The Faraday Effect

Objective

To observe the interaction of light and matter, as modified by the presence of a magnetic field, and to apply the classical theory of matter to the observations. You will measure the Verdet constant for several materials and obtain the value of e/m, the charge to mass ratio for the electron.

Equipment

Electromagnet (Atomic labs, 0028), magnet power supply (Cencocat. #79551, 50V-5A DC, 32 & 140 V AC, RU #00048664), gaussmeter (RFL Industries), High Intensity Tungsten Filament Lamp, three interference filters, volt-ammeter (DC), Nicol prisms (2), glass samples (extra dense flint (EDF), light flint (LF), Kigre), sample holder (PVC), HP 6235A Triple output power supply, HP 34401 Multimeter, Si photodiode detector.

I. Introduction

If any transparent solid or liquid is placed in a uniform magnetic field, and a beam of plane polarized light is passed through it in the direction parallel to the magnetic lines of force (through holes in the pole shoes of a strong electromagnet), it is found that the transmitted light is still plane polarized, but that the plane of polarization is rotated by an angle proportional to the field intensity. This "optical rotation" is called the Faraday rotation (or Farady effect) and differs in an important respect from a similar effect, called optical activity, occurring in sugar solutions. In a sugar solution, the optical rotation proceeds in the same direction, whichever way the light is directed. In particular, when a beam is reflected back through the solution it emerges with the same polarization as it entered before reflection. In the Faraday effect, however, the direction of the optical rotation, as viewed when looking into the beam, is reversed when the light traverses the substance opposite to the magnetic field direction; that is, the rotation can be reversed by either changing the field direction or the light direction. Reflected light, having passed twice through the medium, has its plane of polarization rotated by twice the angle observed for single transmission.

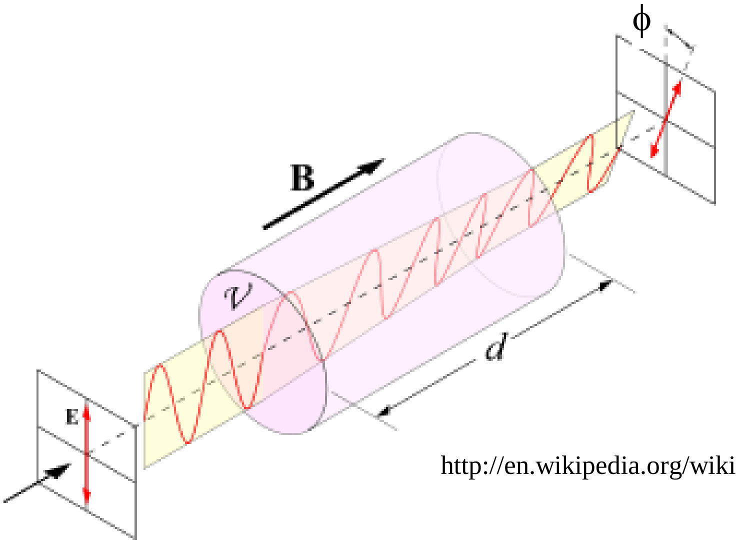
By placing the sample between two pieces of Polaroid or two Nichol prisms, it can be arranged (with sufficient magnetic field strength) that little light is transmitted through the system in one direction, while it can pass, eventually with undiminished intensity, in the opposite direction. The effect is unique in this respect: it permits the construction of an irreversible optical instrument with which observer A can see observer B, while A cannot be seen by B.

Read the theory of the Faraday rotation in the appendix and consult the references given at the end of this writeup. Many references can also be found online.

The relation between the angle of rotation of the polarization and the magnetic field in the transparent material is given by Becquerel's formula:

1. $\phi = \mathcal{V}Bd$

Where ϕ the is the angle of rotation, *d* is the length of the path where the light and magnetic field interact (d is the sample thickness for this experiment), B is the magnetic field component in the direction of the light propagation and $\mathcal{V}$ is the Verdet constant for the material (MKS units: radian/Tesla meter). This empirical proportionality constant varies with wavelength and temperature and is tabulated for various materials.

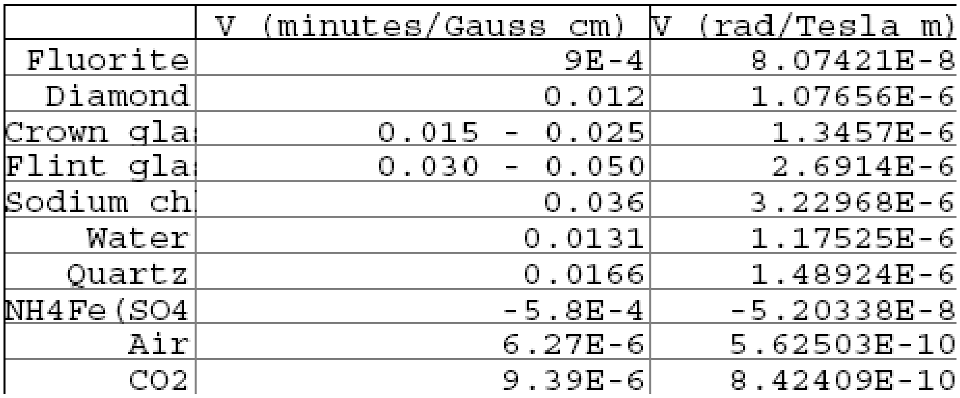


http://en.wikipedia.org/wiki/Faraday_effect

The Verdet Constant, $\mathcal{V}$, depends on the dispersion of the refractive index, $dn/d\lambda$ where n is the index of refraction λ is the wavelength. As shown in the appendix:

2. $$\mathcal{V} = \phi/dB = -\frac{1}{2}\frac{e}{m}\frac{\lambda}{c}\frac{dn}{d\lambda}$$

Here e/m is the charge to mass ratio of the electron and c is the speed of light. Some values of $\mathcal{V}$ are listed in the table

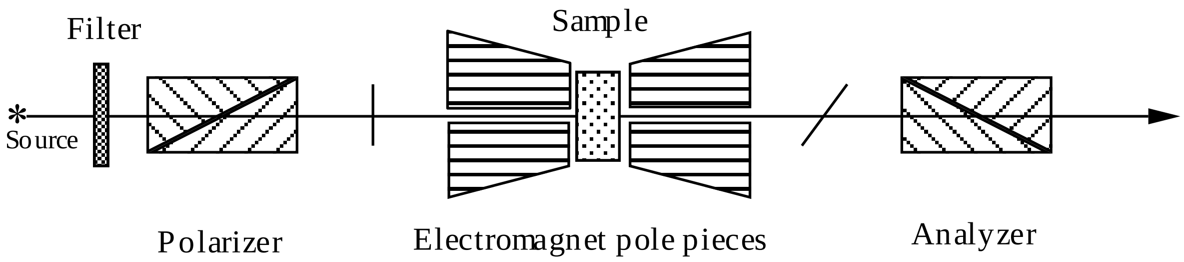


	V (minutes/Gauss cm)	V (rad/Tesla m)
Fluorite	9E-4	8.07421E-8
Diamond	0.012	1.07656E-6
Crown gla	0.015 - 0.025	1.3457E-6
Flint gla	0.030 - 0.050	2.6914E-6
Sodium ch	0.036	3.22968E-6
Water	0.0131	1.17525E-6
Quartz	0.0166	1.48924E-6
NH4Fe(SO4	-5.8E-4	-5.20338E-8
Air	6.27E-6	5.62503E-10
CO2	9.39E-6	8.42409E-10

Substances that are noted for their large dispersion (large dn/dλ), such as heavy flint glasses, and CS$_2$, also show a large Faraday effect as predicted by the theory. In the visible range the refractive index of common substances, such as air, water; lead and soda glasses, etc., decreases rapidly with increasing wavelength (increasing frequency separation from the governing ultraviolet absorption resonances; normal dispersion); hence, dn/dλ is negative and it follows that light traveling in the direction of **B** has its plane of polarization turned counterclockwise for an observer looking into the beam. The theory discussed in the appendix (cross product expression for magnetic force on a moving charged particle) explains the reversal of the rotation when either the field or the light direction is reversed (but not both). These theoretical conclusions are confirmed by the observations. The fact that the theory predicts the correct sign is a direct proof that the effect is related to the motion of negative charges, the electrons.

With the exception of some paramagnetic materials, the quantitative observations are in excellent agreement with Becquerel's equation. Typically, from $\lambda = 6 \times 10^{-5}$ to 7×10^{-5} cm, the refractive index changes by about 10^{-2}. Hence (in mks units) with dn/d$\lambda \approx 10^5$ m^{-1}, $\lambda/c \approx 2 \times 10^{-15}$ seconds, and $\frac{e}{m} = 1.76 \times 10^{11}$, V is therefore about 17.6 radians per tesla-meter, or $\approx$ 0.06 minutes of arc per gauss-cm. With a good Nichol prism, rotations of about one-half minute can be observed. Since paths of several centimeters length and fields of a few thousand gauss can be used, the Faraday effect is quite easy to observe and is measurable with good accuracy.

II. Equipment



a. Samples

Various glass samples are included with the Faraday Effect attachment:

One extra dense flint (marked EDF)
One light flint (marked LF)
One labeled Kigre.

b. Polarimeter and electromagnet

Tapered magnet pole tips are used on the electromagnet to concentrate the magnetic field and raise it to as high a value as possible. The pole tips are attached with hollow bolts so that the light may travel through the magnet parallel to the magnetic lines of force.

Identical Polaroid filters are mounted with split clamps at the ends of the magnet frame (not Nichol prisms, as indicated in the diagram). These are used as polarizer and analyzer respectively. They have a radial handle for coarse rotation, with dial and vernier scale to measure angle. Fine control is provided by a locking screw and drive screw. **Unlock this before coarse angular adjustment.**

These are identical, so either may serve a polarizer, and the other as analyzer. The fine adjust screws of one has been damaged however, so it is best to use this as the polarizer. **Do not change the setting of this during a series of measurements.**

Anisotropy provides selective absorption in Polaroid material (reference, Serway). Long chain hydrocarbon molecules (e.g., polyvinyl alcohol) are aligned by stretching during manufacture, and subsequently made conducting along the chains only by dipping into an iodine solution which provides free electrons. Strong, selective absorption of light then occurs for the electric field component in the molecular direction. It follows that the polarizing direction is independent of wavelength, although the degree of polarization (absorption) may not be. The absorption is incomplete, leading to a non-zero constant term in the Malus fit of the ϕ vs. analyzer angle data curve.

Collimators (plastic washers) have been inserted into the optical path entrance and exit tunnels, to reduce polarization shift by grazing incidence scattering in the magnet "tunnels".

c. Light sources

The transparent glass is placed between the pole faces with the plastic support resting on the tapered portion of the pole faces. A high-intensity incandescent lamp with filters is used as a light source. A bright light source is desirable since greater contrast of light levels is helpful in obtaining accurate data.

There are three filters. The wavelengths are engraved on the outside, front end of the filter barrels.

Blue: 4495 Å Yellow: 5490 Å Red: 6500 Å

These are very narrow band interference filters ($\Delta\lambda$ = 15-50 Å). Use these peak wavelengths as the effective wavelengths.The filters are in cells that slide into the front end of the apparatus. The filter should face the high-intensity tungsten lamp. Do not force the filter cell into the apparatus.

IT IS IMPORTANT THAT THE FILTER DOES NOT COME INTO CONTACT WITH THE HOT BULB. LEAVE A TWO INCH AIR SPACE BETWEEN THE FILTER AND THE BULB.

Put the lamp at maximum intensity for best results. If you see saturation put the lamp on medium brightness and move the lamp away from the filter end.

d. Magnet

The magnet is controlled with a variable power supply. The voltage output of the power supply, proportional to the magnet current, is measured with a multi meter (HP 34401).

III. Experimental procedure

The aim of this experiment is to test the Bequerel relation and to measure the Verdet constant and its dependence on the wavelength of light. You will collect data to plot the light intensity vs. polarizer angle for B=0, and several nonzero B fields. You will then fit the data to Malus law which will allow you to extract the values of the polarization rotation angle as a function of field, B. From the field dependence of the polarization rotation you will check the linearity with field and calculate the Verdet constant for three wavelengths. Finally you will use your data to obtain the value of e/m.

1. Light detection

The intensity minimum can be determined by eye, which has excellent, non-linear sensitivity, but with poor precision and cumulative eye fatigue. Using a photosensitive detector is more objective. Among the possibilities are a photodiode, a photoresistive device and a phototransistor.

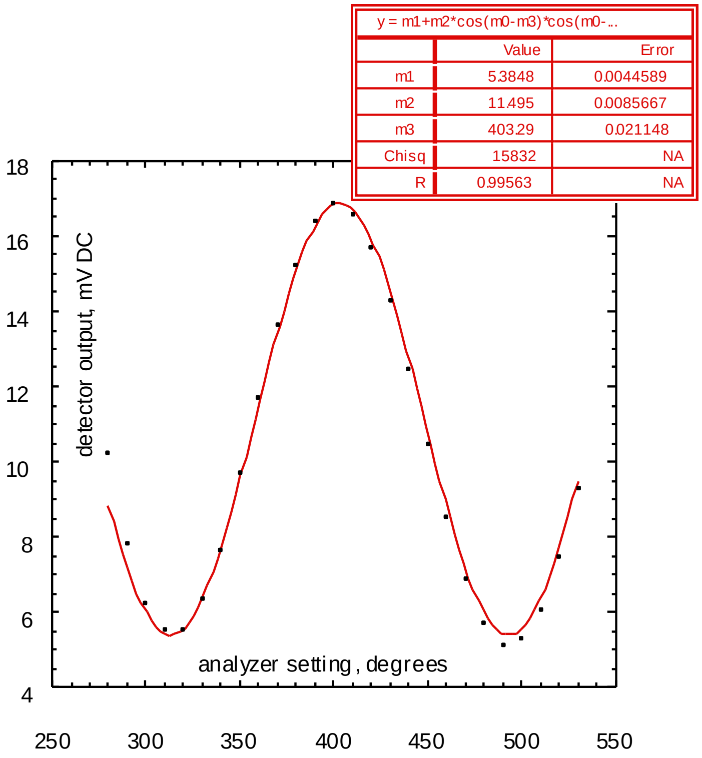
The eye can be very sensitive to low light levels, due to its approximately logarithmic intensity response. A photoresistor also has a non-linear response. The photo transistor and photodiode can exhibit a fairly linear response over a range of intensity, making them suitable for the Malus law (cosine squared) behavior of intensity vs. analyzer angle which is the basis of the present method of determining the minimum of intensity. This law, in turn, follows and exhibits directly the vector character of the electric field.

At present a silicon photodiode is employed in the photovoltaic mode (no bias). DC output voltage is read with a HP 34401 multimeter. Range should be set for minimum fluctuation, sacrificing resolution. Choose "Slow 6 digit" mode (integration time 2 seconds) on the MEASurement (5: resolution) menu. (The display asterisk will blink at the end of each measurement cycle.) Refer to the HP manual menu tutorial; also to pages 13, 34 and 81-83.

The photodiode is about 0.1" square with an integral collecting and focusing lens. The diode housing is clamped with four set screws to the analyzer housing, and rotates with it. One detector lead is grounded to the case (and thus to the coax braid via the UHF-BNC adapter); the other connects to the coax center wire. The other end of the coax cable terminates in a double banana plug. The pin adjacent to the plastic lug is connected to coax braid and thus to Faraday apparatus ground; be sure this plug is inserted into the lo input of the HP multimeter. (Check for proper and for reverse input orientation the meter reading response to touching the cable ground.) If any other devices (e.g., a notebook computer) are plugged into the meter power strip, check meter fluctuation, reading shift, etc. Try reversing the power plug if there seems to be a problem; better, run the device on battery.

The theoretical variation of detector output would follow Malus $\cos^2(\theta)$ "law". This is a direct consequence of the fact that the amplitude of the electromagnetic wave passed by the analyzer is proportional to $|\cos(\theta)|$, where θ is the angle between polarizer and analyzer. Thus the transmitted intensity is proportional to $\cos^2(\theta)$), with an additional constant term that is present when the polarization and analysis are incomplete. When using visual detection of the analyzed intensity observing the minimum angle is preferable to observation of the maximum (less saturation of eye sensitivity), but rates of change of either are small.

The Malus "law" offers a simple alternative way to detect with considerable precision the "crossed" or the "parallel" angle of the analyzer (minimum or maximum detector voltage) by non-linear least square fitting of a curve of detector voltage vs. angle. Application of the Malus theory requires that the detector response remain linear over the range of light intensities fit, and that the source remain fixed in position and constant in intensity during the measurement. Test and application of Malus law is shown below for data taken with an unfiltered mercury discharge source. The fit was done with Kaleidograph. Note the zero offset.



y = m1+m2*cos(m0-m3)*cos(m0-...

	Value	Error
m1	5.3848	0.0044589
m2	11.495	0.0085667
m3	403.29	0.021148
Chisq	15832	NA
R	0.99563	NA

Non-linear weighted least square fit of the data to Malus law. Note that the two "crossed" positions of the analyzer (minima) are 180 degrees apart, as expected.

The constant background (m1 parameter) can be understood as resulting from incomplete polarization and analysis.

The Malus curve at zero field can be used to extract the Faraday rotation angle as a function of field. After the zero field measurement and without moving any part of the apparatus the field is gradually ramped up through a set of desired values. At every field, you will measure the detector voltage versus magnet power supply voltage V. You will then use the hysteresis curve B(V) which is obtained by measuring the field with a Hall probe gaussmeter..

2. Data acquisition

Source and detector familiarization. With the Si photovoltaic detector quickly find and record at zero magnet current approximate maximum and minimum detector meter readings for each filter. The silicon response improves rapidly at shorter wavelengths. Are these detector and the polaroid materials suitable for all of the wavelengths of the filters?

Law of Malus verification Take detector current readings at zero magnet current every ~10 degrees over a range of ≥ 360 degrees to test the Law of Malus ($\cos^2$ intensity behavior). Estimate an average detector current error from observed meter fluctuations. Plot detector current vs. analyzer angle, θ, and use your favorite fitting program (Origin or matlab etc.) to fit the data to the form $m_1 - m_2 * \cos^2(\theta - m_3)$ corresponding to the Malus formula. The fitting parameters m_1, m_2 and and m_3 correspond respectively to the minimum detector current value, the amplitude of the detector current variation and the angle between polarizer and analyzer at the minimum of detector current (remember to use the same units in the simulation as in the measurement, ie degrees, not radians). Include in your report the plot and the fit equation showing fit parameters and minimum chi square value. You will note that the quality of fits improves considerably if you limit the range of fitting angles to bracket a minimum or a maximum.

Shifts of the entire curve with magnetic field are the desired data. This shift is measured by the parameter m3. The polarizer setting must not be changed during a sequence of measurements.

Magnet calibration. Measure the magnetic field with the gaussmeter, removing the sample and orienting the plane of the probe perpendicular to the field for maximum reading. Handle the gaussmeter with care. Note 10x gain.

Establish a reproducible hysteresis loop for both positive and negative magnet supply voltages. Follow the procedure described below.

WARNING : Be certain that the power supply is zeroed and turned off before disconnecting the magnet in order to reverse current leads. **Failure to observe this precaution may result in personal injury or damage to the equipment.**

Hysteresis loop. Turn on the magnet power supply (be sure to set the output to zero!). Turn on the gaussmeter and calibrate it, following the instructions on top of the instrument

Remove any residual magnetism in the magnet ("degauss") as follows: Raise the voltage in the magnet power supply to the maximum value you expect to use ~ +50 V and then back to zero, switch off the power supply and reverse polarity; raise voltage to -40 V and back to zero, switch off and reverse polarity; raise to +30 amperes and back to zero, switch off and reverse polarity, etc.(-20, +10, -5,0).

Hereafter you will maintain a definite magnetic hysteresis curve by always increasing the voltage up to 50V in the same sense, and by always decreasing it back down to zero before raising it again. Turn voltage down to zero before switching off-do not switch off with current flowing in the magnet. This is good practice with any magnetic circuit, to avoid inductively generated high voltages and possible arcing.

Now calibrate your magnet by following by recording B(V) for a few voltages on the up branch (0 u to +50 V) and then same voltages on the down branch (+50, 0, -50) and then back to B=0. If you maintain the same hysteresis curve the B(I) calibration will remain valid throughout the experiment and you will no longer need to use the Hall probe

Take Faraday data as described below on a single branch of the magnet hysteresis curve (+ going or - going). Always maintain the same hysteresis curve as for your calibration (same current maximum and minimum).

Then measure <u>B</u>(<u>V</u>) and plot the curve of transmission minimum angle shift vs. B for increasing and decreasing B (positive and negative currents). Alternatively, with the analyzer fixed at the angle of maximum sensitivity (45 degrees from zero field transmission minimum), measure detector voltage vs magnet current voltage for + and - currents; then convert to angle shift as a function of field using a previously determined Malus curve. Recheck to verify intensity stability.

Rotation of polarization. The data collection procedure described should be repeated for all samples, for all three filters and for at least 3 field values in addition to zero field.

1. Set the field current (run both + and - to double the data range) and then record the angle for minimum transmission measured by eye.

2. Now switch to using the photodetector and Malus law. Measure the detector voltage vs. analyzer angle readings at intervals in the selected region, without attempting to observe the exact minimum; then find the minimum (or maximum) by least square fitting the data. Voltage readings every 10 degrees in a range of about ± 45 degrees around a min (or max) will provide a good determination, if the data is smooth. If you take data around a minimum, you may fit to the form

   m1-m2*cos(m0-m3)*cos(m0-m3); m1=?; m2=; m3=?

   (input initial parameters by inspection of the response curve). Here you would input m3 = approximate observed minimum angle. You could equally well fit to m1-m2*cos(m0-m3)*cos(m0-m3); m1=?; m2=; m3=? . Then your starting m3 would be approximately that of an adjacent maximum. Both best fits (same data set) will be the same, except for a difference of exactly 90 degrees in final m3, so the minimum angle can easily be recovered from a fit to the latter form of Malus law.

3. An alternative method to using the photodetector and Malus law described in 2. Establish the Malus curve at zero field. Set the analyzer to the minimum angle. Sweep the field through a set of values and observe the change in transmitted intensity with field. Using the Malus curve and magnet calibration extract the Farady rotation angle as a function of field. Compare your results to those obtained in 1 and 2.

IV. Analysis

1. Verdet constant

Tabulate your rotational shifts vs. magnetic field (include the tables in an appendix). For all your data plot the rotation shift versus B. Use a least square linear fit to obtain the Verdet constant and the error for all samples and for each wavelength. Include the plot and equation in your report.

Compare the results obtained with each one of the 3 methods and comment on their accuracy. Compare the measured Verdet constant to accepted values.

(use consistent (mks and radian) units when combining experimental values of $\mathbb{V}$ with theory to obtain $\frac{e}{m}$.)

2. Comparison of Faraday inferred dispersion with Cauchy formula prediction

If sample dispersion data is available (check appendix 5), fit to an inverse even power series ($n = A + B/\lambda^2$ does fairly well) and differentiate to obtain

$dn/d\lambda_{dispersion} = -2B/\lambda^3$ at your wavelength. From the measured value of $\mathbb{V}$ and using accepted values for *e/m* and *c* calculate $\frac{dn}{d\lambda}_{\text{Verdet}}$. Make a table with the values of $\mathbb{V}$, $\frac{dn}{d\lambda}_{\text{Verdet}}$ and $dn/d\lambda_{dispersion}$. Compare $\frac{dn}{d\lambda}_{\text{Verdet}}$ to $dn/d\lambda_{dispersion.}$ and comment on the differences.

If corresponding values of $\mathbb{V}$ and the $dn/d\lambda_{dispersion}$ can be obtained at several wavelengths or for several samples, a linear fit to a plot of $2\mathbb{V}/(\lambda/c)$ vs. $dn/d\lambda_{dispersion}$ permits an experimental determination of e/m. Discuss your result.

Report:

Include a theoretical background, description of procedure, the data presented in graphic format whenever possible, analysis and discussion of results. Raw data should be presented in the appendix. In discussion section, space permitting, you may also include other relevant topics such as: the physical (microscopic) origin of the Faraday rotation; why does the classical treatment work for the Faraday rotation (why don't we need Planck's constant to explain the phenomenon), an explanation of the physical origin of hysteresis in magnets.

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Appendix 1. Optical rotation and circular birefringence

1. Decomposition of plane-polarized light into coherent, counter-rotating, circularly polarized components

Optical rotation can be thought of in terms of circular birefringence. By the latter is meant that the propagation velocity, or the refractive index, is different for right and left circular polarized light. For ordinary or "linear" birefringence there is a difference of the refractive indices for two plane polarized components of light which are normal to each other. To show that optical rotation is equivalent to circular double refraction, we note that a plane polarized beam can always be considered as the coherent superposition of two coherent circularly polarized components of the same frequency and of equal amplitude. For instance, a vibration $E = 2A \cos \omega t$ can be considered as the sum of

a. a right circular component, and	**b.** a left circular component
$E_x = A \sin \omega t$	$E_x = -A \sin \omega t$
$E_y = A \cos \omega t$	$E_y = A \cos \omega t$.

If these equations represent the light incident on an active medium, the light emerging after passing through a distance $\underline{D}$ is given by similar equations that differ from the above only by the fact that the right circular components are shifted in phase by $\frac{2\pi n_r D}{\lambda}$, while the phase shift of the left circular component is $\frac{2\pi n_l D}{\lambda}$. $\underline{n}_r$ and $\underline{n}_l$ are the two indexes of refraction, and $(\underline{n}_r-\underline{n}_l) = \underline{\Delta}$ is the index of birefringence. The transmitted beam is therefore given by:

Eq. 1

$$E_x = A \left[\sin\left(\omega t - \frac{2\pi n_r D}{\lambda}\right) - \sin\left(\omega t - \frac{2\pi n_l D}{\lambda}\right)\right]$$

$$= -2A \sin\left\{\left[\frac{2\pi D}{\lambda}\right] \text{x} \left[\frac{1}{2} (n_r-n_l)\right]\right\} \quad \text{x} \quad \cos\left\{\omega t - \left[\frac{2\pi D}{\lambda}\right] \text{x} \left[\frac{1}{2}(n_r + n_l)\right]\right\}$$

Eq. 2

$$E_y = A \left[\cos\left(\omega t - \frac{2\pi n_r D}{\lambda}\right) + \cos\left(\omega t - \frac{2\pi n_l D}{\lambda}\right)\right]$$

$$= 2A \cos\left\{\left[\frac{2\pi D}{\lambda}\right] \times \left[\frac{1}{2}(n_r - n_l)\right]\right\} \quad \times \cos\left\{\omega t - \left[\frac{2\pi D}{\lambda}\right] \times \left[\frac{1}{2}(n_r + n_l)\right]\right\}$$

2. Rotation of the plane of polarization if $n_r \neq n_l$

The x and y components are in phase; the transmitted light is therefore plane polarized, but the direction of vibration is changed from the y direction to a direction in the second quadrant which forms with the y direction an angle given by $\tan\phi = \frac{E_x}{E_y}$, whence a rotation (in radians, with λ and D in the same units) of

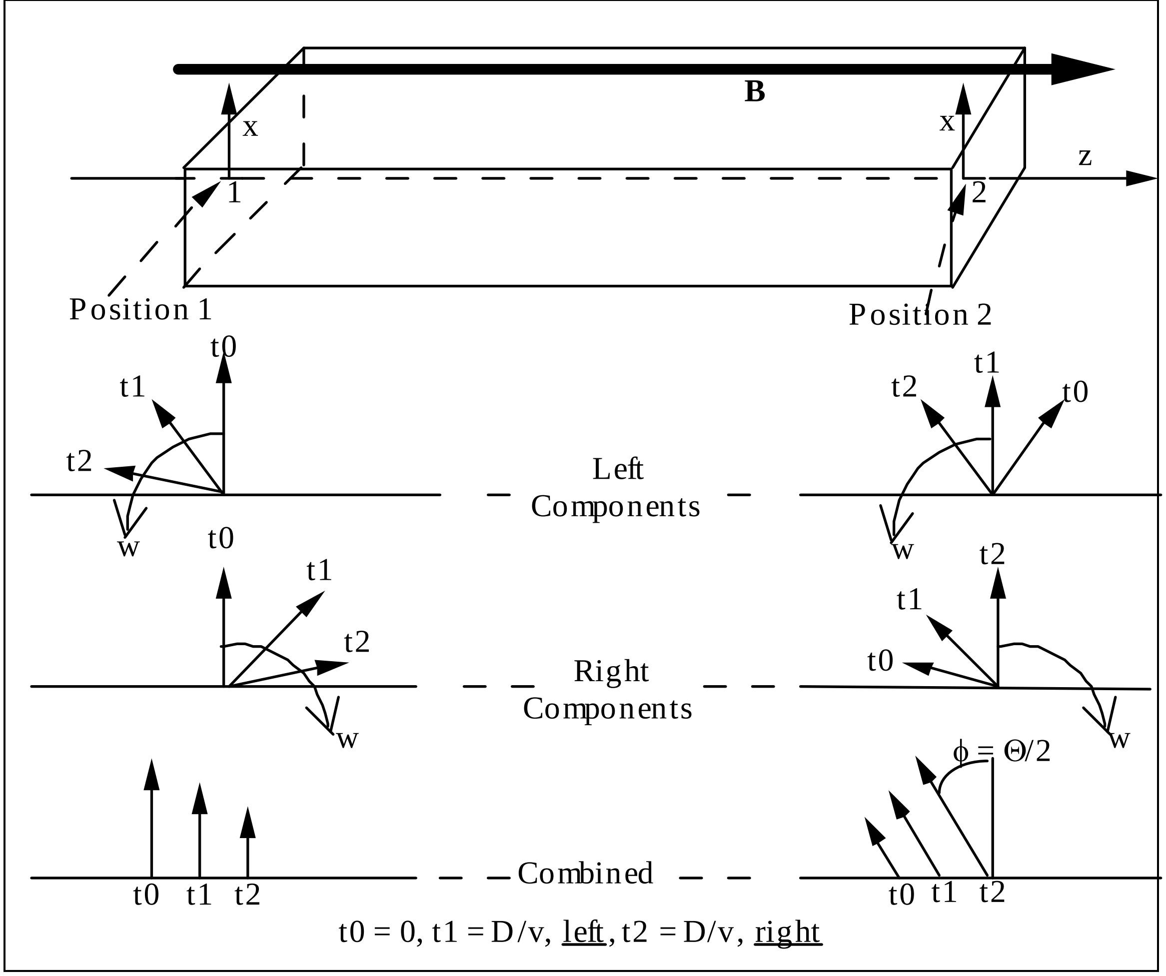
Eq. 3 $$\phi = \pi \frac{D(n_r - n_l)}{\lambda}.$$

The rotation is counterclockwise when $n_r > n_l$.

The figure below illustrates how circular birefringence can produce optical rotation. Light linearly polarized along the x-axis is incident at point 1. The light can be thought of as being formed of two circularly polarized components, and the vectors representing the electric fields of these components at point 1 are shown at t_0 (when our observation starts) and at two later times, t_l and t_2 .

The diagrams on the right side of the figure show the light emerging from the substance at the corresponding times. The left circular component is assumed to travel faster through the substance ($n_r > n_l$); and by time t_1, the electric field of that component is along the z-direction at point 2. The right circular component travels more slowly, and it takes until time t_2 before the electric field of that component is along the z-axis. The left circular component at point 2 has continued to rotate and the angle Θ between the field vectors of the two components at the time t_2 is just the angular velocity multiplied by the difference in time taken for the two components to travel the distance D:

Eq. 4 $$\Theta = \omega\,(\text{time right - time left}) = \omega \frac{D}{v_{right}} - \omega \frac{D}{v_{left}} = \omega \frac{D(n_r - n_l)}{c}.$$



Incident (position 1) and transmitted (position 2) light showing counterclockwise rotation of polarization plane due to slower propagation speed of right circularly polarized component, for $\underline{n}_r > \underline{n}_l$. The resultant field makes an angle Θ/2 with the z-axis, and the net result for this case is that the polarization plane is rotated <u>counterclockwise</u>. (All observations are taken looking into the oncoming beam.) The rotation angle is given as before by:

$$\phi = \Theta/2 = \frac{\omega D(n_r - n_l)}{2c} = \frac{\pi D(n_r - n_l)}{\lambda}.$$

Thus different propagation speeds for the left and right circular components can account for the rotation observed in the Faraday effect. The cause of the different propagation speeds will be discussed next.

Appendix 2. Classical model for light propagation in a medium

1. Spring-mass model of forced electron oscillation

A convenient model for explaining the propagation of light in a substance involves the absorption and re-radiation (delayed and re-directed) of the incoming light by electrons of the medium. The details of this model give an explanation for the absorption, scattering, and index of refraction of the medium.

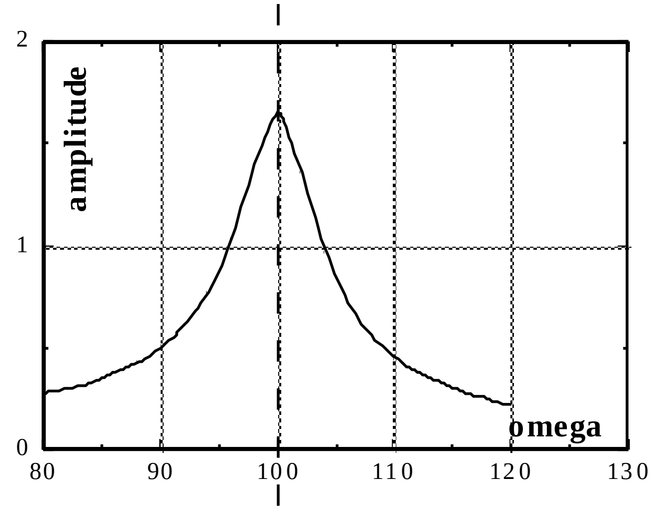
In a classical model, when electrons are disturbed from their equilibrium orbits, they behave as if they were bound by a linear, radial restoring force, and they exhibit the familiar mass-spring damped resonance phenomenon. The absorption and phase shift characteristics of a linear oscillator are shown below.

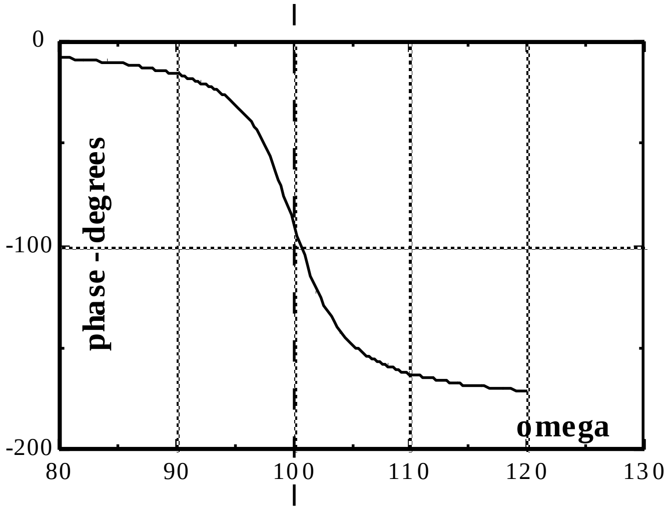
The phase shift curve provides an explanation for the reduced speed of light in a medium. It corresponds to a delay in re-radiation of the light. The re-radiated light combines coherently with the incoming beam, which is further absorbed and re-radiated deeper in the medium. The net effect is a lower propagation speed. The change in the index of refraction with frequency (dispersion) can be explained by the change in phase shift with frequency.

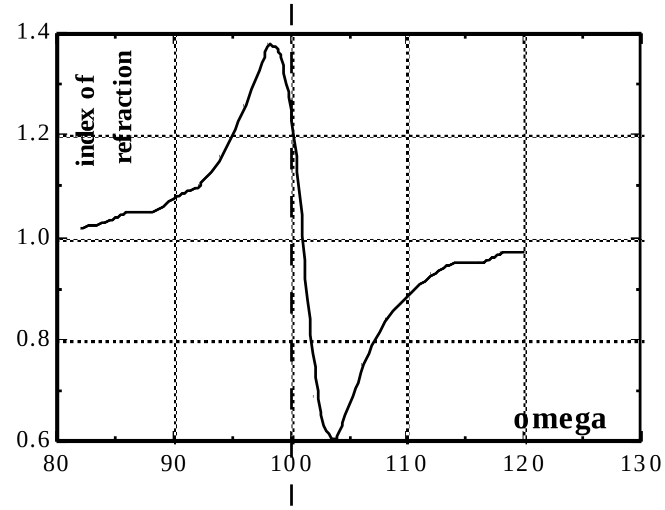
A more quantitative relationship between the refractive index and phase shift is discussed in Rossi, Section 8-4. Briefly, the velocity of light depends upon

$$\frac{1}{\sqrt{em}}$$

where the dependence upon ε (and hence the relationship between the **D** and **E** vectors) results from the displacement current contribution to **B**. If we consider the electrons of the material to have very little damping, the phase relation between **E** and the electron motion (displacement) will shift by almost 180° as the driving (light) frequency passes through resonance. Whereas at lower frequencies the polarization of the atoms reduces the electric field **D** in the medium, at frequencies above the resonance (unsplit, no magnetic field) the phase of the electron motion is reversed, which tends to increase the electric field. Since $\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P}$, ε becomes less than ε_0, and the index of refraction is less than 1 above the resonance. (This implies that $v > c$, the relativistic limiting velocity for causal information. However, as is shown in many texts, the phase velocity (which we are discussing) can exceed c, but not the group or signal velocity.)







Also, recalling that the influence of the medium should diminish as the driving frequency moves away from resonance, we get a qualitative explanation for the curves shown previously (amplitude, phase relation and index of refraction) for an oscillator without an external magnetic field.

2. Analysis of forced, sinusoidal electron oscillations about equilibrium

a. Unsplit resonance at ω_0, with B = 0. [$n_l = n_r$ (--> $v_l = v_r$)]

The equations governing the motion of the electron are:

Eq. 5 $$m\ddot{\mathbf{r}} = -m\omega^2\mathbf{r} + e\mathbf{E} \quad \text{or, with } K=\omega^2 m,$$

$$m\ddot{x} = - eE\cos(\omega t) - Kx$$

$$m\ddot{y} = - eE\sin(\omega t) - Ky\ .$$

The resonance occurs for either of $\omega = \pm\sqrt{K/m} = \pm\,\omega_0$, so at this point (no magnetic field) both right and left circular components are affected in the same way. The introduction of an external magnetic field along the direction of propagation of the light removes this symmetry and leads to the following classical explanation of the Faraday effect.

b. Split "left" and "right" resonances when B ≠ 0; the Larmor frequency shift $\pm\,\omega_L$ from ω_0. [$n_l \neq n_r$ (--> $v_l \neq v_r$)]

Refraction is the result of the interaction of the light with the electrons. Considering a right circular beam passing through a diamagnetic medium in the direction of an externally applied magnetic field $\underline{\mathbf{B}}$ ($\underline{z}$ axis), the equations of motion of the electrons now become:

Eq. 6 $$m\ddot{\mathbf{r}} = - m\omega^2\,\mathbf{r} + e\mathbf{E} + \frac{e}{c}\,(\dot{\mathbf{r}} \times \mathbf{B}) \quad ===>$$

$$m\ddot{x} = - eE\cos(\omega t) - Kx - \frac{e}{c}\,B\dot{y}$$

$$m\ddot{y} = - eE\sin(\omega t) - Ky + \frac{e}{c}\,B\dot{x}\ .$$

Disregarding, momentarily, the incoming electric field, it is easy to show that there are solutions corresponding to circular motion of the electrons with angular frequency

Eq. 7 $$\omega_L = \frac{eB}{(2m)} = 2\pi\nu_L,$$

where ν_L is called the "Larmor" frequency.

For an observer looking into the beam, the light vector rotates clockwise, and the electronic structure rotates counterclockwise (negative charge) with the Larmor frequency, relative to the circularly polarized light. If ν_0 is the frequency of light it is apparent that, to the rotating electrons, the light vector appears to be rotating at a higher frequency $\nu_0 + \nu_L$ and it will act accordingly; that is the refractive index for this light will have the same value as the unmagnetized medium has for light of frequency $\nu_0 + \nu_L$. Hence

Eq. 8 $$n_r(\nu) = n(\nu_0+\nu_L)$$

where n is the refractive index as ordinarily measured without magnetic field. In a similar way left circular light passing in the same direction as B appears to the atoms to have a lower frequency and hence

Eq. 9 $$n_l(\nu) = n(\nu_0-\nu_L) .$$

Since ν_L for visible light is much smaller than ν_0 , we can write

Eq. 10 $$n(\nu_0 \pm \nu_L) = n(\nu_0) + \frac{dn}{d\nu}\nu_L$$ and, since

$$\lambda = \frac{c}{\nu} \text{ and } \frac{d\lambda}{d\nu} = -\frac{c}{\nu^2} \Rightarrow \frac{dn}{d\nu} = -\frac{dn}{d\lambda}\frac{c}{\lambda^2} = -\frac{\lambda^2}{c}\frac{dn}{d\lambda},$$ we obtain

$$(n_r\text{-}n_l) = 2\,\frac{dn}{d\nu}\nu_L \text{ and, with } \nu_L = \frac{1}{2\pi}\omega_L = \frac{1}{2\pi}\frac{eB}{2m}(<< n_0)$$

Eq. 11 $$(n_r - n_l) = -\frac{2\lambda^2}{c}\frac{dn}{d\lambda}\frac{eB}{4\pi m}$$

where $\frac{dn}{d\lambda}$ is evaluated at frequency ν_0.

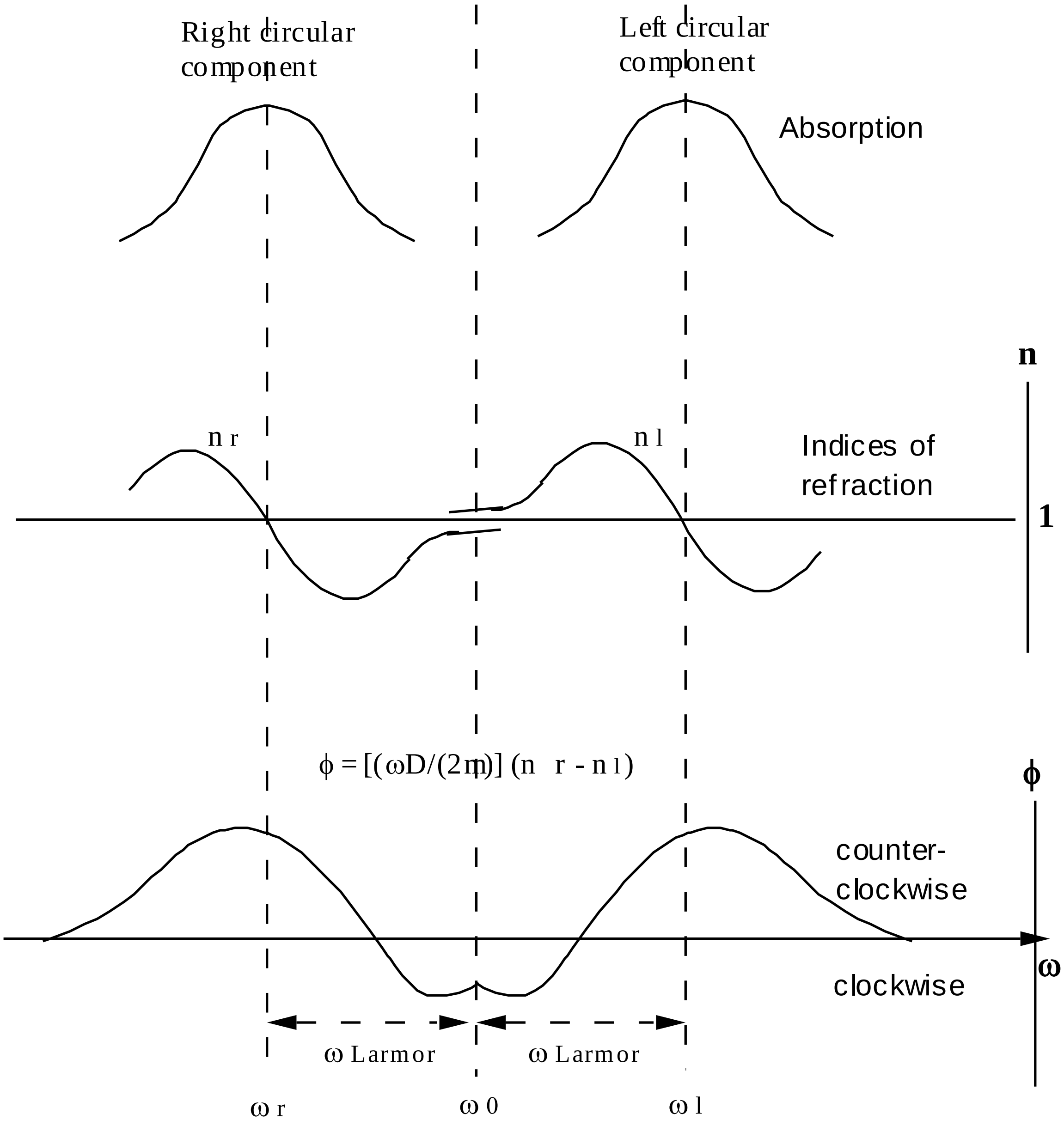
Finally, the optical rotation per unit length is

Eq. 12 $$\frac{\mathrm{f}}{\mathrm{D}} = -\frac{1}{2}\frac{e}{m}\frac{\lambda}{c}\frac{dn}{d\lambda}\mathrm{B} .$$

This is called Becquerel's equation. The quantity $V = \phi/(DB) = -\frac{1}{2}\frac{e}{m}\frac{\lambda}{c}\frac{dn}{d\lambda}$ is called the Verdet constant of the material. It is usually given in degrees of rotation per gauss, per centimeter of light path. (Note that the unit degree, although dimensionless, nevertheless involves mixed units of length. Angular measure is defined as the ratio of arc length to radius length; with both in the same units, the angular unit is the radian. When using degrees, the arc length unit is [360/(2π)] times that of the radius. In comparing experiment and theory, it is best to convert the Verdet constant to radians, rather than degrees.)

Appendix 3. Anomalous variation of n between the Larmor-split resonances

Finally, combining the varying index of refraction with the small splitting due to an external magnetic field ($\pm \omega_L$) from the zero-field resonant frequency ω_0 , we obtain for the two circular components the results shown below (greatly exaggerating the shift due to the Larmor rotation):



showing anomalous dispersion ($n < 1$), and the origin of the rotation angle vs. $\underline{\omega}$, in the region of the split resonances. Of course, our data is taken far from these classical spring-mass ultraviolet electron resonances (at lower ω), where $n_r > n_l$, as discussed previously.

This model describes the normal, diamagnetic Faraday effect quite well. Paramagnetic and ferromagnetic substances also exhibit a Faraday effect. The present experiment, however, involves measurement of the Faraday rotation for a diamagnetic substance.

Appendix 4. Why does the classical picture work?

Since we know that the spring-mass classical picture of electron binding to a positively charged nucleus leads to prediction of an atom unstable to radiative energy loss, an interesting question is: Why does the classical picture above work so well? Another is: Why are the important, governing resonances in the ultraviolet?

As to the first question, all we need are strong and heavily damped (broad) absorption resonances. Quantized excitation energies provide them. For the second question, we must conclude that atomic and solid state quantum level structures are such that resonances with the required properties lie in the ultraviolet. This involves such fundamental quantities as the strength of the Coulomb force, the mass of the electron, the stability of nuclei, the Pauli principle (quantum statistics of the electron), etc.

Appendix 5. Dominant effect of ultraviolet resonances on normal dispersion in the visible. Cauchy formula for n(λ).

The dispersion, $\underline{dn}/\underline{d\lambda}$, is also a function of $\underline{\lambda}$. For most glasses, the λ behavior of n is dominated by "distant" scattering , strongly absorptive (therefore broad) ultraviolet resonances. An exception is the Kigre glass sample used in this experiment, for which erbium doping produces infrared absorption dominance in the visible variation of n. The sign of the Verdet constant is thus different for the flint and Kigre glasses.

The first successful attempt to represent the curve of normal dispersion was made by Cauchy in 1836 using the form

Eq. 13 $$n = A + B/\lambda^2 + C/\lambda^4 ,$$

where $\underline{A}$, $\underline{B}$, and $\underline{C}$ are constants characteristic of any one substance ($\underline{B} \neq$ magnetic field here, of course). Very often it is sufficient to keep the first two terms only. Then

Eq. 14 $$n = A + B/\lambda^2 \quad \text{and} \quad dn/d\lambda = -2B/\lambda^3$$

The table below gives the index of refraction $\underline{n}$ vs. wavelength for various materials, in the visible range from <u>400</u> to <u>750</u> nanometers. A quadratic fit in

$\frac{1}{l^2}$ (using KaleidaGraph on a Macintosh, for example) will yield the Cauchy parameters for n vs. λ. dn/dλ can then be determined by differentiation of this analytic fit formula. Around 200 nm, n curves sharply upward, indicating the distant absorption and scattering resonances which cut off transmission in the ultraviolet.

In specifying glass the manufacturer usually gives n_D (the index for the yellow sodium D lines) and also the value of $(n_D-1)/(n_F-n_C)$, the indices for several other lines of common sources, and the differences between these and a number of other lines. Detailed information concerning many glasses can be found in the International Critical Tables. The table below is from Monk. The numerical part can be copied and pasted directly into a KaleidaGraph data file.

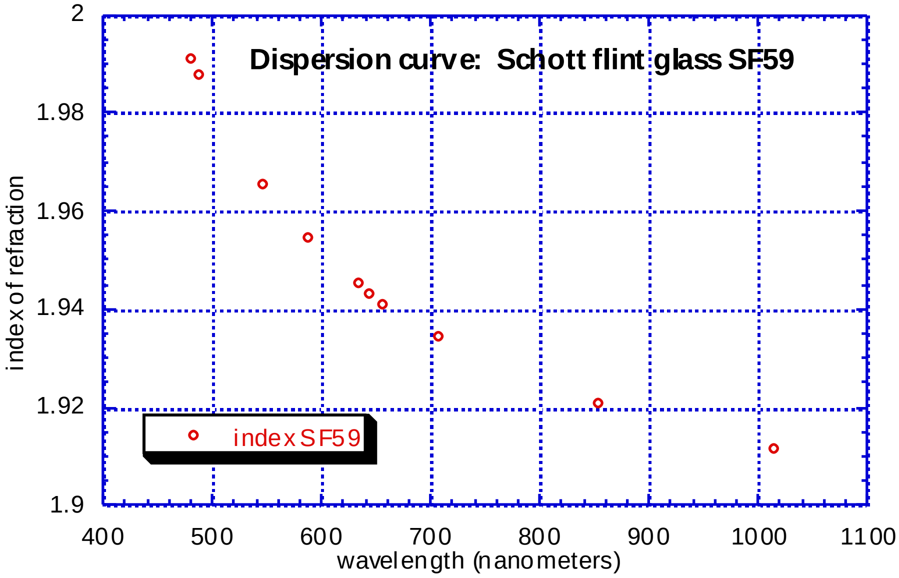
λ (Å)	**Light Crown**	**Dense crown**	**Light Flint**	**Dense Flint**	**Heavy Flint**	**Fused Quartz**	**Fluorite**
4000	1.5238	1.5854	1.5932	1.6912	1.8059	1.4699	1.4421
4600	1.5180	1.5801	1.5853	1.6771	1.7843	1.4655	1.4390
5000	1.5139	1.5751	1.5796	1.6770	1.7706	1.4624	1.4366
5600	1.5108	1.5732	1.5757	1.6951	1.7611	1.4599	1.4350
6000	1.5085	1.5679	1.5728	I.6542	1.7539	1.4581	1.4336
6500	1.5067	1.5651	1.5703	I.6503	1.7485	1.4566	1.4324
7000	1.5051	1.5640	1.5684	1.6473	1.7435	1.4553	1.4318
7500	1.5040	1.5625	1.5668	1.6450	1.7389	1.4542	1.4311

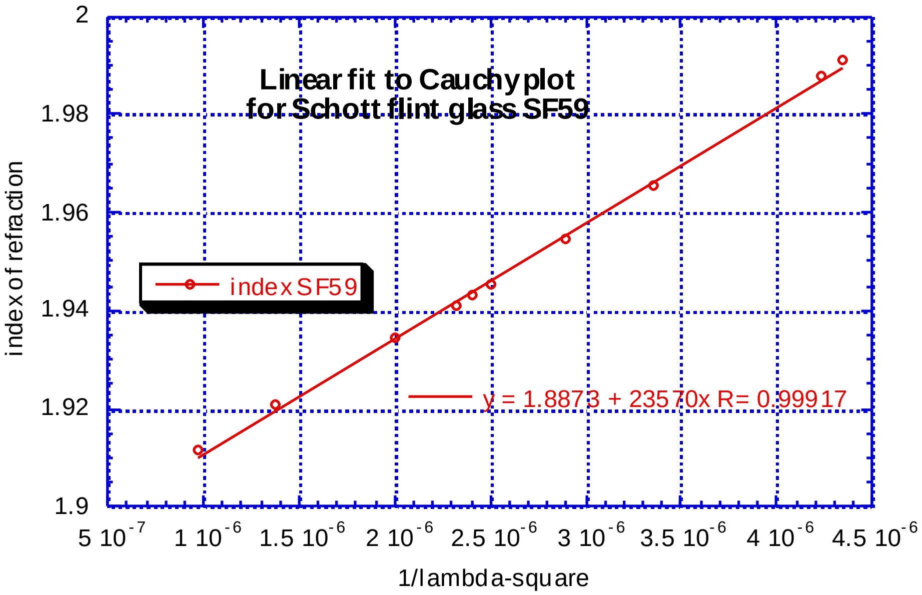
It is also interesting to analyze the Faraday rotation effects in terms of the phase shifts encountered by the light beam as it traverses the substance. If the incoming light has a frequency below both resonance frequencies ($\omega_0 \pm \omega_L$), it is clear that the phase shift will be greater for the right circular component ($\frac{dn}{dl}$ negative). Thus its propagation speed will be lower, and a situation as illustrated will result.

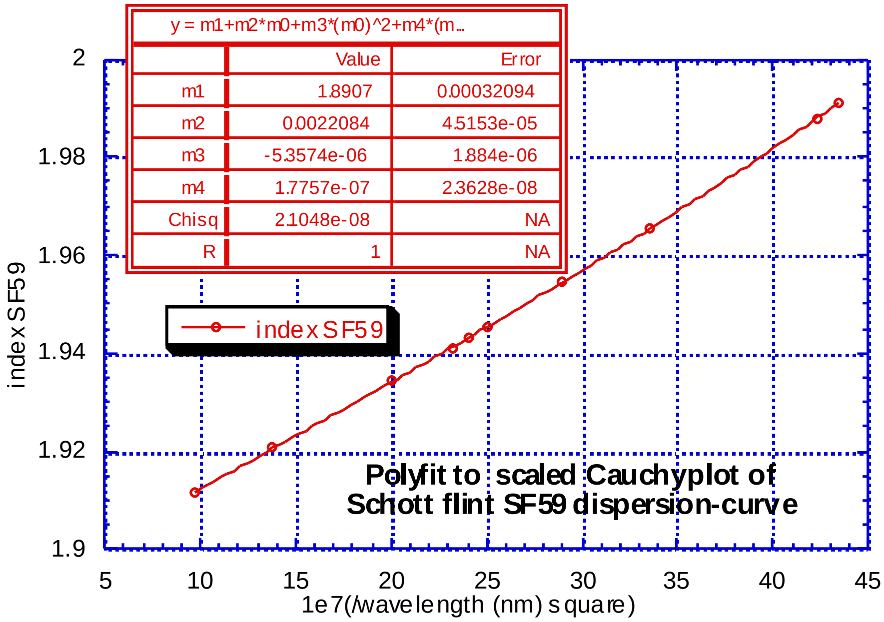
Manufacturers data (Schott Glass technologies, Inc.) for the flint glass sample (type SF59, melt #703495/I, order #Q346376, international code 953204, Lk #23908, Schott Ref. #D6502, 20 mm diameter x 10 mm thick) includes the following indices of refraction:

line	**λ (nm)**	**$1/\lambda^2$**	**1e7 x $1/\lambda^2$**	**index n**
nt	1014.0	9.7258e-07	9.7258	1.9117
ns	852.10	1.3773e-06	13.773	1.9206
nr	706.50	2.0034e-06	20.034	1.9342
nC	656.30	2.3216e-06	23.216	1.9412
nC'	643.80	2.4127e-06	24.127	1.9433
n	632.80	2.4973e-06	24.973	1.9452
nd	587.60	2.8963e-06	28.963	1.9545
ne	546.10	3.3532e-06	33.532	1.9654
nF	486.10	4.2320e-06	42.320	1.9880
nF'	480.00	4.3403e-06	43.403	1.9909

Plots and fits to this data are shown below.


Dispersion curve: Schott flint glass SF59
index of refraction
2
1.98
1.96
1.94
1.92
1.9
index SF59
400
500
600
700
800
900
1000
1100
wavelength (nanometers)


Linear fit to Cauchy plot
for Schott flint glass SF59
index of refraction
2
1.98
1.96
1.94
1.92
1.9
index SF59
y = 1.8873 + 23570x R= 0.99917
5 10^-7
1 10^-6
1.5 10^-6
2 10^-6
2.5 10^-6
3 10^-6
3.5 10^-6
4 10^-6
4.5 10^-6
1/lambda-square



y = m1+m2*m0+m3*(m0)^2+m4*(m...

	Value	Error
m1	1.8907	0.00032094
m2	0.0022084	4.5153e-05
m3	-5.3574e-06	1.884e-06
m4	1.7757e-07	2.3628e-08
Chisq	2.1048e-08	NA
R	1	NA