Physics 602 **Homework Set 4** **Spring 2017**

Due date: Monday, April 24, at the end of the lecture.

Note: you are allowed to use only the textbook (Grosso & Parravicini), lecture notes, materials that I specifically provided, and Mathematica or other similar software to do the homework.

1. *Conductivity of a superconductor in the two-fluid model.* In the two-fluid model for a superconductor, it is assumed that there exist both normal and superconducting electrons. The normal electrons obey the Drude-like equation

$$\frac{d\vec{j}_n}{dt} = \frac{n_n e^2}{m}\vec{E} - \frac{\vec{j}}{\tau},$$

where n_n and $\vec{j}_n$ are the number and current densities of the normal electrons, respectively. The superconducting electrons obey the London equation

$$\frac{d\vec{j}_s}{dt} = \frac{n_s e^2}{m}\vec{E},$$

where n_s and $\vec{j}_s$ are the number and current densities of the superconducting electrons, respectively.

1. Find the (total) frequency-dependent complex conductivity $\sigma(\omega)$ for a superconductor. Use time dependence of the form $e^{-i\omega t}$ for time-dependent quantities and assume that the normal and superconducting fluids respond independently to the electric field.
2. Show that, in the low-frequency limit, the response of the normal fluid is purely ohmic, while the response of the superconducting fluid is purely inductive.

2. *Magnetic field inside an infinite superconducting plate.* Solve the London equations for an infinite superconducting plate of finite thickness $2t$. Assume that the magnetic field of magnitude B_0 is applied parallel to the plate. Find both the magnetic field and the supercurrent inside the plate. Plot the magnetic field and supercurrent for $2t = \lambda_L$ and $2\lambda_L$.

3. *Critical field of a type-I superconductor in the Ginzburg-Landau theory.* Find the temperature dependence $H_c(T)$ of the critical filed of a type-I superconductor for T close to T_c within the Ginzburg-Landau theory.

4. *Proximity effect between two planar superconductors.* Two planar superconductors 1 and 2 are placed with their flat faces in very good contact. Their critical temperatures are T_{c1} and T_{c2}, respectively, with $T_{c2} > T_{c1}$ and $T_{c2} - T_{c1} \ll T_{c1}$. The system is cooled to a temperature T between T_{c1} and T_{c2}, so that only superconductor 2 is superconducting.

1. Show that the Ginzburg-Landau equation for superconductor 1 can be written as

$$-\xi_1^2 \frac{d^2\phi}{dx^2} + \phi + \phi^3 = 0,$$

   where ϕ is the dimensionless wave function, and determine ξ_1.

2. Making use of the fact that $|\phi| \ll 1$ in a normal metal so that the cubic term in the above equation can be neglected, show that the wave function decays according to $\phi = \phi_0 e^{-|x|/\xi_1}$ in superconductor 2, where $x = 0$ is at the interface between the two superconductors and superconductor 2 occupies the $x < 0$ region.

5. *Finite-momentum BCS state.* The finite-momentum BCS state is given by

$$|\Psi_S(\mathbf{K})\rangle = \prod_{\vec{k}} (u_\mathbf{k} + v_\mathbf{k} c^\dagger_{\mathbf{k}+\mathbf{K}/2\uparrow} c^\dagger_{-\mathbf{k}+\mathbf{K}/2\downarrow})|0\rangle,$$

where $u_\mathbf{k}$ and $v_\mathbf{k}$ are the same as for $\mathbf{K} = 0$.

1. Compute the order parameter $\Delta_\mathbf{K} = g \sum_\mathbf{k} \langle c_{-\mathbf{k}+\mathbf{K}/2\downarrow} c_{\mathbf{k}+\mathbf{K}/2\uparrow}\rangle$ in $|\Psi_S(\mathbf{K})\rangle$ (in terms of the energy gap Δ_0 for the zero-momentum BCS state).

2. Evaluate the current density carried by the finite momentum BCS state, i.e. the expectation value of the current density operator

$$\hat{\mathbf{j}}(\mathbf{q}) = -\frac{e\hbar}{mV} \sum_{\mathbf{k}\sigma} (\mathbf{k} + \mathbf{q}/2) c^\dagger_{bfk\sigma} c_{\mathbf{k}+\mathbf{q},\sigma},$$

   where V is the system volume, in $|\Psi_S(\mathbf{K})\rangle$. What is the current density in the zero momentum BCS state we used in class?