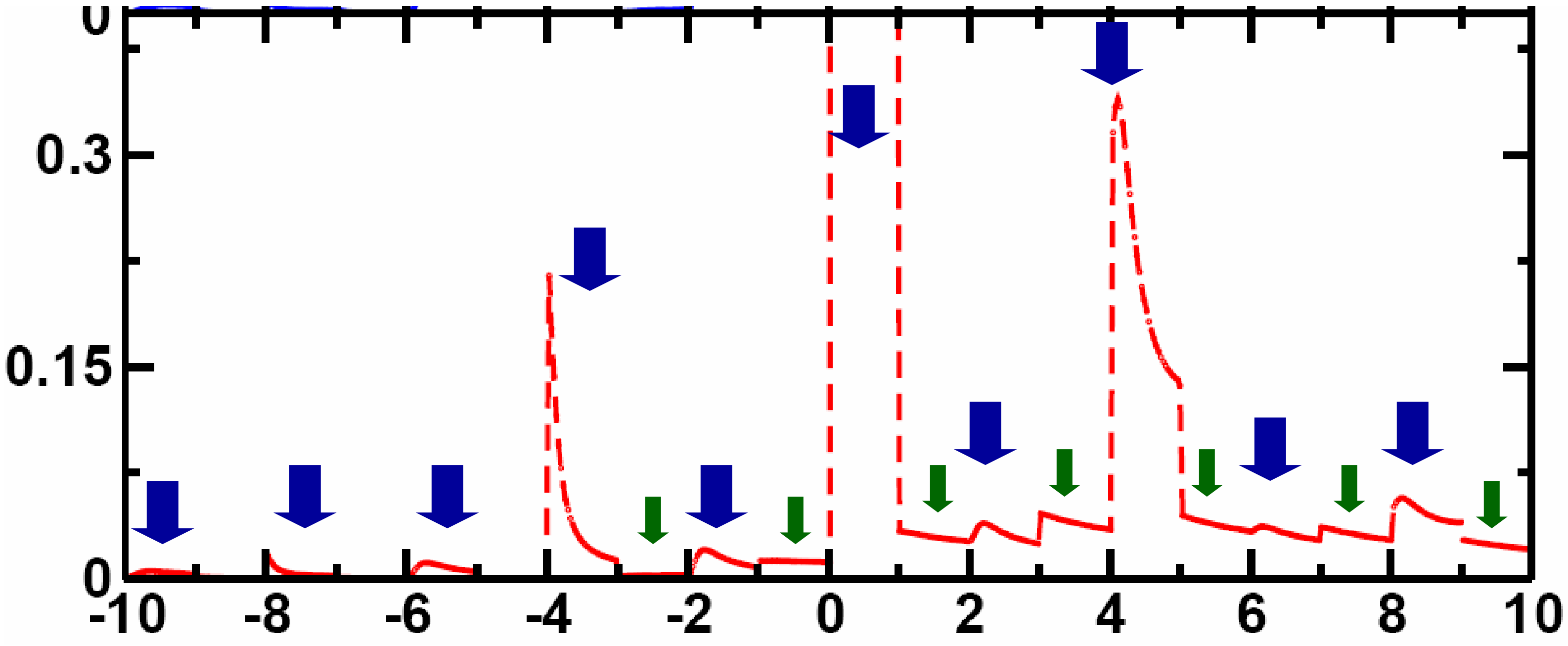
Regime of oscillating order parameter

two series of equidistant peaks!

$$\hbar\omega_n = \Delta_s^2/2\varepsilon_F + 2n\Delta_s$$

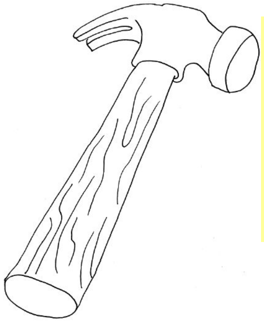
$$\hbar\tilde{\omega}_n = -\Delta_s + 2n\Delta_s$$





Cooper pair turbulence (in progress)

$\Psi_{cond}\left(t=0\right)$ – spatially homogeneous



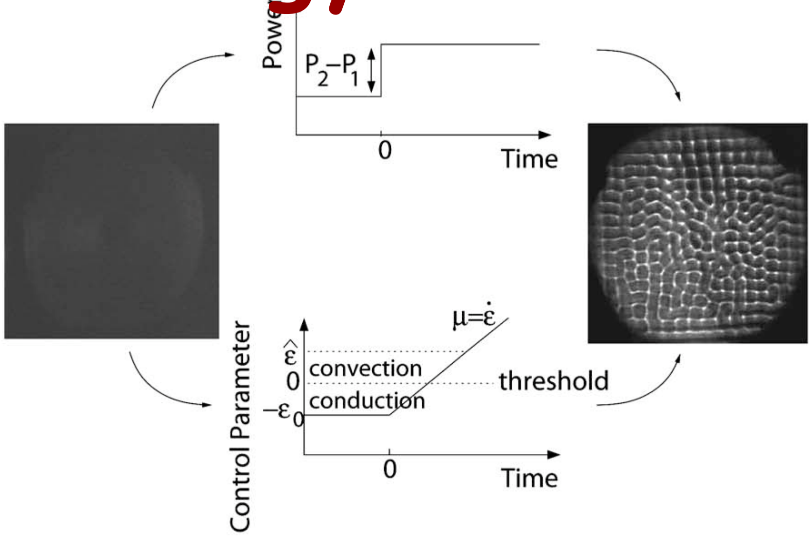
$g_i \to g_f$ – (spatially) uniform quench

$\Delta(t)$ – spatially homogeneous solution

How can spatial inhomogeneities be induced by a uniform quench?

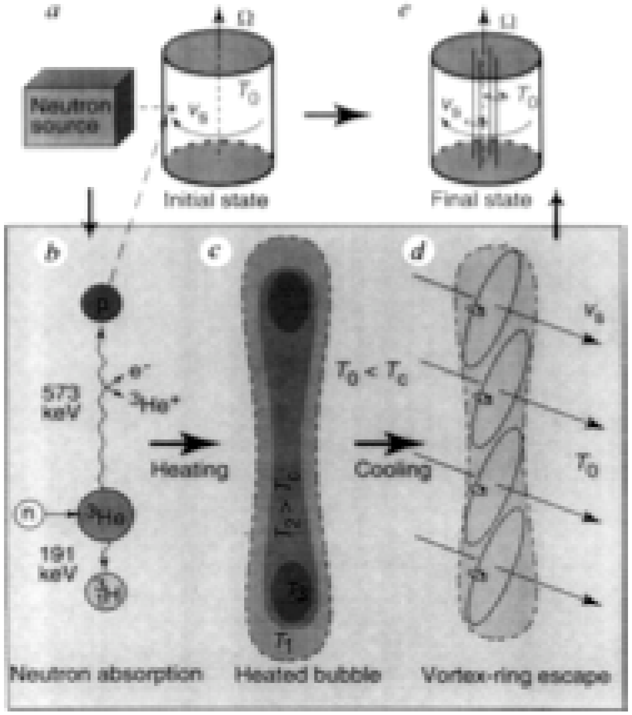
Pattern formation: cosmology in a lab

Parameter quench – “Big Bang”



Convection-conduction in a Rayleigh-Benard system. Pattern formation develops as a result of a sudden change of a control parameter (external power)

S. Casado *et al.*, Phys. Rev. E, 2006

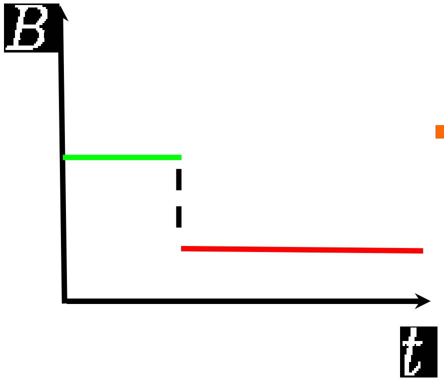


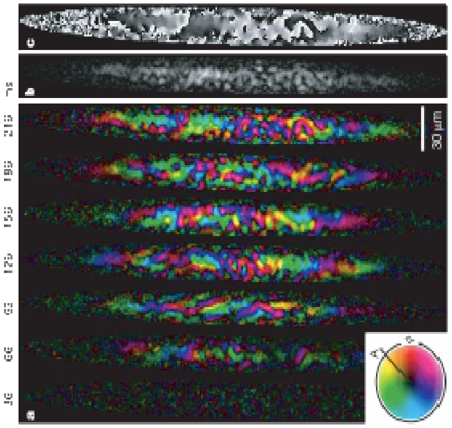
neutron scattering in superfluid ^{3}He: production of the “hot” spots. Cooling of the “hot” spots gives rise to the formation of vortices

V. M. H. Ruutu *et al.*, Nature (London), 1996

Pattern formation: cosmology in a lab

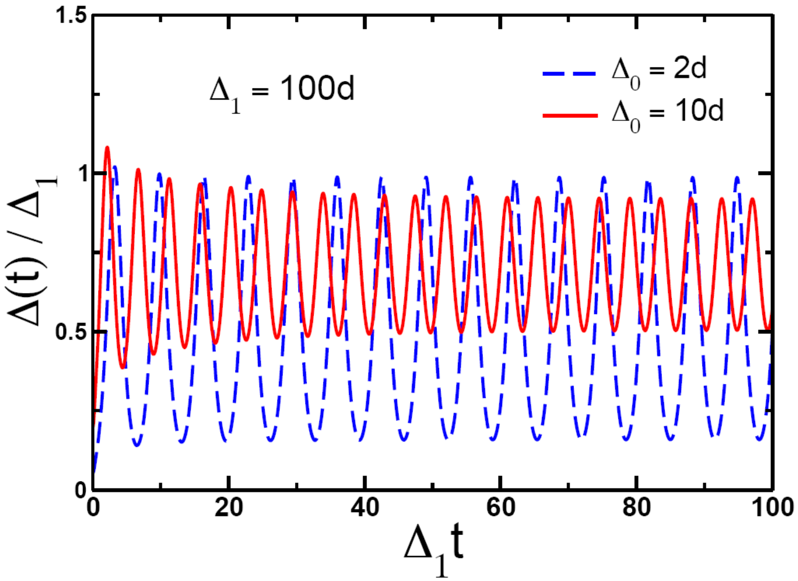
Parameter quench – “Big Bang”





magnetic domain formation in ferromagnetic BEC following a sudden quench of the applied magnetic field, Sadler et al., Nature (London), 2006

Cooper pair turbulence

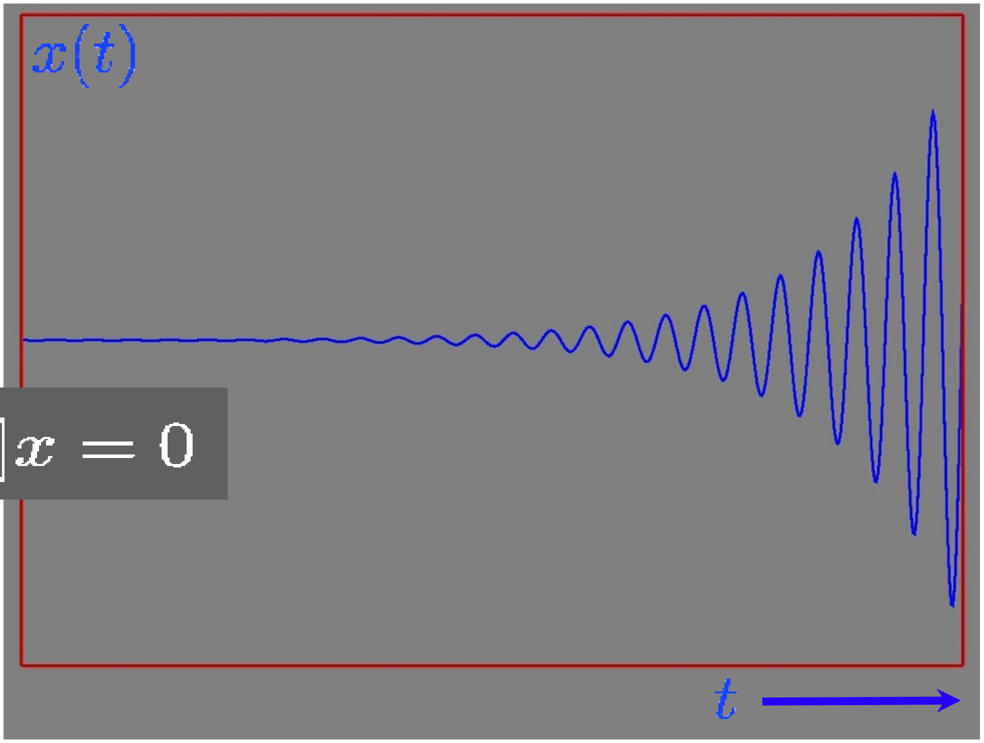


$$\frac{d\vec{K}_p}{dt} = (-2\Delta(t), 0, 2\varepsilon_p) \times \vec{K}_p$$

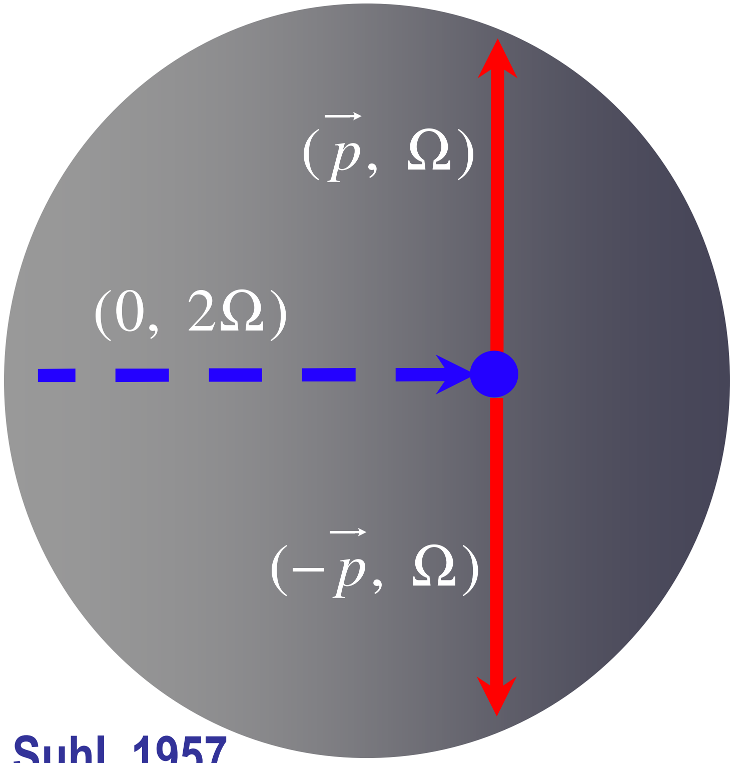
$$\omega(\varepsilon) = 2\sqrt{\varepsilon^2 + \Delta^2}$$

Parametric resonance in continuous media?

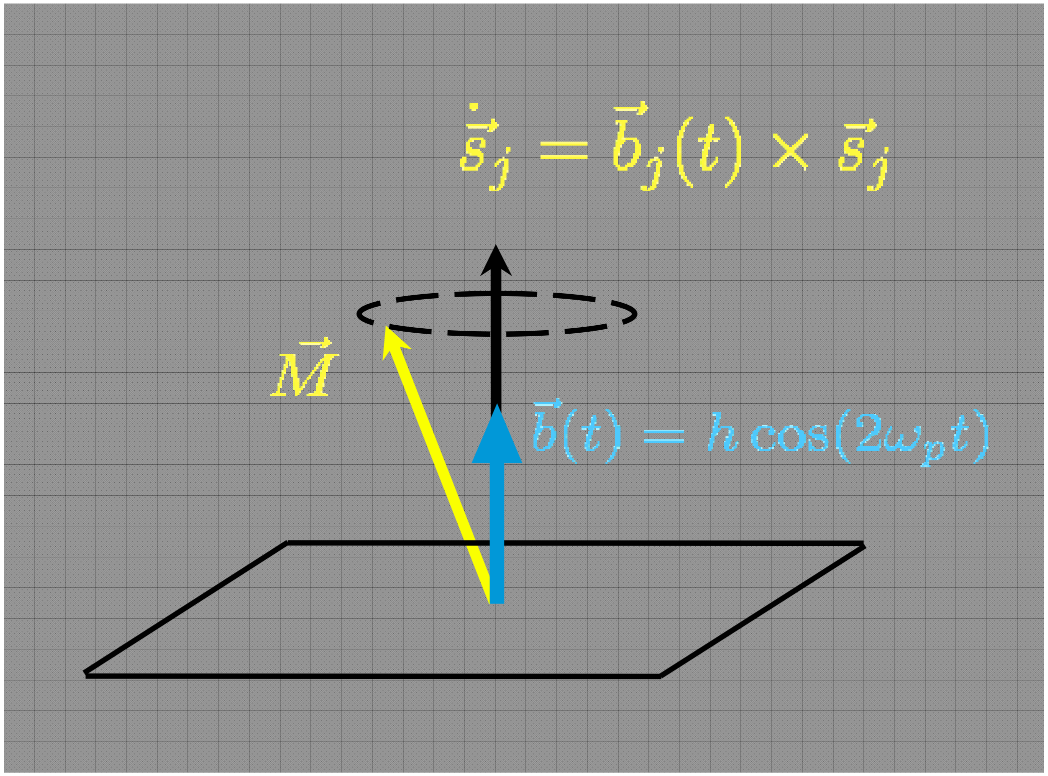
$$\ddot{x} + \omega_0^2[1 + h\cos(2\omega_0 + \varepsilon)t]x = 0$$



Wave turbulence



H. Suhl, 1957



dielectric ferromagnet in a uniaxial field (YIG)

microscopic theory of spin wave turbulence

Zakharov, L’vov & Starobinets, 1974

Cooper pair turbulence

$$\delta\Delta(\vec{r},t) = \frac{\sqrt{q}\,\Delta_s c_s \sin(k_s|\vec{r}-\vec{r}_0|)}{k_s|\vec{r}-\vec{r}_0|}A(t)$$

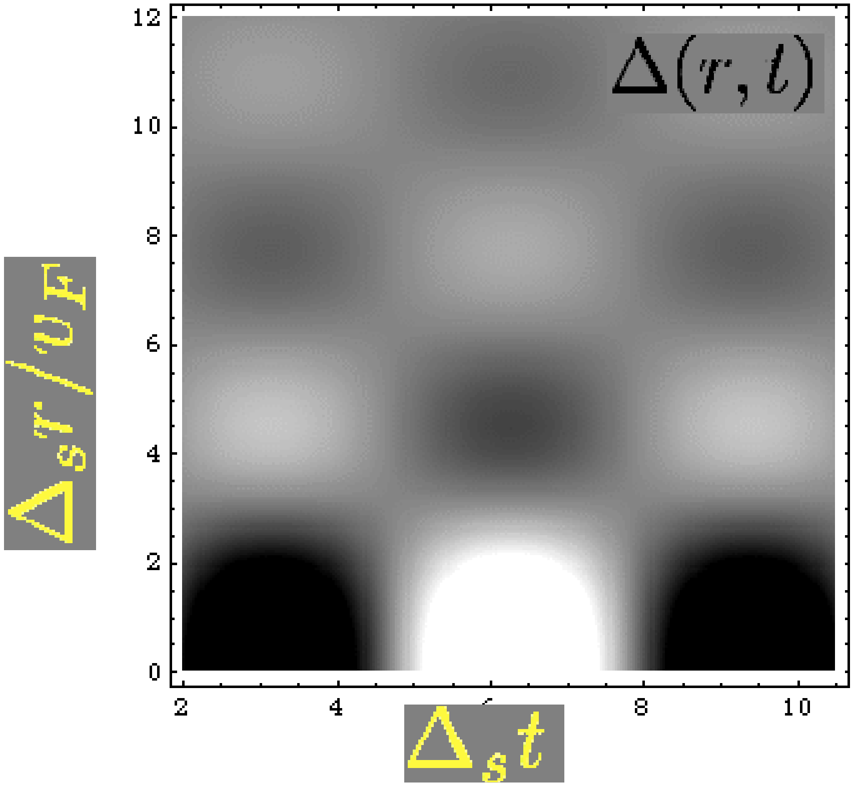
$$L \gg \xi$$

"bubble" of superfluid

$\vec{r}_0$ – position

$A(t)$ – periodic with random amplitude

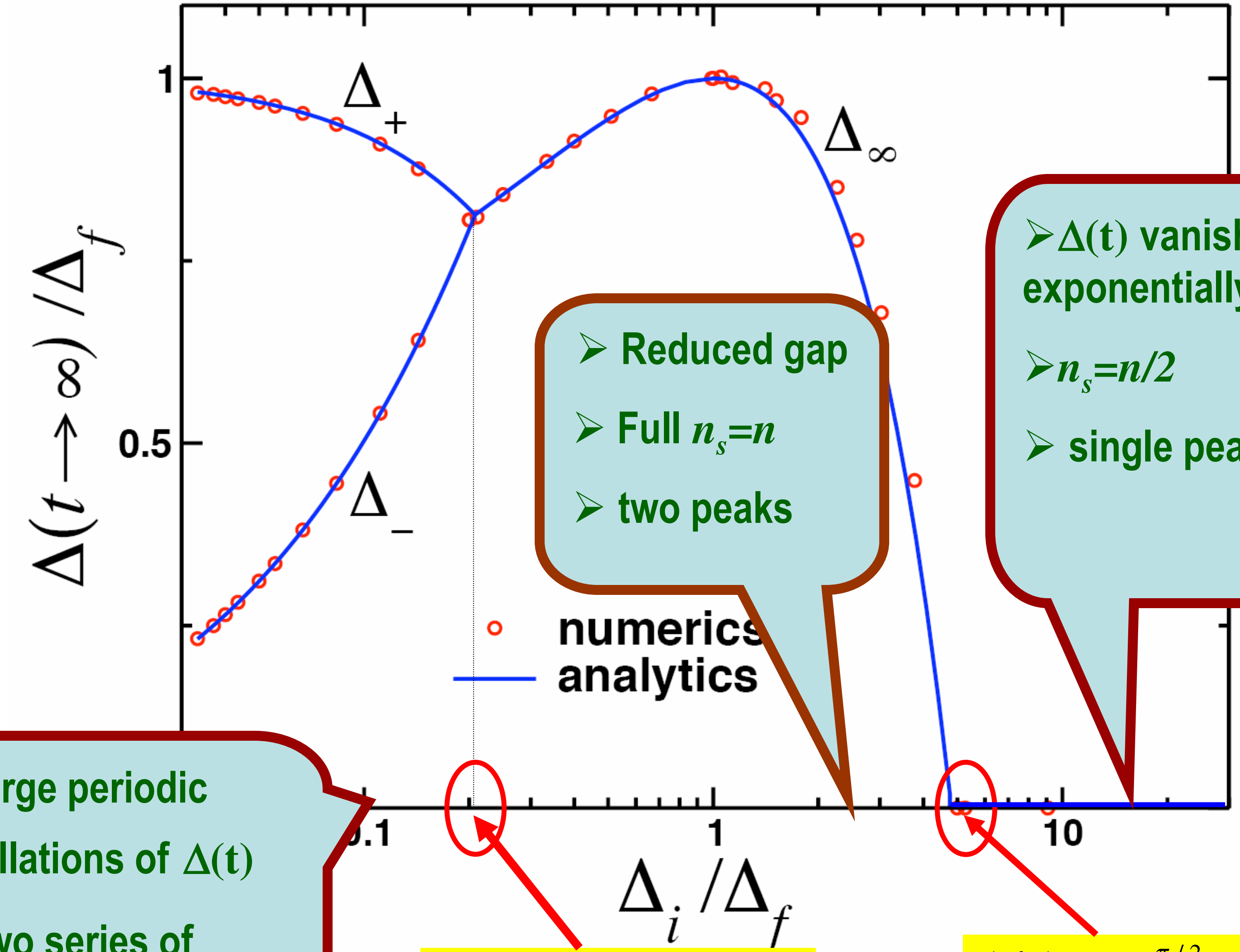
Flow of energy to progressively smaller length scales



Typical situation – random superposition of bubbles

M. Dzero, E.Y., B. Altshuler, arXiv:0805.2798 (2008)

Far from equilibrium “phase” diagram



➢ Reduced gap

➢ Full $n_s=n$

➢ two peaks

➢Δ(t) vanishes exponentially

➢$n_s=n/2$

➢ single peak

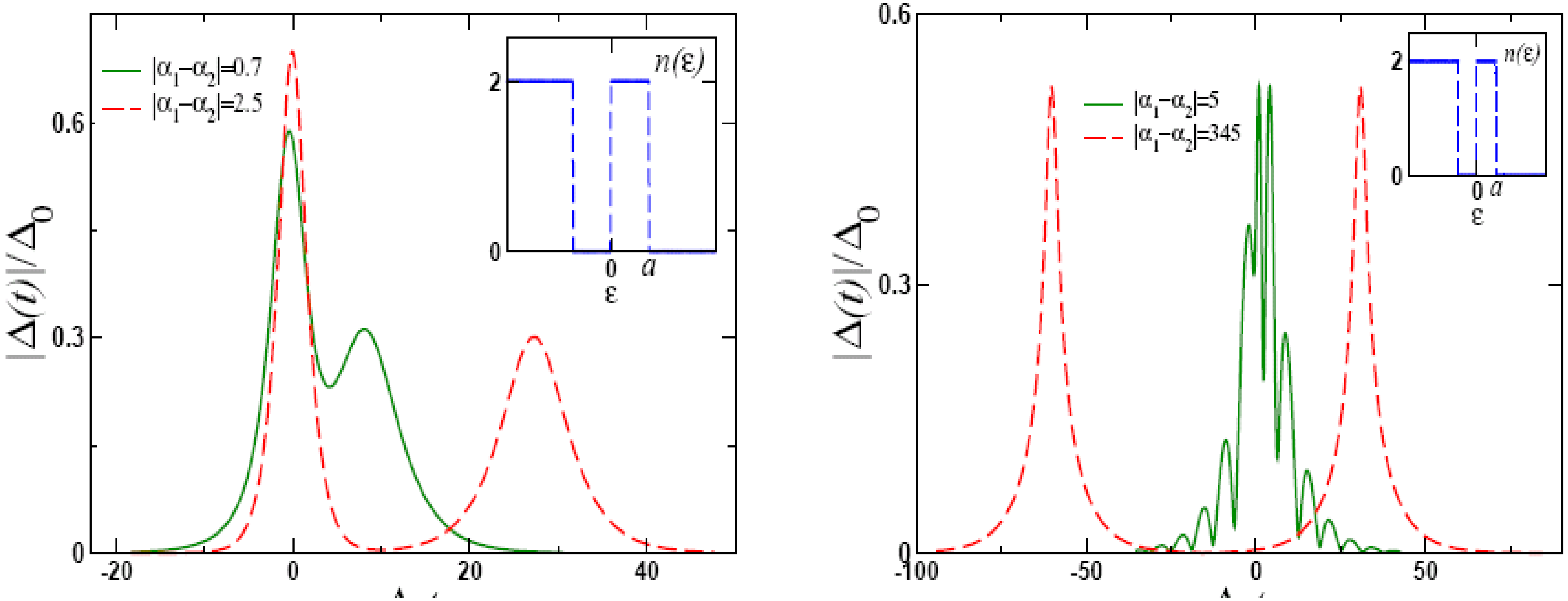
➢ large periodic oscillations of Δ(t)

➢ two series of peaks

$\Delta_i/\Delta_f = e^{-\pi/2} \approx 1/5$

$\Delta_i/\Delta_f = e^{\pi/2} \approx 5$

Normal and anomalous solitons



E.Y., arXiv:0807.3181 (2008)

Cooper pair turbulence

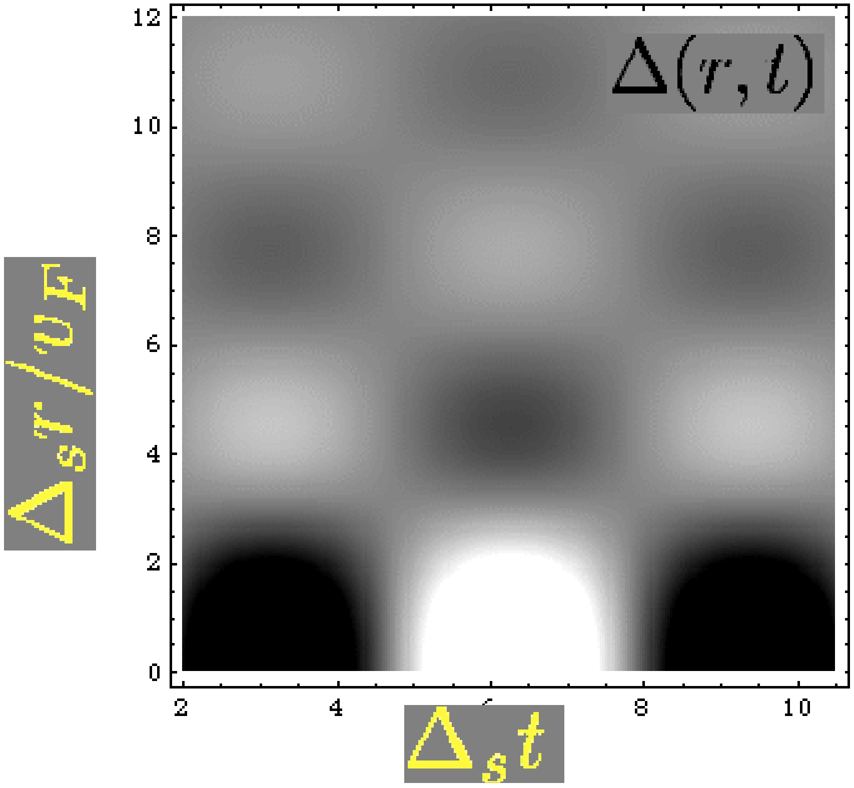
$$\delta\Delta(\vec{r},t)=\frac{\sqrt{q}\,\Delta_s c_s \sin(k_s|\vec{r}-\vec{r}_0|)}{k_s|\vec{r}-\vec{r}_0|}A(t)$$

$$L \gg \xi$$

"bubble" of superfluid

$\vec{r}_0$ – position

$A(t)$ – periodic with random amplitude



Flow of energy to progressively smaller length scales

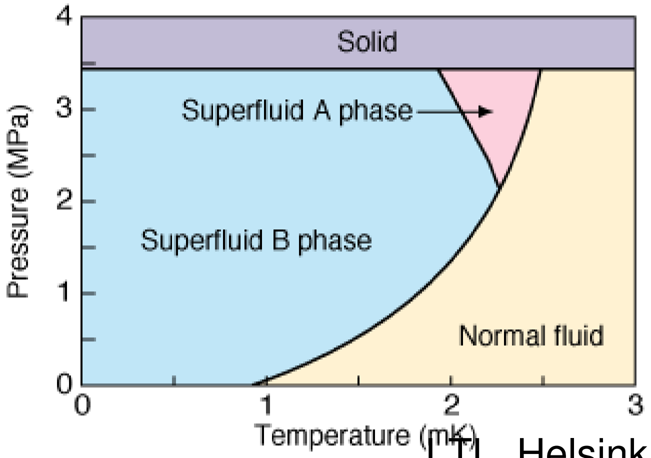
Typical situation – random superposition of bubbles

M. Dzero, E.Y., B. Altshuler, arXiv:0805.2798 (2008)

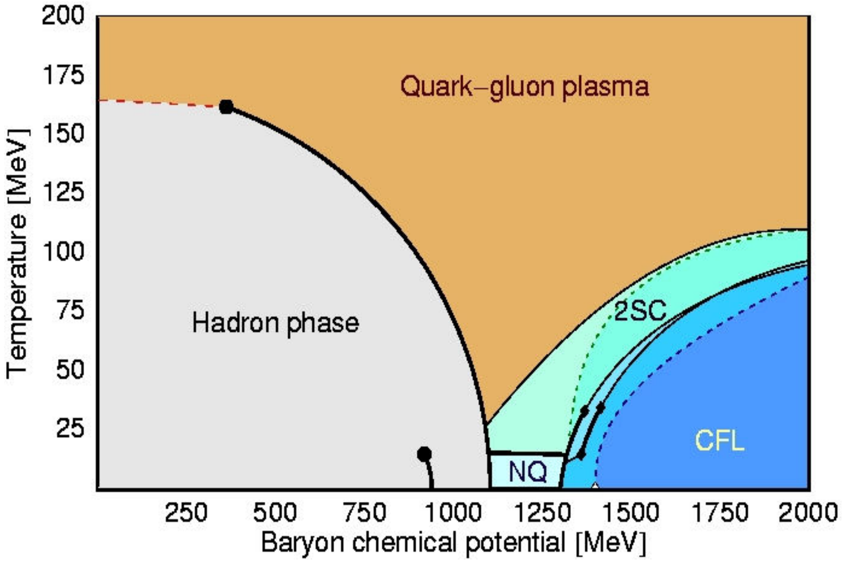
Three component superfluidity in cold gases

Why multiple components?

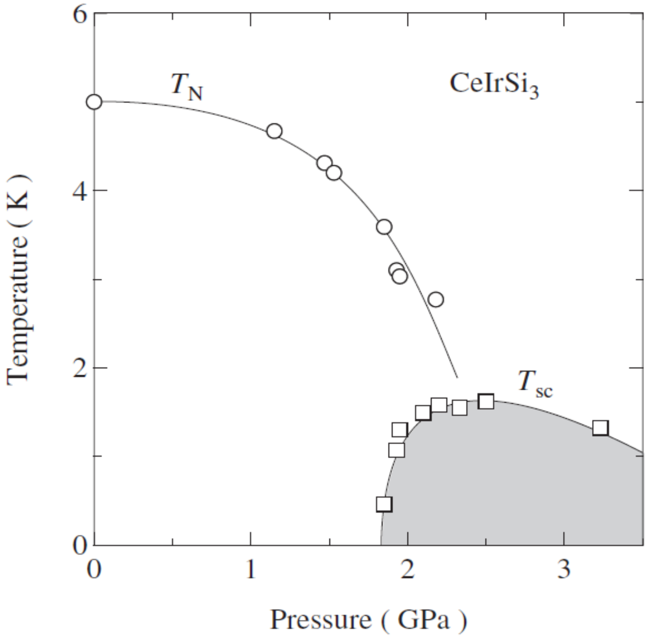
- superfluid ^{3}He
- unconventional superconductivity in heavy fermion compounds
- color superconductivity in nuclear matter



LTL, Helsinki



see S. Ruster et al., PRD 72, 034004 (2005)



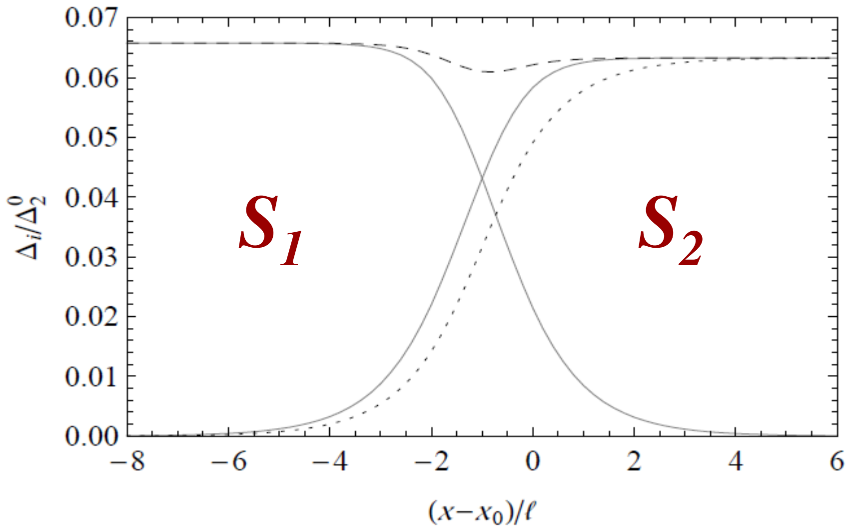
R. Settai et al., JPSJ 76, 051003 (2007)

Three component superfluidity

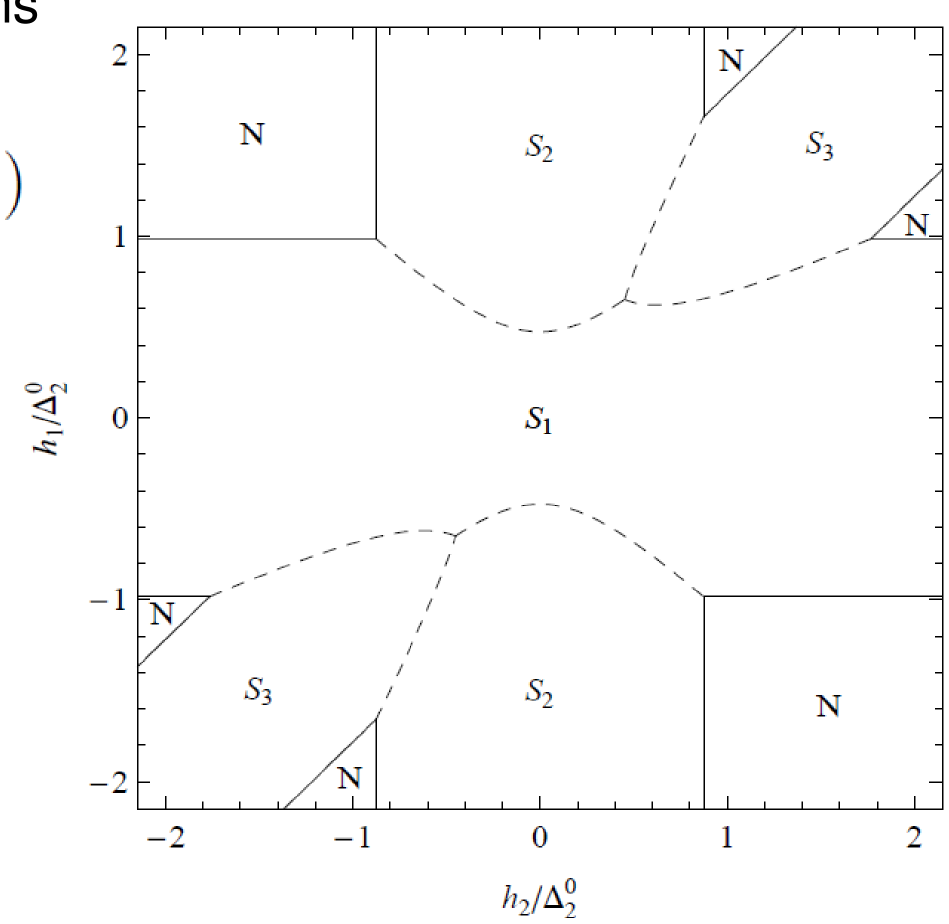
$$H_{int} = g_1 \Psi_2^\dagger \Psi_3^\dagger \Psi_3 \Psi_2$$ + permutations

vector order parameter: $\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$

extended coexistence regions (domain walls):



phase diagram



G. Catelani, E.Y., *Phys. Rev. A* (2008)