Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

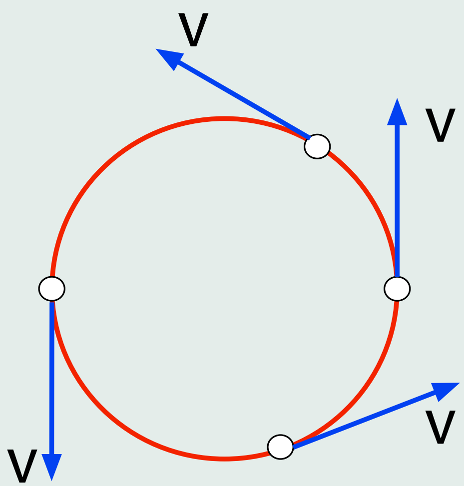
Lecture 5

Announcements

- i-Clicker mailing list: all names on the roster who have not been assigned an i-clicker remote id. Please answer before Monday September 26th. Will start counting next week.

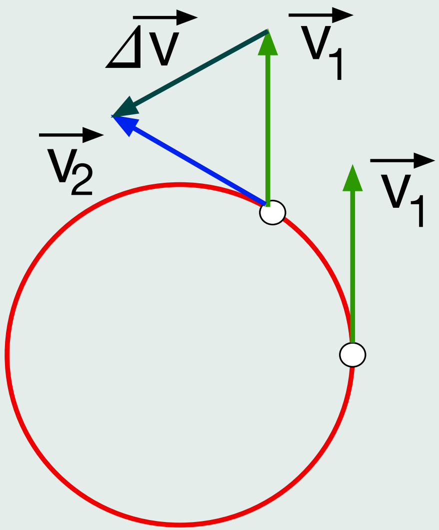
4. Motion in 2D and 3D. Uniform circular motion

- Particle moving around a circle or circular arc
- The **speed** $|\vec{v}|$ is constant.



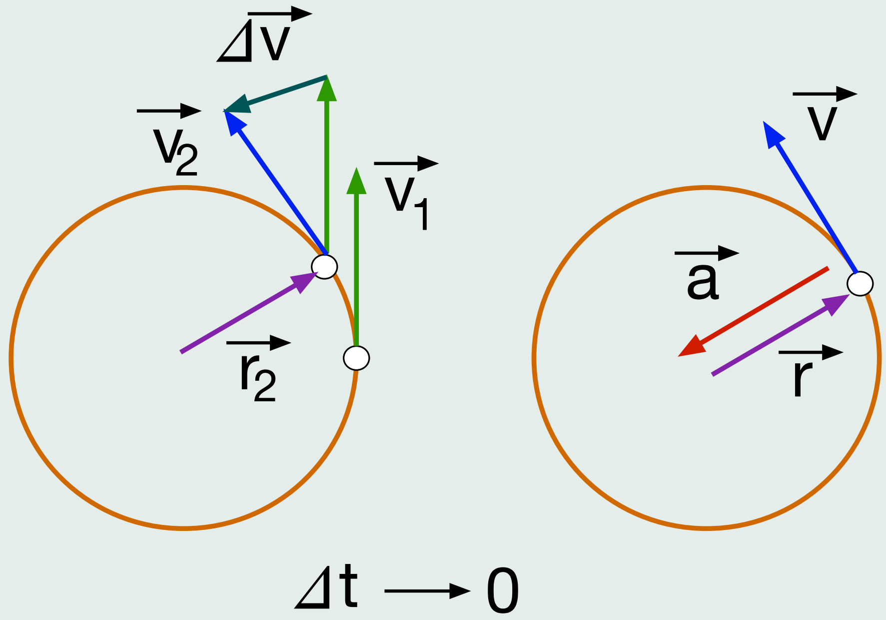
Is the acceleration vector zero?

$$\vec{a} = \frac{d\vec{v}}{dt} \qquad \vec{a} = 0 \;\Leftrightarrow\; \vec{v} \text{ has } \begin{cases} \textbf{constant magnitude} \\ \textbf{and} \\ \textbf{constant direction} \end{cases}$$



Average acceleration:

$$\vec{a}_{\mathsf{avg}} = \frac{\Delta\vec{v}}{\Delta t} \neq 0$$

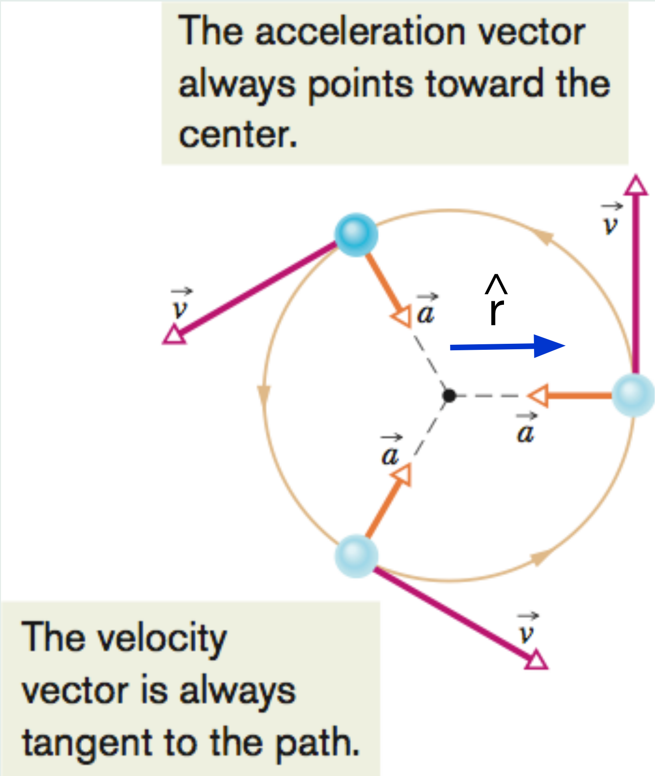


$\vec{a}$ is parallel to $\vec{r}$ and points into opposite direction.

Instantaneous acceleration:

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} \neq 0$$

- **Centripetal (center-seeking) Acceleration**



$$\vec{a} = -\frac{v^2}{r}\hat{r}$$

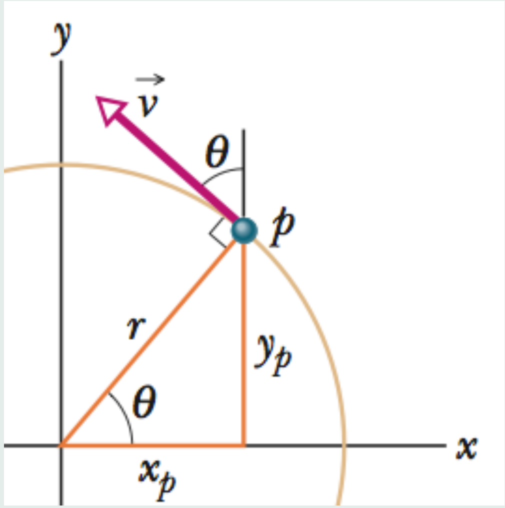
$$|\vec{a}| = \frac{v^2}{r}$$

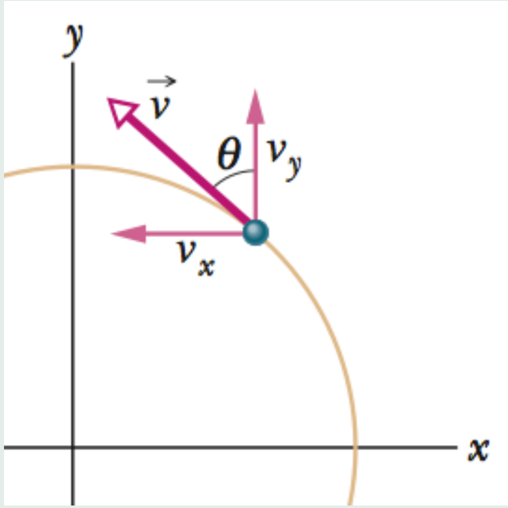
$$r = \text{ radius of the circle}$$

$$v = \text{ speed}$$

$$\hat{r} = \frac{\vec{r}}{r} \quad \textbf{unit} \text{ radial vector}$$

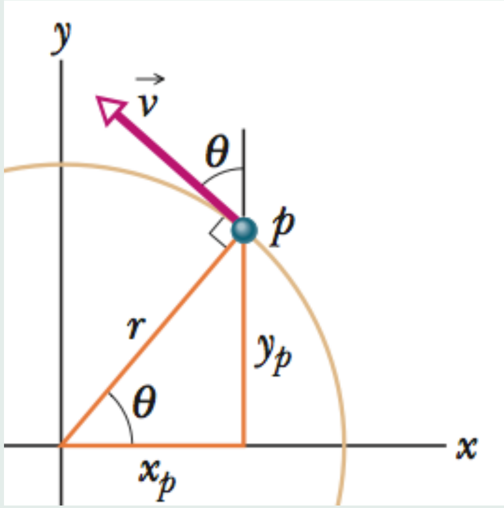
- **Proof:**

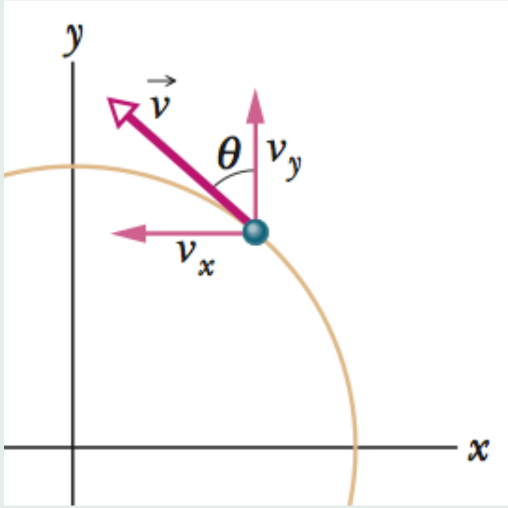




$$v_x = -v\sin\theta = -v\frac{y_p}{r} \qquad v_y = v\cos\theta = v\frac{x_p}{r}$$

$$a_x = \frac{dv_x}{dt} = -\frac{v}{r}\frac{dy_p}{dt} \qquad a_y = \frac{dv_y}{dt} = \frac{v}{r}\frac{dx_p}{dt}$$





$$\frac{dy_p}{dt} = v_y = v\cos\theta = v\frac{x_p}{r}$$

$$\frac{dx_p}{dt} = v_x = -v\sin\theta = -v\frac{y_p}{r}$$

$$a_x = -\frac{v}{r}\frac{dy_p}{dt} = -\frac{v^2}{r}\frac{x_p}{r} \qquad a_y = \frac{v}{r}\frac{dx_p}{dt} = -\frac{v^2}{r}\frac{y_p}{r}$$

$$a_x = -\frac{v}{r}\frac{dy_p}{dt} = -\frac{v^2}{r}\frac{x_p}{r} \qquad a_y = \frac{v}{r}\frac{dx_p}{dt} = -\frac{v^2}{r}\frac{y_p}{r}$$

$$\vec{a} = a_x\widehat{i} + a_y\widehat{j} = -\frac{v^2}{r}\frac{x_p\widehat{i} + y_p\widehat{j}}{r} = -\frac{v^2}{r}\frac{\vec{r}}{r} = -\frac{v^2}{r}\widehat{r}$$

$$\boxed{\begin{aligned} \vec{a} &= -\frac{v^2}{r}\widehat{r} \\ |\vec{a}| &= \frac{v^2}{r} \end{aligned}}$$

- **Period**

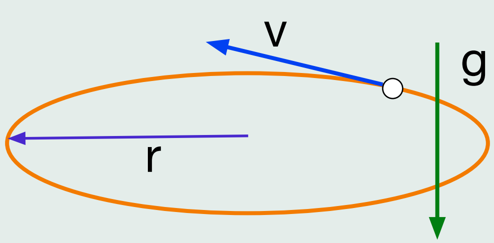
$$T = \frac{2\pi r}{v} \qquad \text{how long for one rotation}$$

- **Frequency**

$$\nu = \frac{1}{T} = \frac{v}{2\pi r} \qquad \text{number of rotations per unit time}$$

i-Clicker

An object moving on a circular trajectory of radius r with constant speed v is at the same time falling freely near the Earth's surface. Which of the following statements holds for the **total** acceleration vector?



A) It is horizontal.

B) It is vertical.

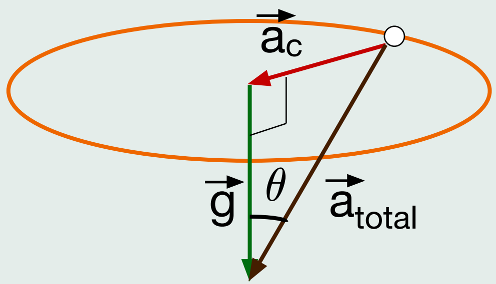
C) Its magnitude is g.

D) Its magnitude is v^2/r.

E) It makes an angle $\theta = \arctan(v^2/rg)$ with $\vec{g}$.

Answer

An object moving on a circular trajectory of radius r with constant speed v is at the same time falling freely near the Earth's surface. Which of the following statements holds for the **total** acceleration vector?



$\vec{a}_c = -\frac{v^2}{r}\widehat{r}$

A) It is horizontal.

B) It is vertical.

C) Its magnitude is g.

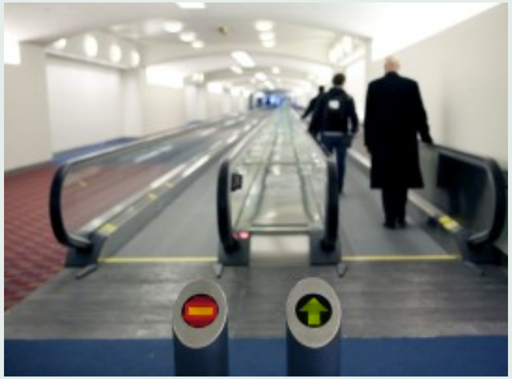
D) Its magnitude is v^2/r.

E) It makes an angle $\theta = \arctan(v^2/rg)$ with $\vec{g}$.

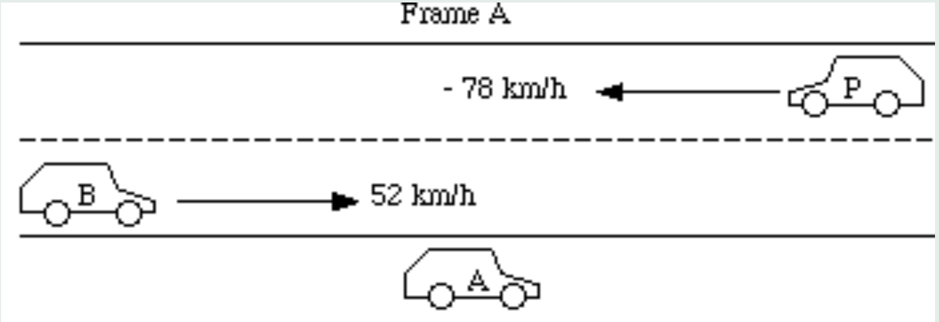
Relative motion in one dimension

- cars travel at 50-60 mph yet they barely move with respect to each other.

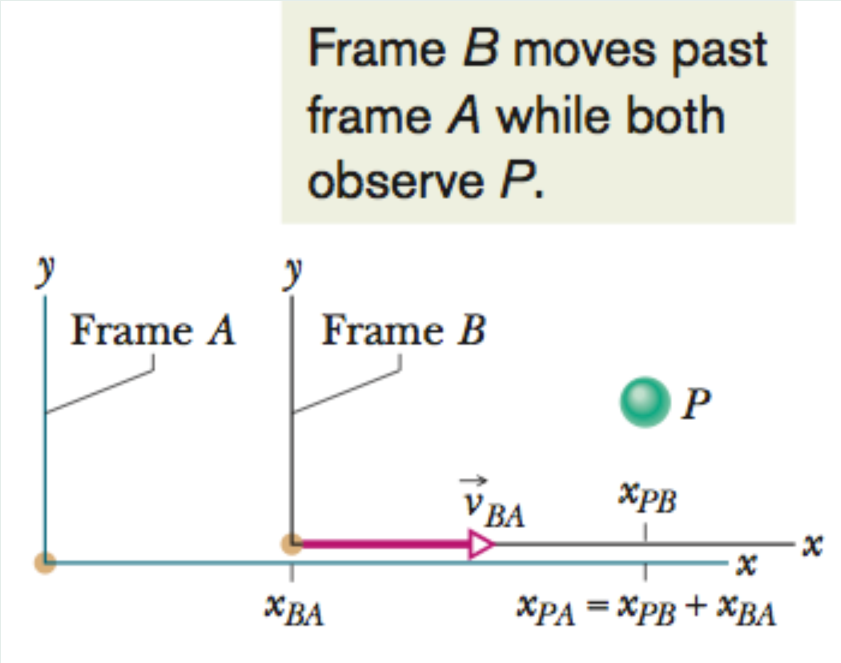
• passengers stand still on moving walkway yet they move with respect to the ground.



The velocity of a particle depends on the reference frame of the observer.



- What is the velocity of car P measured by the driver of Car B?

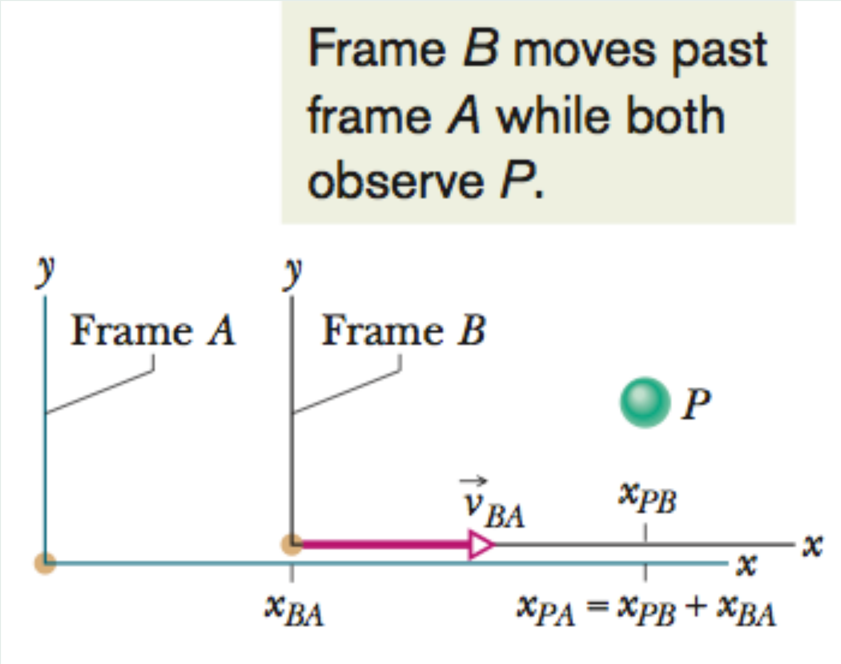


x_{BA} coordinate of B in frame A

x_{PA} coordinate of P in frame A

x_{PB} coordinate of P in frame B

$$x_{PA} = x_{PB} + x_{BA}$$



x_{BA} coordinate of B in frame A

x_{PA} coordinate of P in frame A

x_{PB} coordinate of P in frame B

$$x_{PA} = x_{PB} + x_{BA}$$

$$\frac{dx_{PA}}{dt} = \frac{dx_{PB}}{dt} + \frac{dx_{BA}}{dt} \quad \Rightarrow \quad \boxed{v_{PA} = v_{PB} + v_{BA}}$$

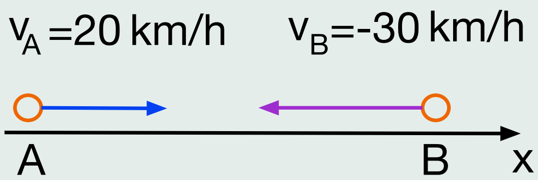
$$\frac{dv_{PA}}{dt} = \frac{dv_{PB}}{dt} + \frac{dv_{BA}}{dt} \;\Rightarrow\; \boxed{a_{PA} = a_{PB} + a_{BA}}$$

- **Note:** if $a_{BA} = 0$ then $a_{PA} = a_{PB}$.

Different observers moving at constant velocity relative to each other will measure the same acceleration for a moving particle P.

i-Clicker

Two cars A and B move on straight line with constant velocities $v_A = 20\,\mathrm{km/h}$ and $v_B = -30\,\mathrm{km/h}$ relative to a stationary observer. Recall that v_{BA} is the velocity of B measured by the driver A while v_{AB} is the velocity of A measured by the driver B.



$A)$ $v_{AB} = 20\,\mathrm{km/h}$, $v_{BA} = -30\,\mathrm{km/h}$

$B)$ $v_{AB} = 50\,\mathrm{km/h}$, $v_{BA} = -50\,\mathrm{km/h}$

$C)$ $v_{AB} = 50\,\mathrm{km/h}$, $v_{BA} = 50\,\mathrm{km/h}$

$D)$ $v_{AB} = -50\,\mathrm{km/h}$, $v_{BA} = -50\,\mathrm{km/h}$

$E)$ $v_{AB} = -30\,\mathrm{km/h}$, $v_{BA} = 20\,\mathrm{km/h}$

Answer

Two cars A and B move on straight line with constant velocities $v_A = 20\,\mathrm{km/h}$ and $v_B = -30\,\mathrm{km/h}$ relative to a stationary observer. Recall that v_{BA} is the velocity of B measured by the driver A while v_{AB} is the velocity of A measured by the driver B.

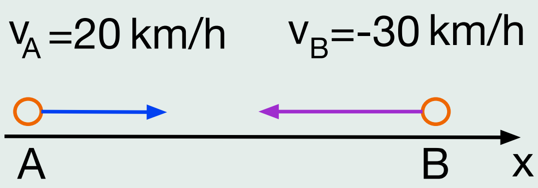
$A)$ $v_{AB} = 20\,\mathrm{km/h},\ v_{BA} = -30\,\mathrm{km/h}$

$B)$ $v_{AB} = 50\,\mathrm{km/h},\ v_{BA} = -50\,\mathrm{km/h}$

$C)$ $v_{AB} = 50\,\mathrm{km/h},\ v_{BA} = 50\,\mathrm{km/h}$

$D)$ $v_{AB} = -50\,\mathrm{km/h},\ v_{BA} = -50\,\mathrm{km/h}$

$E)$ $v_{AB} = -30\,\mathrm{km/h},\ v_{BA} = 20\,\mathrm{km/h}$



$$v_B = v_A + v_{BA} \qquad v_A = v_B + v_{AB}$$

Relative motion in two dimensions

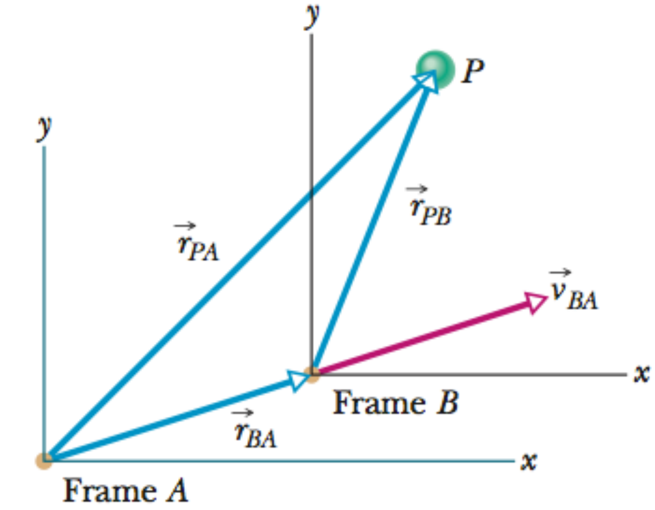
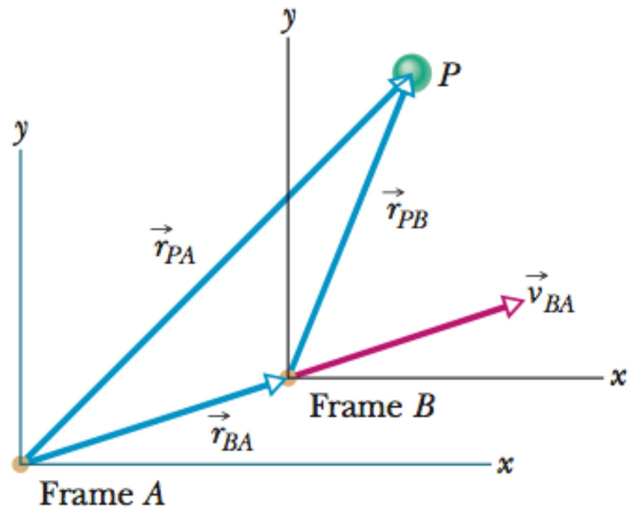


Fig. 4-19 Frame B has the constant two-dimensional velocity $\vec{v}_{BA}$ relative to frame A. The position vector of B relative to A is $\vec{r}_{BA}$. The position vectors of particle P are $\vec{r}_{PA}$ relative to A and $\vec{r}_{PB}$ relative to B.

The motion of particle P is studied by two observers moving at constant velocity relative to each other.

Observer B is moving with constant velocity $\vec{v}_{BA}$ relative to observer A.



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- $\vec{r}_{PA}$ position vector of P relative to A
- $\vec{r}_{PB}$ position vector of P relative to B
- $\vec{r}_{BA}$ position vector of B relative to A

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

- same notation for velocity and acceleration

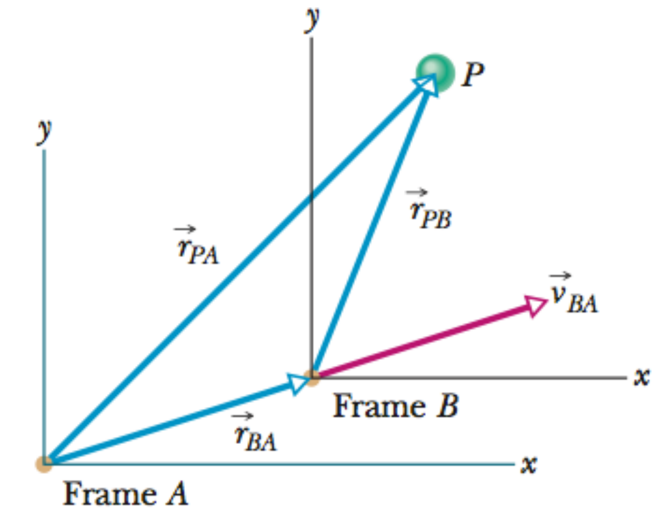


Fig. 4-19 Frame B has the constant two-dimensional velocity $\vec{v}_{BA}$ relative to frame A. The position vector of B relative to A is $\vec{r}_{BA}$. The position vectors of particle P are $\vec{r}_{PA}$ relative to A and $\vec{r}_{PB}$ relative to B.

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$$

$$\Downarrow \; d/dt$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

$$\Downarrow \; d/dt$$

$$\vec{a}_{PA} = \vec{a}_{PB} + \vec{a}_{BA}$$

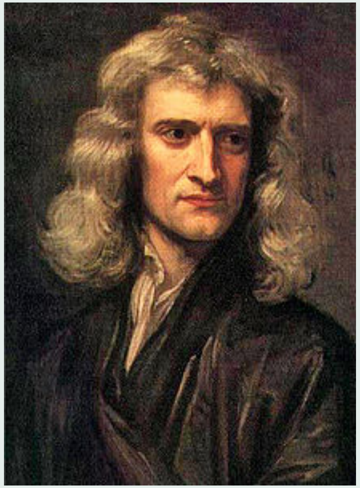
$$\vec{a}_{BA} = 0 \;\Rightarrow\; \vec{a}_{PA} = \vec{a}_{PB}$$

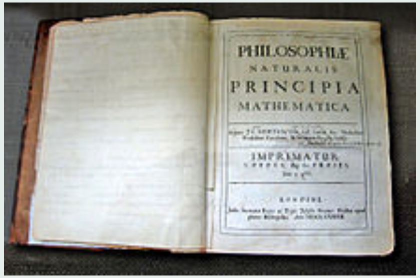
5. Force and Motion I

- What **causes** motion? What changes motion?

- How do objects interact with each other?

- First mathematical theory:





Isaac Newton, *Principia Mathematica*, 1687

Newtonian Mechanics

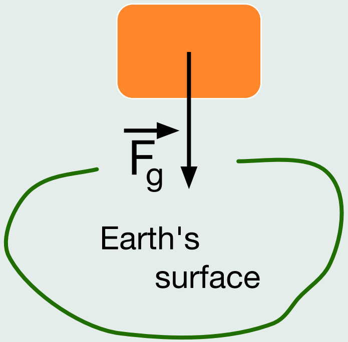
- Relation between **force** and **acceleration**

$$\Downarrow$$

Three fundamental laws.

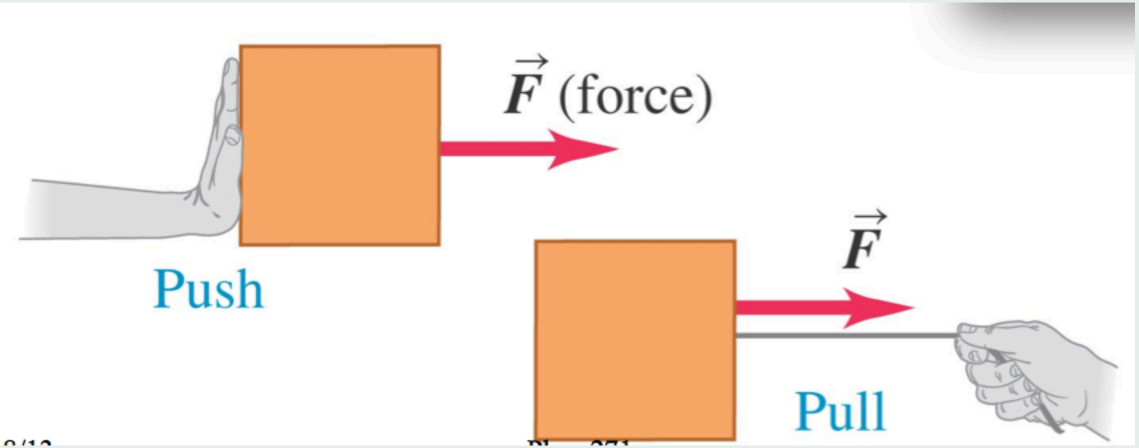
- Does not apply to **all** physical phenomena:
    - Particles travelling close to the speed of light. (Special relativity)
    - Atomic and sub-atomic particles. (Quantum mechanics)

- How can we define **force**?
    - Basic idea: a physical effect which changes the **velocity** of an object.
    - Mathematically: **force** is a **vector quantity**; it has **direction** and **magnitude**
- **Examples:**
    - **Gravitational attraction**

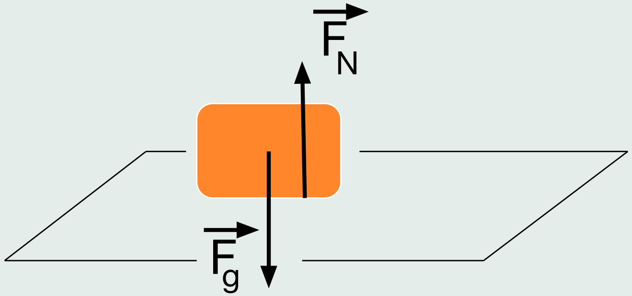


All objects near the Earth's surface are attracted towards the Earth.

- **Push and pull forces**



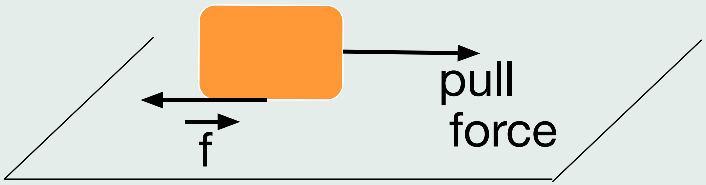
- **Normal force**



Any object placed on a surface is subject to a push force stopping it from falling through.

Always perpendicular to the surface.

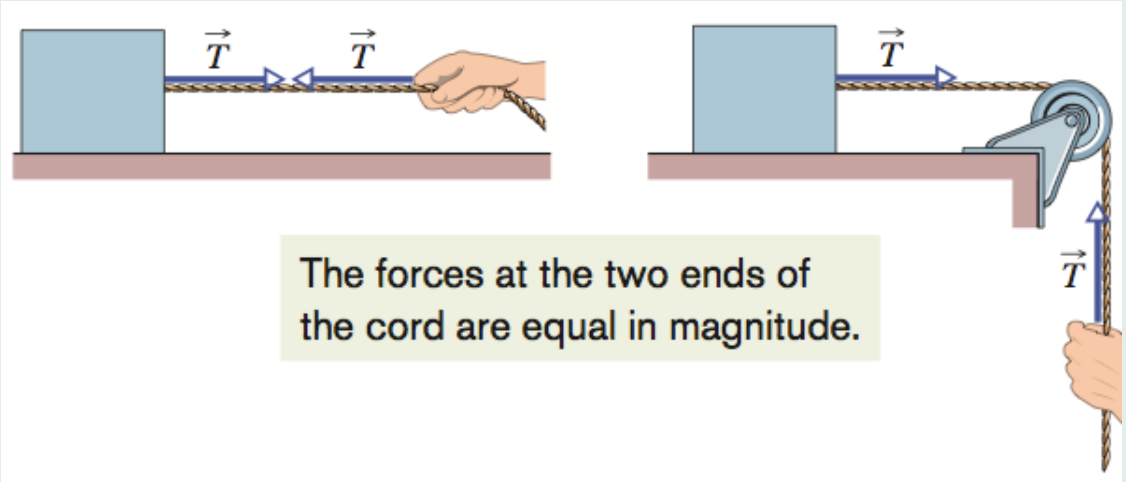
- **Frictional force (friction)**



Resistance force opposing motion.

Always tangent to the surface.

- **Tension**: pull force exerted by a (*massless unstretchable*) cord



- **Other forces in nature:**
  electromagnetic, weak, strong . . .

Newton's First Law

If no force acts on a body, the body's velocity cannot change; the body cannot accelerate.

$$\Downarrow$$

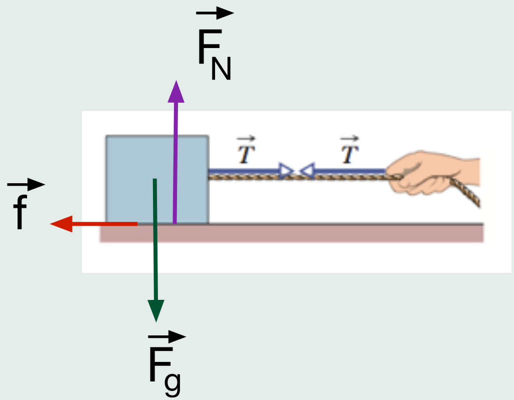
If the body is at rest, it will stay at rest. If it is moving, it will continue to move with the same velocity (same magnitude and direction.)

- **Good approximation:** puck sliding on ice

Friction is negligible for a short ice strip but on a very long ice strip the puck will eventually stop.

- **Forces** are **vectors** (magnitude and direction.)
- The **net force** acting on an object is the **vector sum** of all individual forces acting on that object.

$$\vec{F}_{\text{net}} = \vec{F}_g + \vec{F}_N + \vec{f} + \vec{T} + \cdots$$



Newton's 1st Law (reformulation)

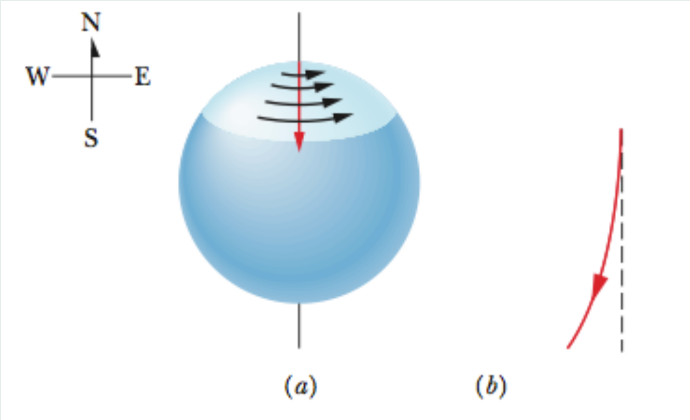
If no net force acts on a body,

$$\vec{F}_{\text{net}} = 0,$$

the body's velocity cannot change;

the body cannot accelerate.

- **Inertial Frame:** a frame in which Newton's laws hold.



Puck sliding on a strip of frictionless ice.

(a) the path of the puck seen by a stationary observer in space (inertial)

(b) the path of the puck seen by a ground observer rotating with the Earth. (noninertial)

Note: apparent deflection caused by gradient in Earth's rotation; for a short strip of ice a a ground observer is $\sim$ inertial.

What is the main effect of a nonzero net force?

- An applied force can set stationary object in motion or can stop a moving object.

$$\Downarrow$$

change in velocity, hence acceleration.

What is the relation between the net force acting on an object and the resulting acceleration?

$$\Downarrow$$

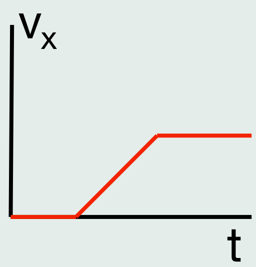
Newton's second law

$$\boxed{\vec{F}_{\text{net}} = m\vec{a},}$$

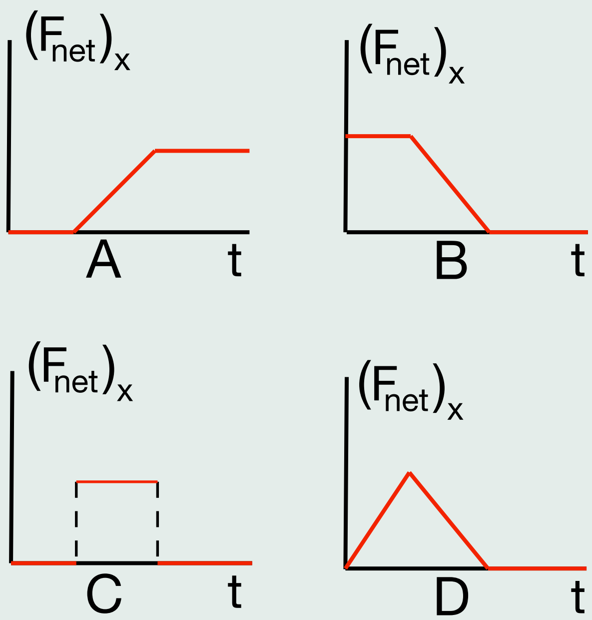
- **m=Mass:** **intrinsic characteristic** of a body relating a force on the body to the resulting acceleration.

i-Clicker

The graph below represents the x-component of the velocity of an object as a function of time.

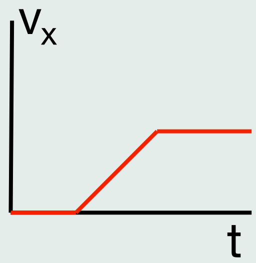


Which of the following graphs represents the time dependence of the x-component of the net force acting on the object?



Answer

The graph below represents the x-component of the velocity of an object as a function of time.



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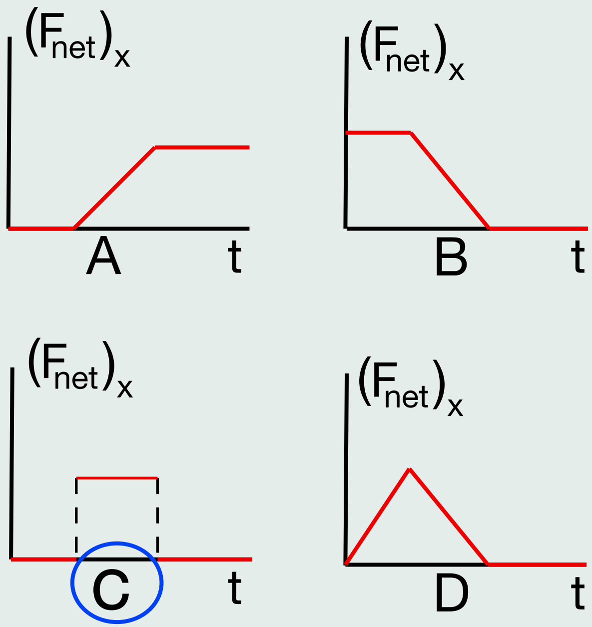

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- **Force unit:** Newton (N)

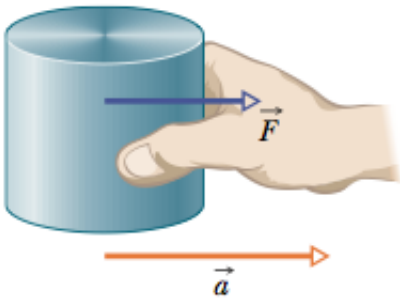

Fig. 5-1 A force $\vec{F}$ on the standard kilogram gives that body an acceleration $\vec{a}$.

1 Newton = the force exerted on a standard mass of 1 kg to produce an acceleration of 1 m/s^2.