Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 21

Final Exam:

- Wednesday, Dec 21st, 8:00-11:00am in PHL.
- Final make up: Thursday, Dec 22nd, 10:00am - 1:00pm in Serrin (physics building) E372.
- Final: 20 questions = 25% final score

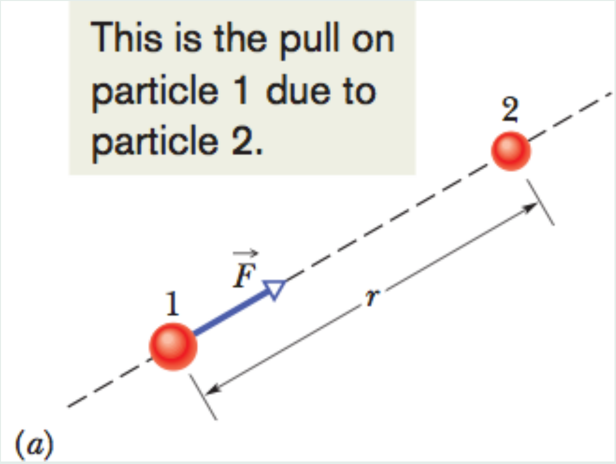
13. Gravitation

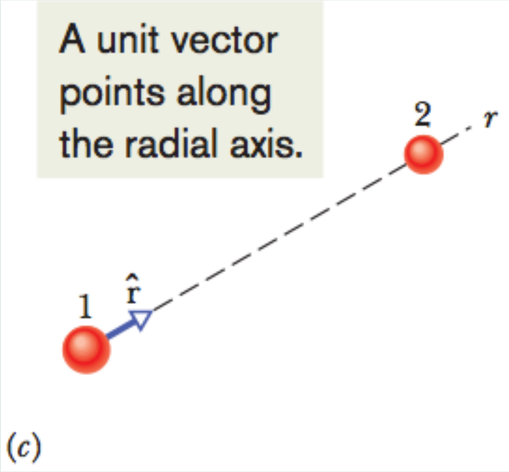
Newton's law of gravitation

Every point particle attracts every other particle with a gravitational force

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

$$G = 6.67 \times 10^{-11}\,\mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2 = 6.67 \times 10^{-11}\,\mathrm{m}^3/\mathrm{kg}\cdot\mathrm{s}^2$$





Gravitational force on particle 1 – vector form:

$$\vec{F}_{2\,\mathrm{on}\,1} = G\frac{m_1 m_2}{r^2}\widehat{r}$$

$\widehat{r}$ is the radial **unit** vector: $|\vec{r}| = 1$.

- **Principle of Superposition**

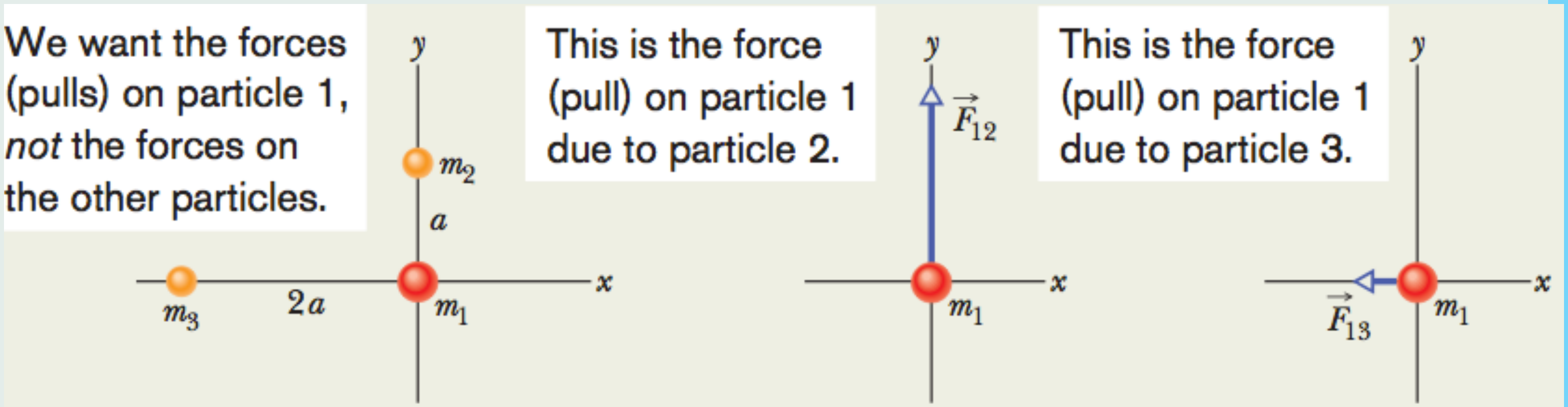
Given n interacting particles:

$$\boxed{\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \cdots + \vec{F}_{1n}}$$

$\vec{F}_{1,\text{net}} =$ net gravitational force acting on particle 1

$\vec{F}_{1,i} =$ gravitational force on particle 1 from particle i

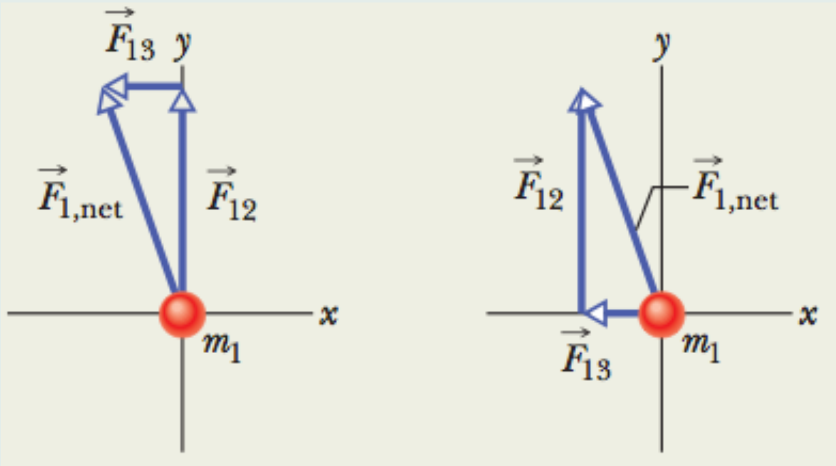
Example



• Isolated system of particles far from other massive objects.

• What is the magnitude of the gravitational force on particle 1 ?

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13}$$



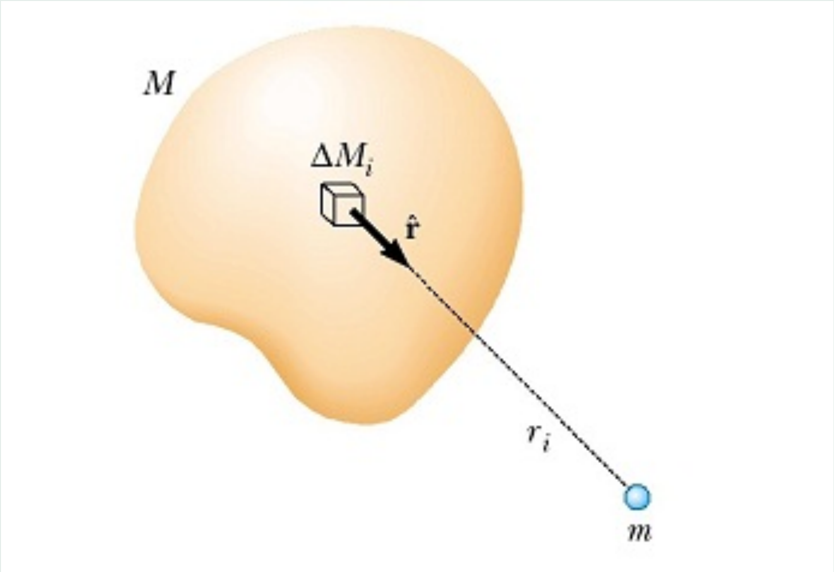
$$\vec{F}_{12} = G\frac{m_1 m_2}{a^2}\hat{j}$$

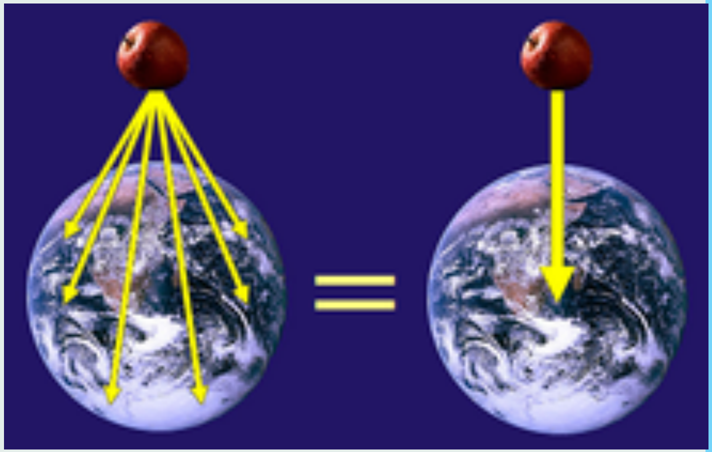
$$\vec{F}_{13} = -G\frac{m_1 m_3}{4a^2}\hat{i}$$

$$\vec{F}_{1,\text{net}} = \frac{Gm_1}{a^2}\left(m_2\hat{j} - \frac{m_3}{4}\hat{i}\right)$$

$$|\vec{F}_{1,\text{net}}| = \frac{Gm_1}{a^2}\sqrt{m_2^2 + \frac{m_3^2}{16}}$$

• Gravitational force on a particle from an extended object:



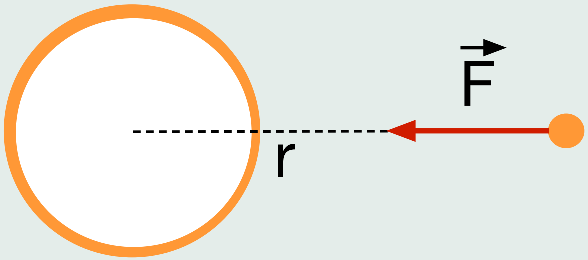


$$\vec{F} = \int d\vec{F} = \int -G\frac{m dM}{r^2}\widehat{r}$$

Shell theorem

A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell's mass were concentrated at its center.

$$F = G\frac{mM}{r^2}$$

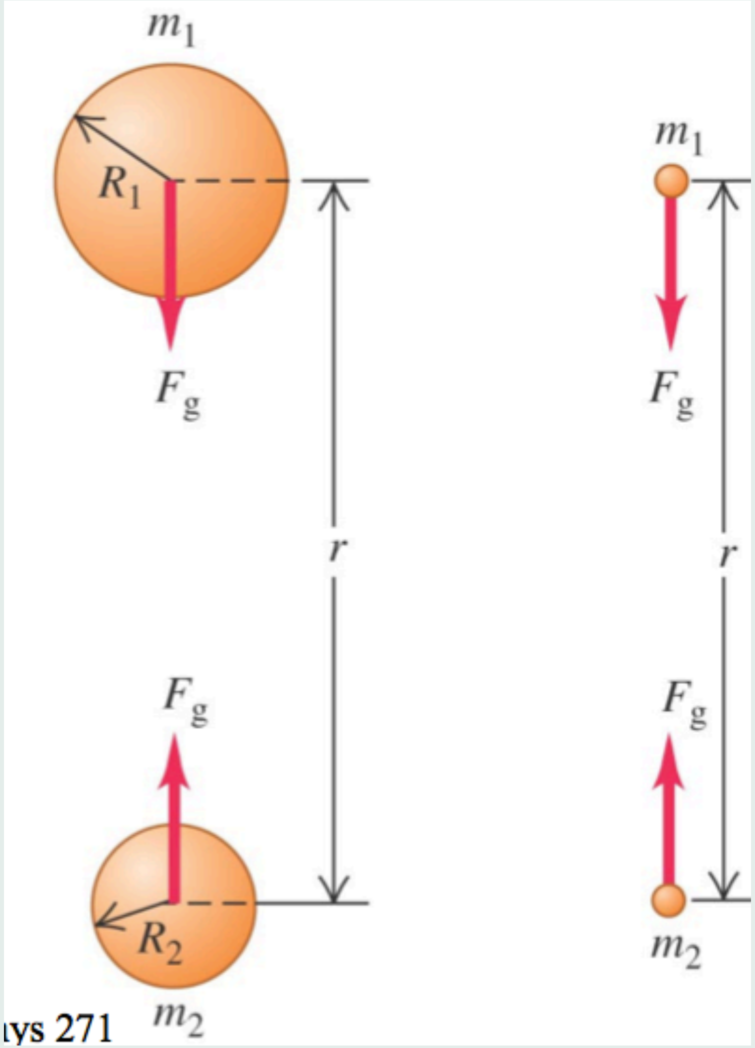


- m mass of particle
- M mass of shell
- r distance from particle to center of spherical shell

(http://hyperphysics.phy-astr.gsu.edu/hbase/ mechanics/sphshell.html)

Consequence:

The gravitational force between two **uniform** spherical distributions of mass is the same as if all the mass of each sphere were concentrated at its center



- **Gravitation near Earth's surface**

- Assume the Earth is exactly spherical.
- Assume uniform distribution of mass throughout the Earth
- Neglect rotation effects.

Gravitational force on a particle near Earth's surface:

$$\vec{F}_g = -G\frac{mM}{r^2}\widehat{r} \;\Rightarrow\; a_g = \frac{GM}{r^2}$$

$r =$ distance between particle and center of the Earth

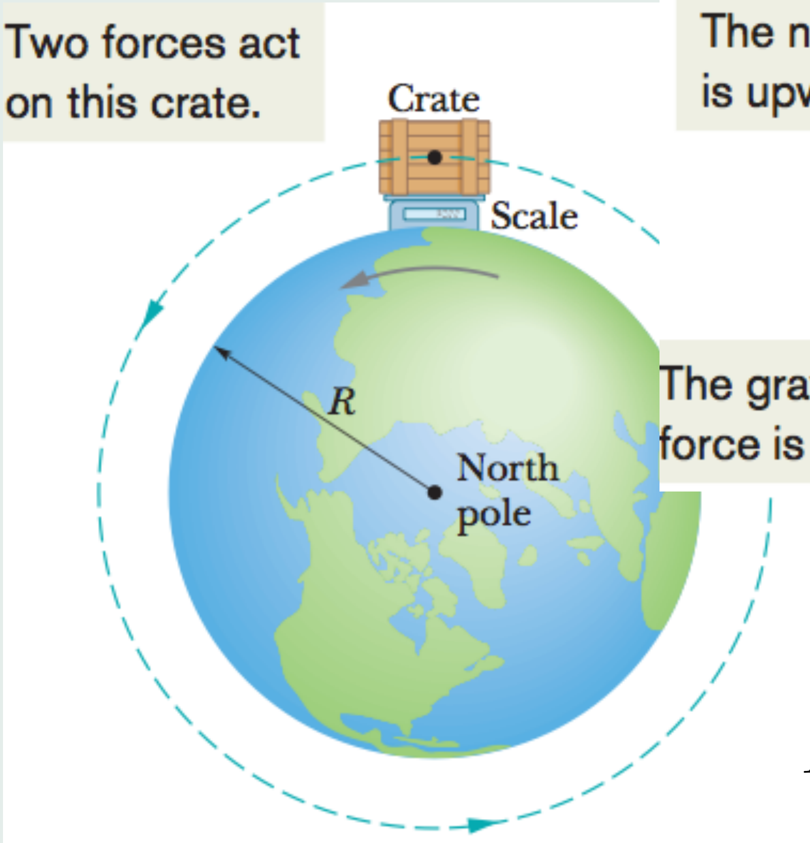
Note: the free fall acceleration decreases with r

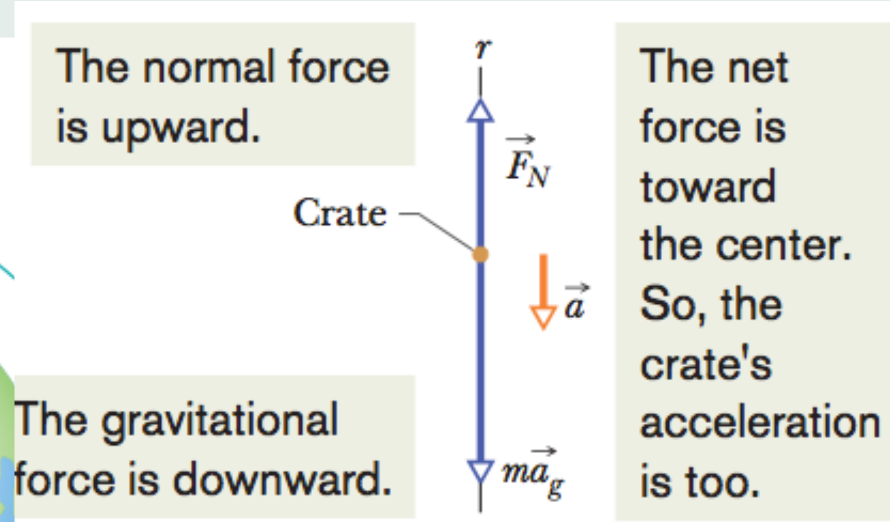
Altitude (km)	a_g (m/s^2)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest crewed balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

$$a_g = \frac{GM}{r^2} = \frac{GM}{(R+h)^2}$$

h = altitude = distance from particle to the surface of the Earth

- **Rotation effects**





$$m\vec{a}_c = m\vec{a}_g + \vec{F}_N$$

$$F_N = ma_g - m\omega^2 R$$

$$g = a_g - \omega^2 R$$

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The gravitational acceleration on the surface of the Earth is g (neglecting rotation.) What will it be on the surface of a planet that has half the mass of the Earth and half its radius?

$A)$ $g/4$

$B)$ $g/2$

$C)$ g

$D)$ $2g$

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A) $g/4$

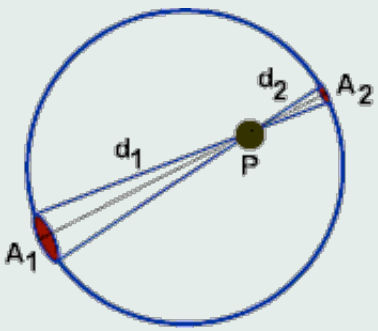
B) $g/2$

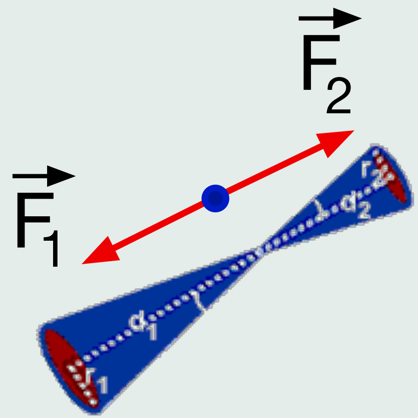
C) g

D) $2g$ $\qquad g = G\frac{M}{R^2} \;\Rightarrow\; g'/g = M'R^2/M(R')^2 = 4/2 = 2$

- **Gravitation inside the Earth**

A **uniform** shell of matter exerts **no** net gravitational force on a particle located **inside** it.





$$\frac{r_1}{d_1} = \frac{r_2}{d_2} \qquad \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

$$\frac{F_1}{F_2} = \frac{A_1/d_1^2}{A_2/d_2^2} = 1$$

$$\vec{F}_1 + \vec{F}_2 = 0$$

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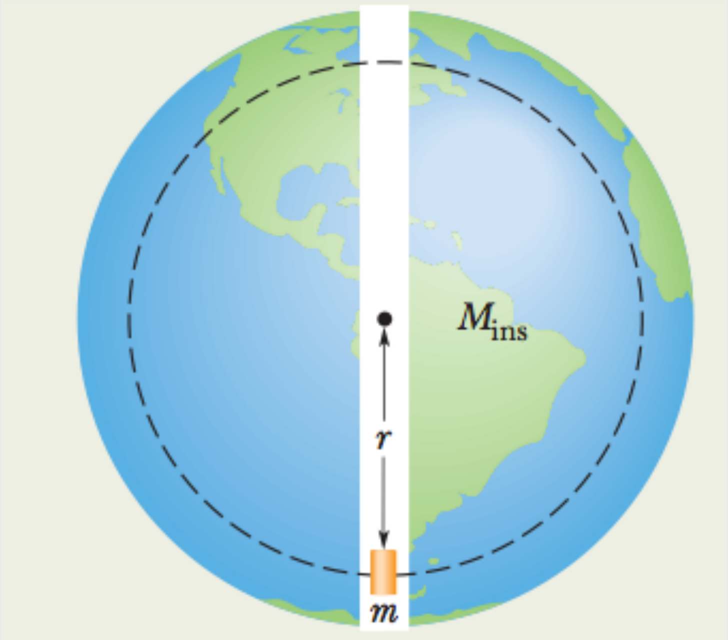
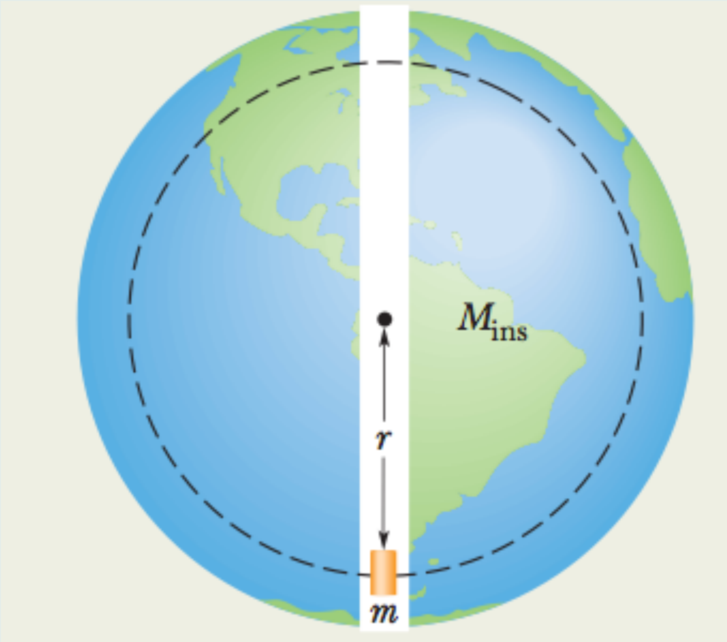


Suppose a very narrow tunnel is dug through the Earth along the North-South axis.

What is the gravitational force on a particle of mass m at distance $r < R$ from the center?

Any thin spherical shell of matter of radius $r_{\text{shell}} > r$ does not yield any net force.

Only thin shells of radius $r_{\text{shell}} < r$ give a nonzero force.



The gravitational force on the particle is the same as the force due to a sphere of radius r.

Assume spherical shape and uniform mass distribution. Mass density

$$\rho = \frac{M}{4\pi R^3/3} = \frac{3M}{4\pi R^3}$$

$$F = \frac{Gm}{r^2} \times \text{Mass inside sphere of radius } r$$
$$= \frac{Gm}{r^2} \times \left(\frac{4\pi r^3 \rho}{3}\right) = \frac{4\pi G m \rho}{3} r = \frac{GmM}{R^3} r$$

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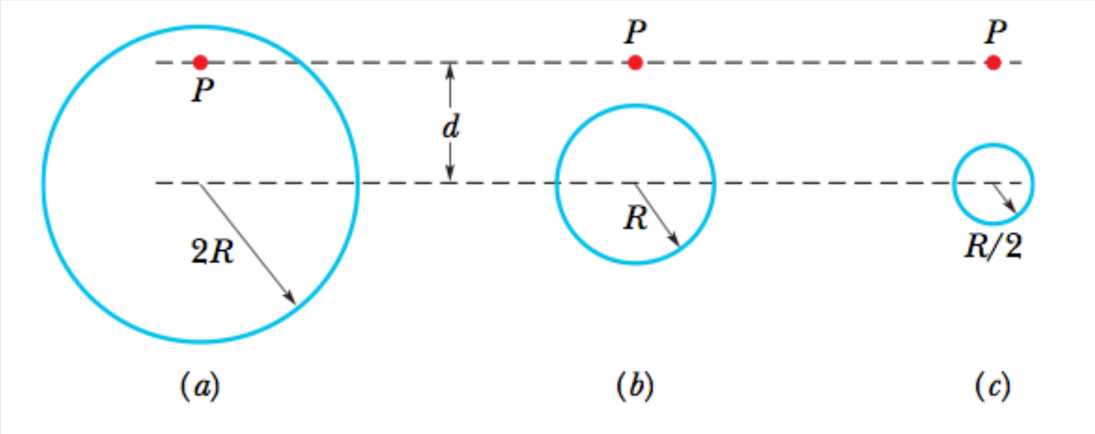

$A)\ F_a > F_b > F_c$

$B)\ F_a = F_b > F_c$

$C)\ F_a < F_b < F_c$

$D)\ F_a < F_b = F_c$

- All spherical shells are uniform and have the same mass M.
- Rank the situations according to the magnitude of the gravitational force on the particle.

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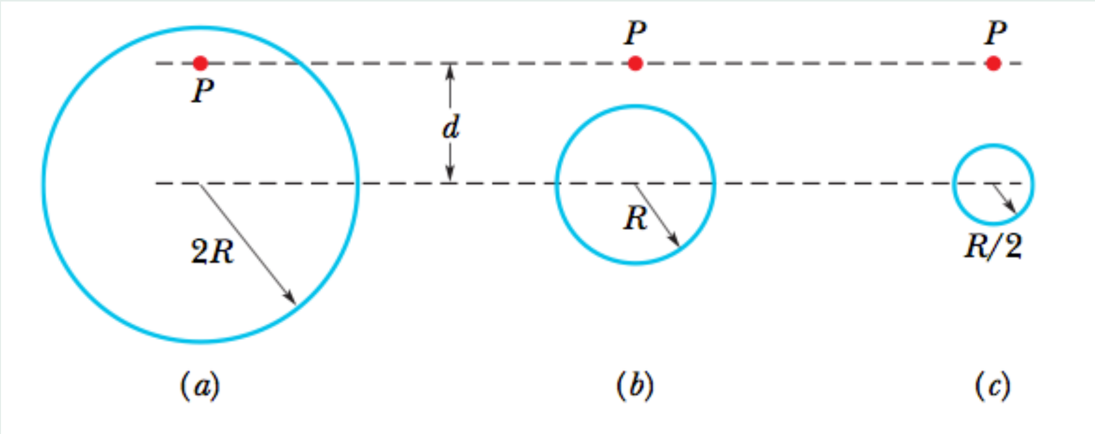


A) $F_a > F_b > F_c$

B) $F_a = F_b > F_c$

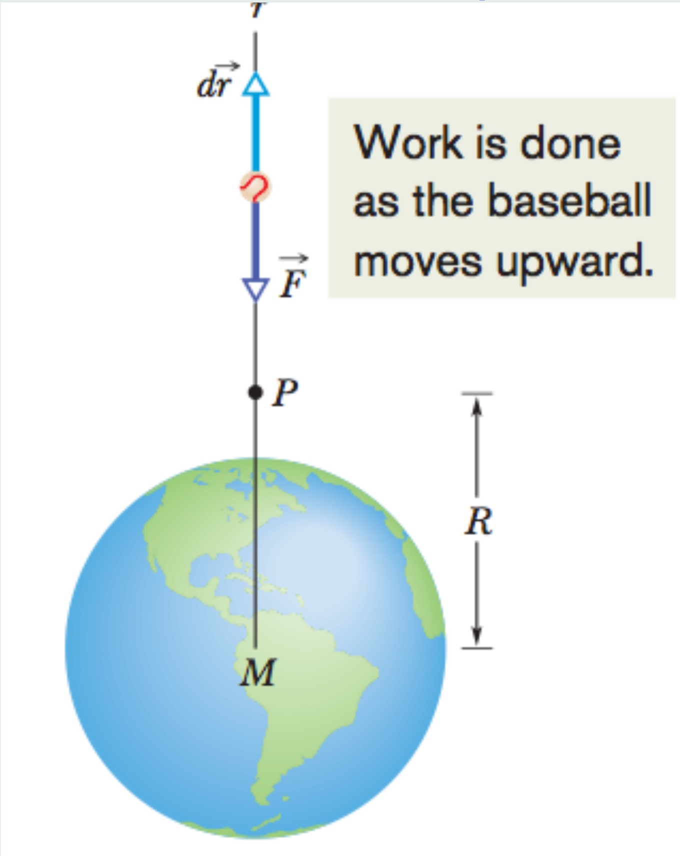
C) $F_a < F_b < F_c$

D) $F_a < F_b = F_c$

$F_a = 0, \; F_b = F_c = GmM/r^2$

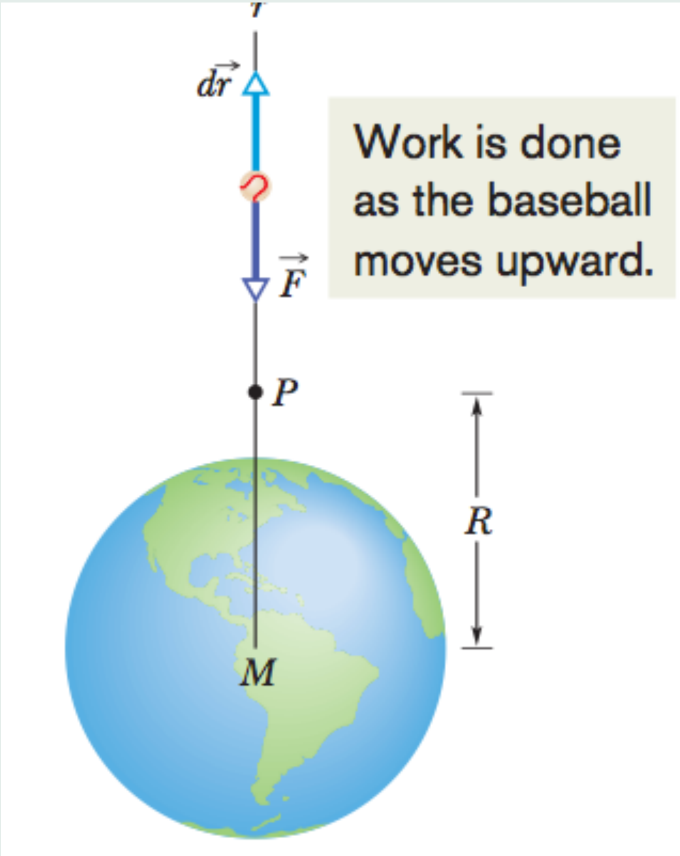
- All spherical shells are uniform and have the same mass M.
- Rank the situations according to the magnitude of the gravitational force on the particle.

- **Gravitational potential energy**



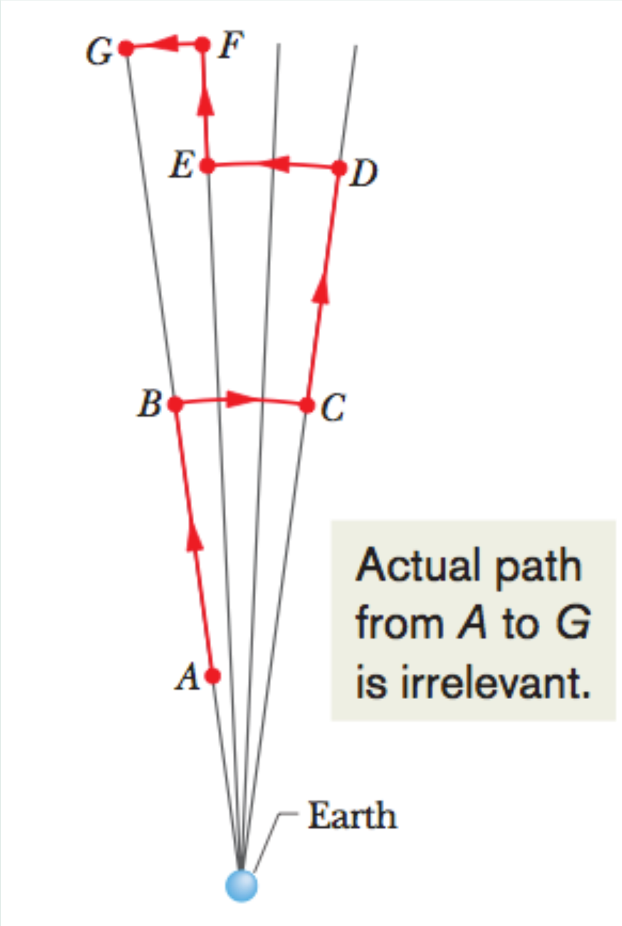
Work done by gravitational force: suppose a baseball moves upward from a distance r_1 to a distance $r_2 > r_1$ in the gravitational field of the Earth.

What is the work done by gravitational force?



$$\begin{aligned} W_{F_g} &= \int_{r_1}^{r_2} \vec{F}_g \cdot d\vec{r} \\ &= -GmM \int_{r_1}^{r_2} \frac{dr}{r^2} \\ &= GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \end{aligned}$$

Does it depend on the path?



W_{F_g} does **not** depend on the path.

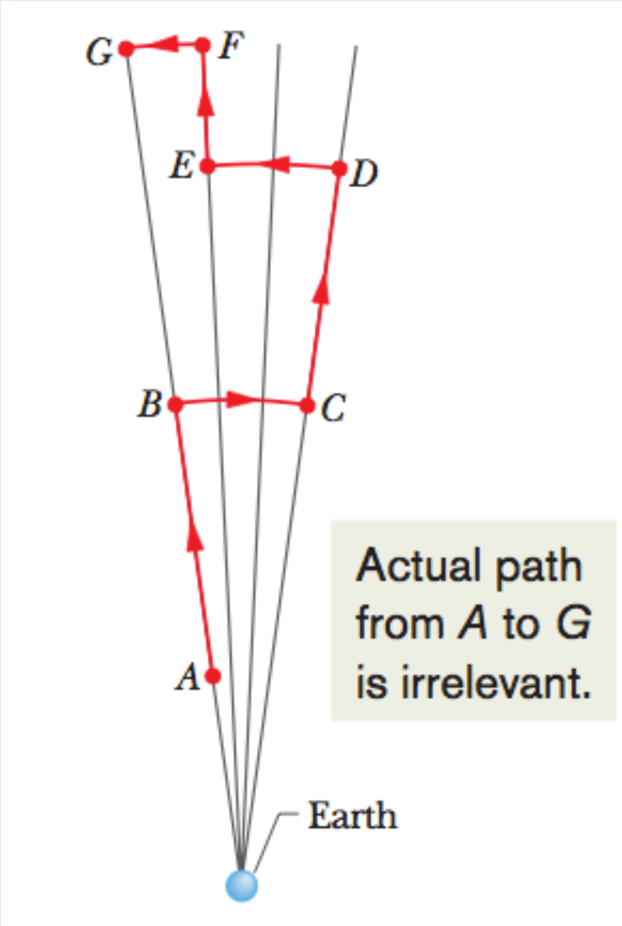
Contribution of **circular** arcs is 0:

$$\vec{F}_g \cdot d\vec{r} = 0$$

Contribution of **radial** arcs:

$$\vec{F}_g \cdot d\vec{r} = -\frac{GmM}{r^2}dr$$

$$W_{F_g,\text{curved path}} = W_{F_g,\text{radial path}}$$

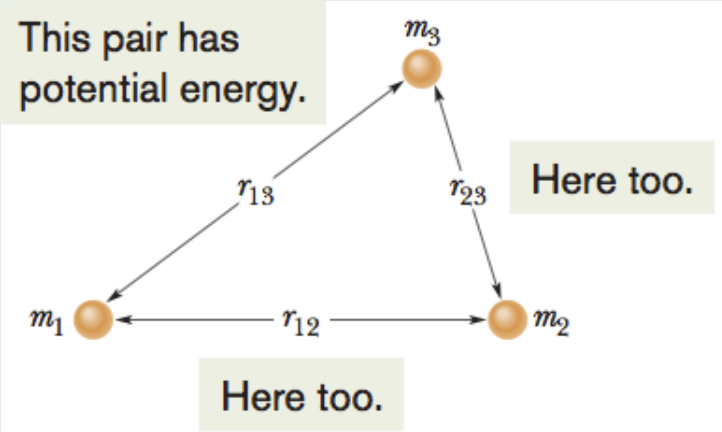


- **The gravitational force is conservative**
- **Gravitational potential energy**:

$$U(\infty) - U(r) = -W_{F_g, r\to\infty}$$
$$= \frac{GmM}{r}$$

$$\boxed{U(r) = -\frac{GmM}{r}}$$

Gravitational potential energy: system of particles



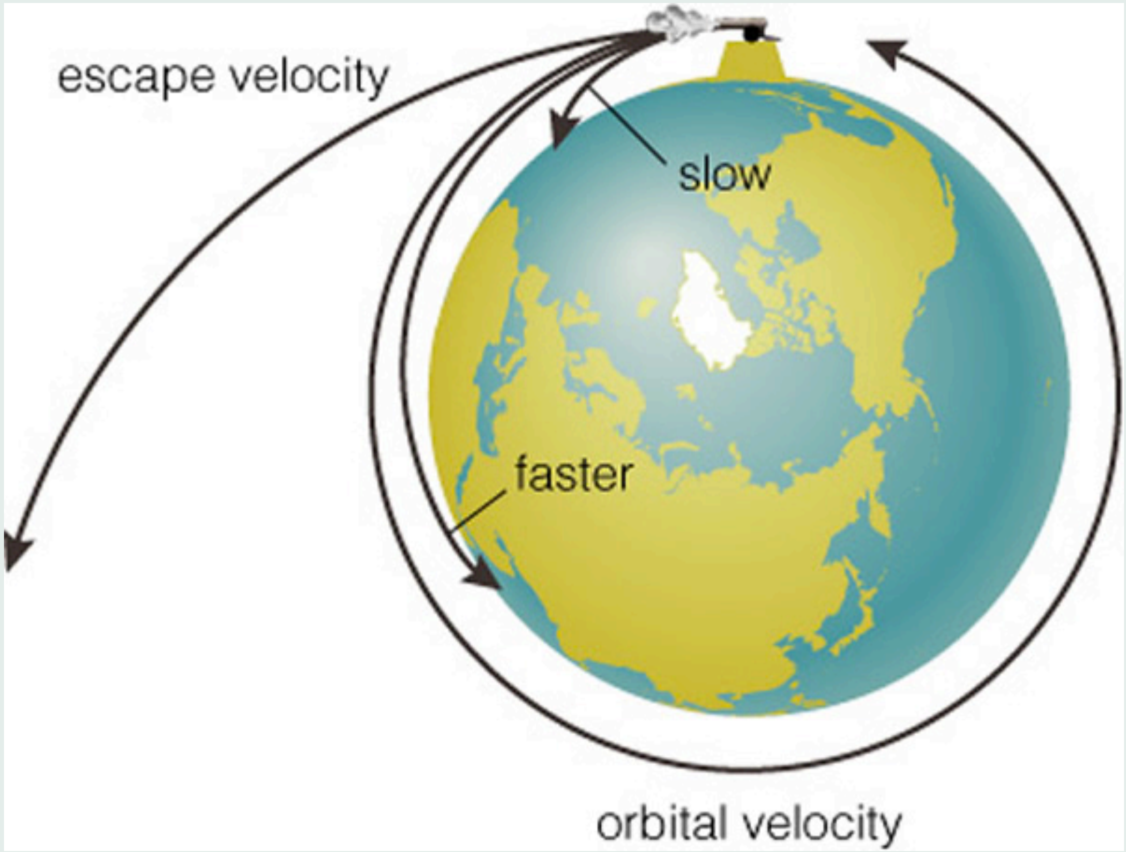
- For any pair of particles (i, j)

$$U_{ij} = -\frac{Gm_i m_j}{r_{ij}}$$

$$U_{\text{total}} = -\sum_{i<j} \frac{Gm_i m_j}{r_{ij}}$$

$$U_{\text{total}} = -\left(\frac{Gm_1 m_2}{r_{12}} + \frac{Gm_2 m_3}{r_{23}} + \frac{Gm_1 m_3}{r_{13}}\right)$$

- **Escape speed**



How fast should a projectile be launched such that it escapes Earth's gravitational field?

How fast should it be launched such that it does not fall back on Earth?

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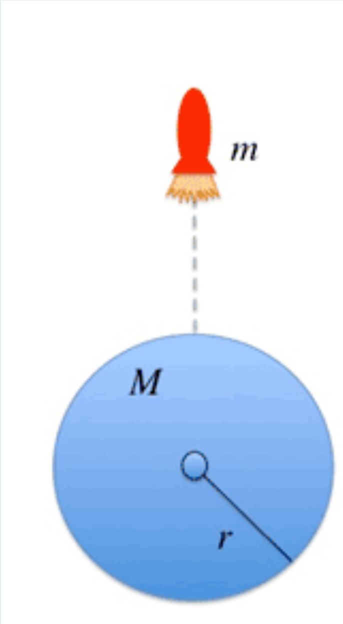

Find the initial speed of the rocket such that it escapes the Earth's gravitational field, moving infinitely far away.

$A)$ $\sqrt{MG/R}$

$B)$ $\sqrt{2MG/R}$

$C)$ $\sqrt{MG/2R}$

$D)$ It is impossible for the rocket to escape.

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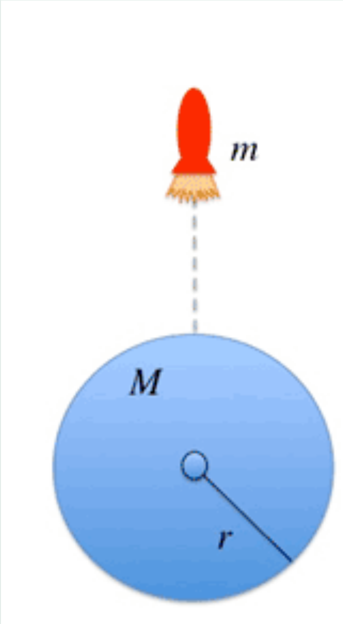

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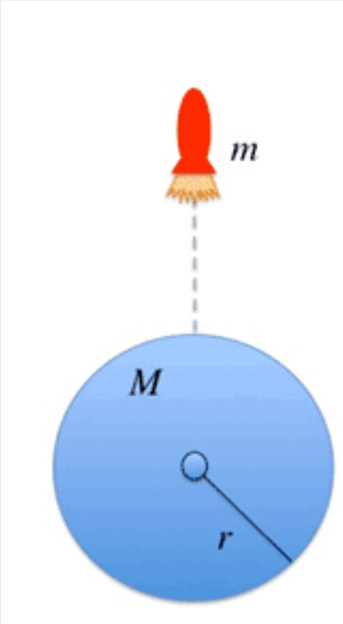


- **Energy conservation**

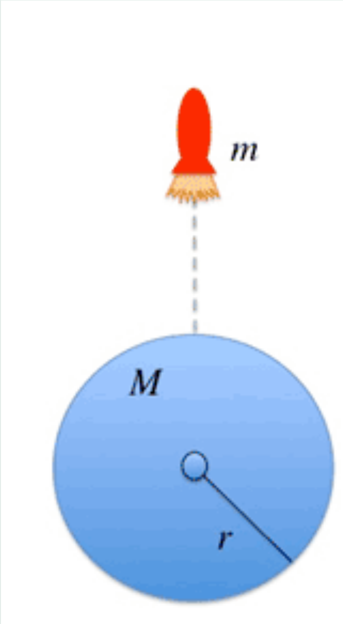
Initially: object on the surface of the Earth; kinetic and potential energy

$$E_{mec} = K_0 + U_{\text{grav}}$$

Finally: object at ∞; kinetic energy

$$E_{mec} = K \geq 0$$

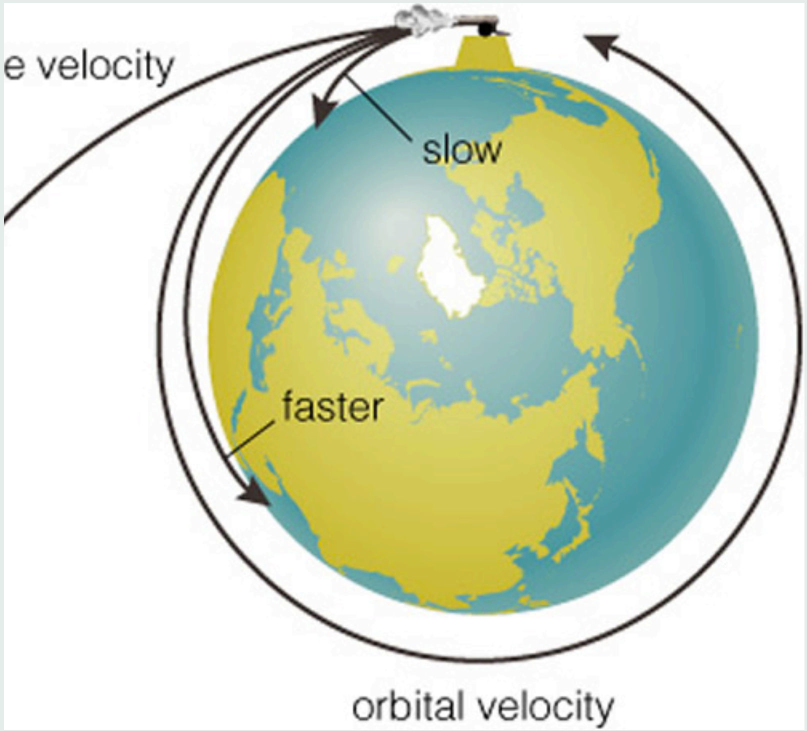
$$\frac{1}{2}mv_0^2 - \frac{GmM}{R} = \frac{1}{2}mv^2 \geq 0 \;\Rightarrow\; v_0 \geq \sqrt{\frac{2GM}{R}}$$



$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

Escape speed

- **i-Clicker**



A projectile is launched horizontally from altitude h above the Earth's surface. How fast should a projectile be launched such that it moves on a circular trajectory around the Earth?

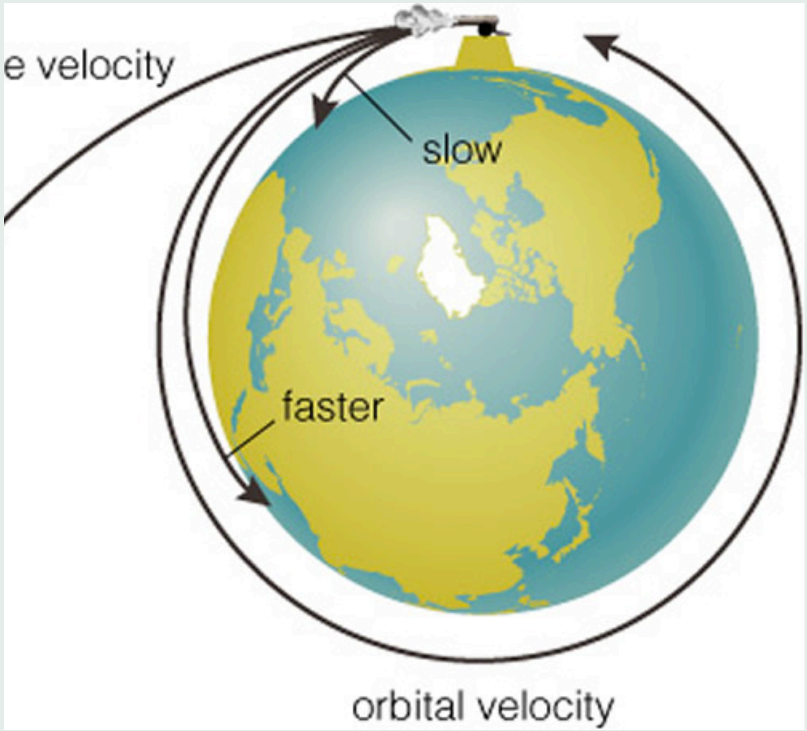
$A)$ $\sqrt{MG/(R+h)}$

$B)$ $\sqrt{2MG/(R+h)}$

$C)$ $\sqrt{MG/2(R+h)}$

$D)$ It is impossible for the projectile to circle the Earth.

- **i-Clicker**



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$D)$ It is impossible for the projectile to circle the Earth.

$$m\vec{a}_c = \vec{F}_g$$

$$\frac{mv^2}{R+h} = \frac{GmM}{(R+h)^2}$$

$$v = \sqrt{\frac{GM}{R+h}}$$

Orbital velocity