

Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 20

[Home Page](#)

[Title Page](#)



[Page 1 of 31](#)

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

1. **No** quizzes during Thanksgiving week. There will be recitation according to the regular schedule using the $Th \rightarrow T$, $Fr \rightarrow Wed$ switch.
2. **No** lecture on Wednesday Nov 23rd (Friday classes)

[Home Page](#)[Title Page](#)[Page 2 of 31](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

Midterm II

100 90 80 70 60 50 40 30 20 10

26 44 53 29 18 7 0 0 0 0

Class average: 80.6

[Home Page](#)

[Title Page](#)



Page 3 of 31

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

- **Simple harmonic motion (SHM)**

One dimensional motion of a point particle given by

$$x(t) = x_m \cos(\omega t + \phi).$$

The diagram shows the equation $x(t) = x_m \cos(\omega t + \phi)$ with labels and leader lines pointing to its parts:

- Displacement at time t** : points to $x(t)$.
- Amplitude**: points to x_m .
- Angular frequency**: points to ω .
- Time**: points to t .
- Phase constant or phase angle**: points to ϕ .
- Phase**: a bracket groups $\omega t + \phi$ and is labeled "Phase".

- x_m **amplitude** = maximum displacement
- t time
- ω **angular frequency**
- ϕ **phase constant** or **phase angle**

- **Period:**

T = time needed to complete one oscillation

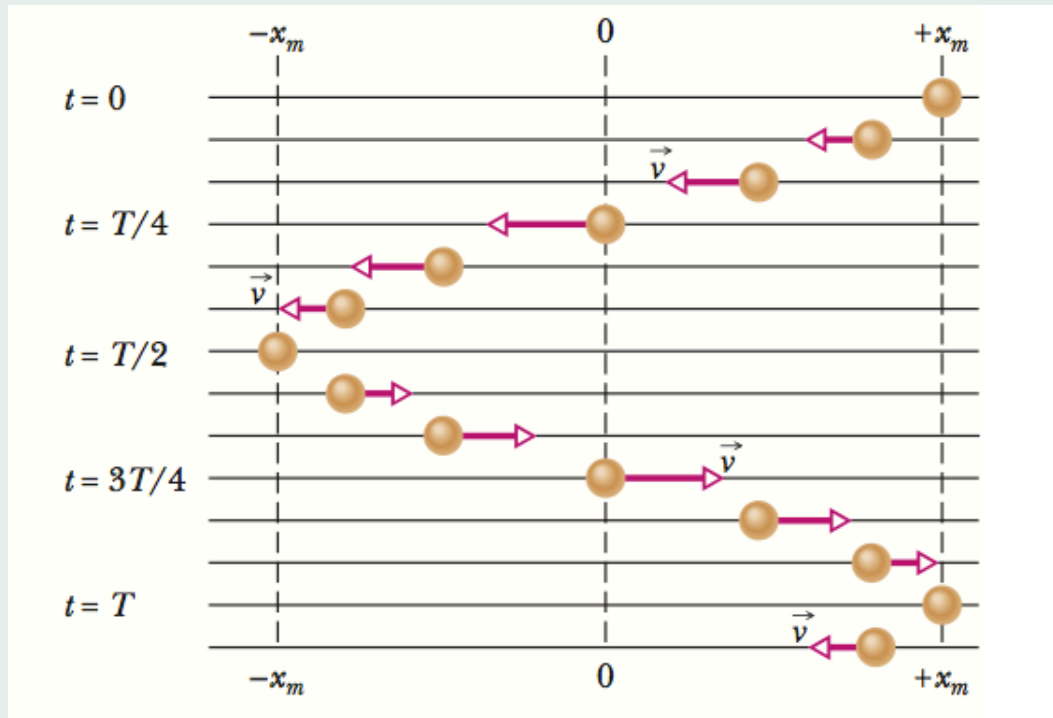
$$T = \frac{2\pi}{\omega}$$

- **Frequency:**

f = number of oscillations per unit time.

$$f = \frac{1}{T} = \frac{\omega}{2\pi}.$$

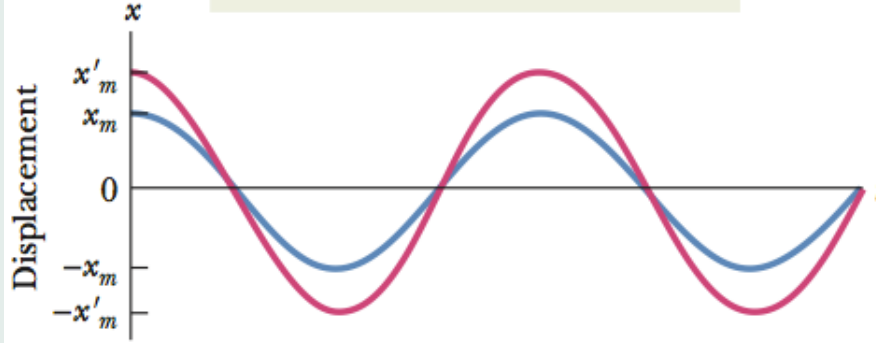
[Home Page](#)[Title Page](#)[<<](#)[>>](#)[<](#)[>](#)[Page 5 of 31](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)



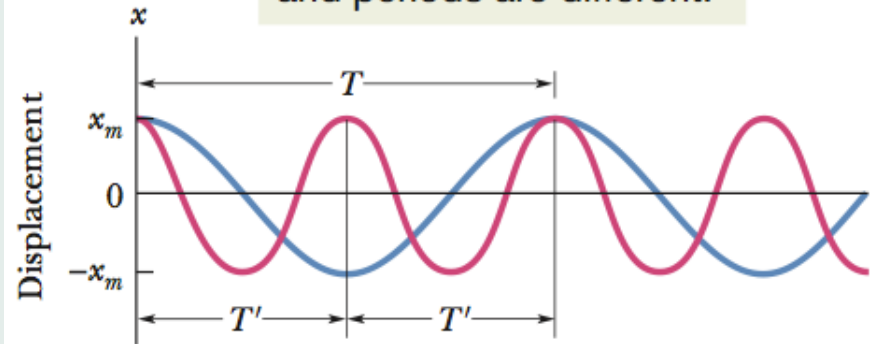
- back and forth from $+x_m$ to $-x_m$
- $\vec{v} = 0$ when $x(t) = \pm x_m$.
- $v = v_{\max}$ when $x(t) = 0$.

[Home Page](#)
[Title Page](#)
[<<](#)
[>>](#)
[<](#)
[>](#)
[Page 6 of 31](#)
[Go Back](#)
[Full Screen](#)
[Close](#)
[Quit](#)

The amplitudes are different,
but the frequency and
period are the same.



The amplitudes are the
same, but the frequencies
and periods are different.



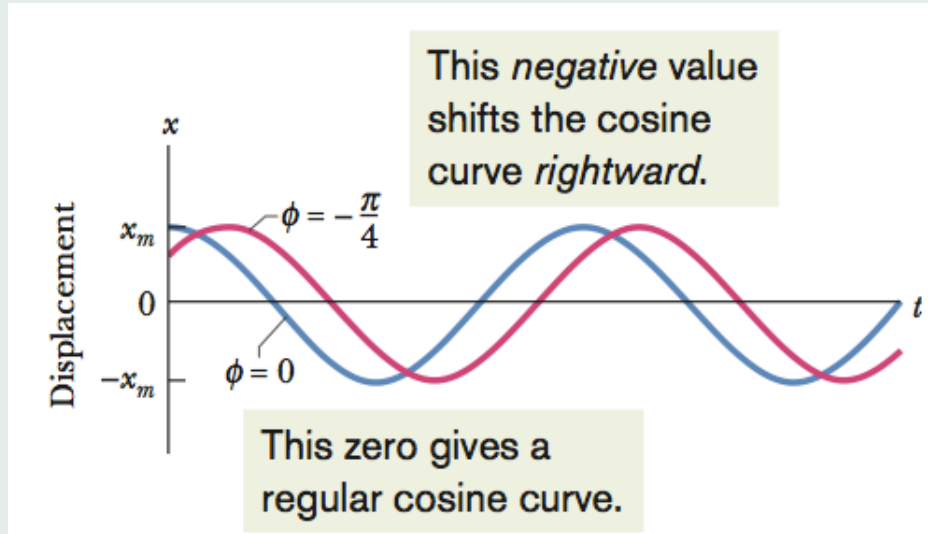
- same ω , ϕ

- $x'_m > x_m$

- same x_m , ϕ

- $T = 2T' \Leftrightarrow \omega' = 2\omega$

[Home Page](#)[Title Page](#)[<<](#)[>>](#)[<](#)[>](#)[Page 7 of 31](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)



- same x_m , ω
- $\phi = 0$, $\phi' = -\pi/4$
- $\phi < 0 \Rightarrow$ **right** shift
- $\phi > 0 \Rightarrow$ **left** shift

The value of ϕ depends on the displacement and the velocity at time $t = 0$:

$$x_m \cos \phi = x(0) \qquad -\omega x_m \sin \phi = v_x(0)$$

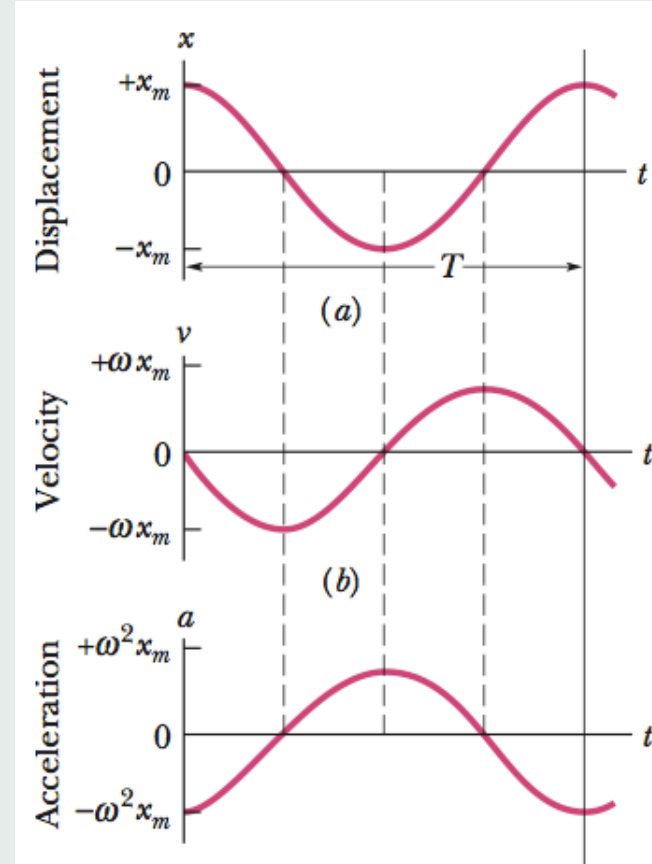
- **Velocity and acceleration**

$$x(t) = x_m \cos(\omega t + \phi)$$

$$v_x(t) = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \phi)$$

$$\begin{aligned} a_x(t) &= \frac{dv_x}{dt} = -\omega^2 x_m \cos(\omega t + \phi) \\ &= -\omega^2 x(t) \end{aligned}$$

$$a_x(t) = -\omega^2 x(t)$$

[Home Page](#)[Title Page](#)[<<](#)[>>](#)[<](#)[>](#)[Page 9 of 31](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

- Force law in SHM

Newton's 2nd law:

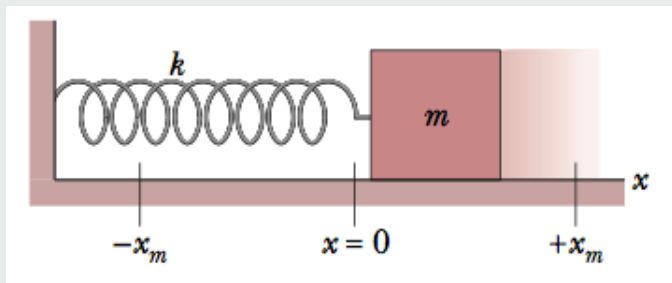
$$F_x = ma_x$$



$$F_x = -m\omega^2 x$$

Spring force!

SHM is the motion executed by a system subject to a force that is proportional to the displacement of the system but opposite in sign.



$$F_x = -kx \Rightarrow k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Note:

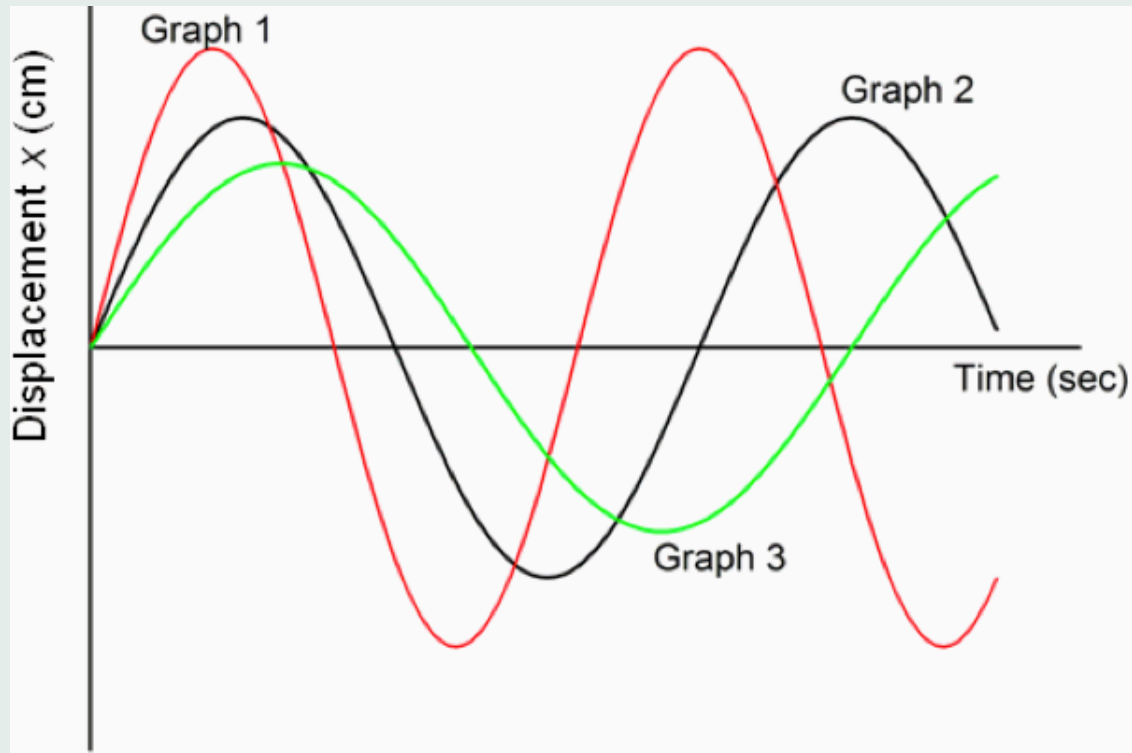
$$m\frac{d^2x}{dt^2} = -kx \Rightarrow x = x_m\cos(\omega t + \phi)$$

$$x_0 = x(0) = x_m\cos\phi \quad v_{x0} = v_x(0) = -\omega x_m\sin\phi$$

[Home Page](#)
[Title Page](#)
[<<](#)
[>>](#)
[<](#)
[>](#)

Page 11 of 31

[Go Back](#)
[Full Screen](#)
[Close](#)
[Quit](#)



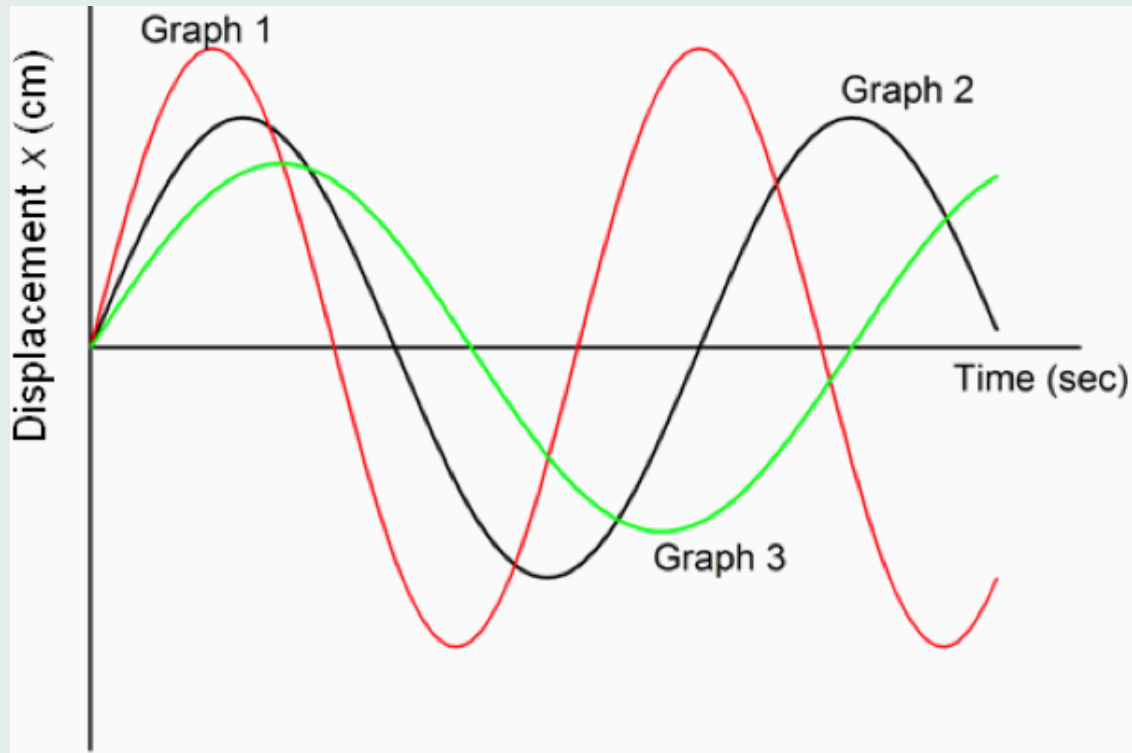
- 3 different springs, identical masses

- which one has the smallest k ?

- A) Graph 1
B) Graph 2

- C) Graph 3
D) All have the same k

[Home Page](#)[Title Page](#)[<<](#)[>>](#)[<](#)[>](#)[Page 12 of 31](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)



- 3 different springs, identical masses

- which one has the smallest k ?

A) Graph 1

B) Graph 2

C) Graph 3 ($T = 2\pi\sqrt{m/k}$)

D) All have the same k

[Home Page](#)

[Title Page](#)

[◀](#)

[▶](#)

[◀](#)

[▶](#)

Page 13 of 31

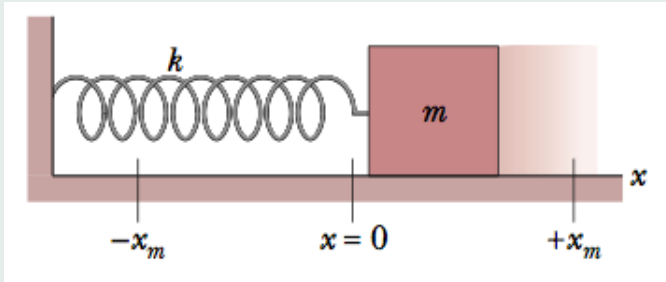
[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

- **Energy in SHM**

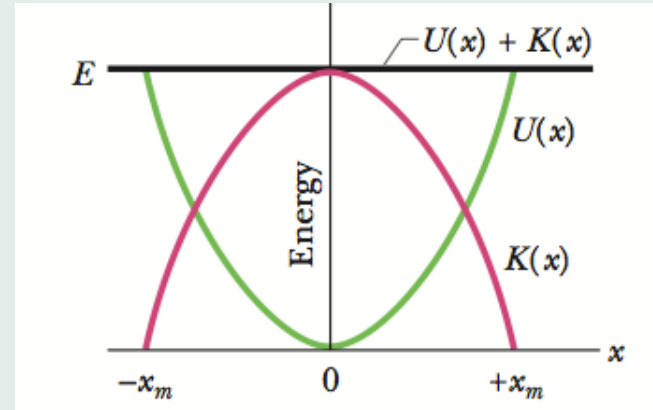
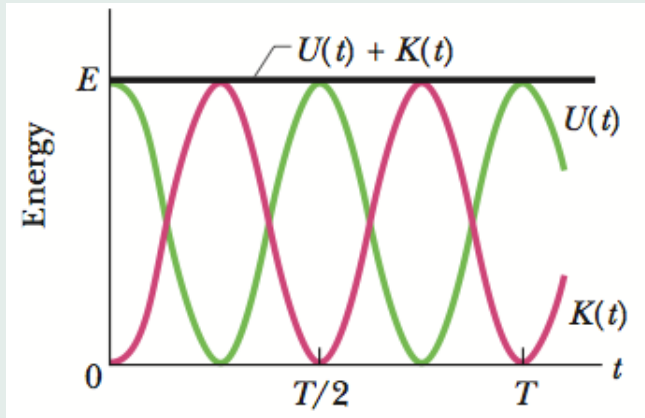


- **Kinetic Energy**

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ &= \frac{m\omega^2 x_m^2}{2} \sin^2(\omega t + \phi) \end{aligned}$$

- **Potential energy**

$$\begin{aligned} U &= \frac{1}{2}kx^2 \\ &= \frac{kx_m^2}{2} \cos^2(\omega t + \phi) \\ &= \frac{m\omega^2 x_m^2}{2} \cos^2(\omega t + \phi) \end{aligned}$$



$$K = \frac{m\omega^2 x_m^2}{2} \sin^2(\omega t + \phi)$$

$$U = \frac{m\omega^2 x_m^2}{2} \cos^2(\omega t + \phi)$$

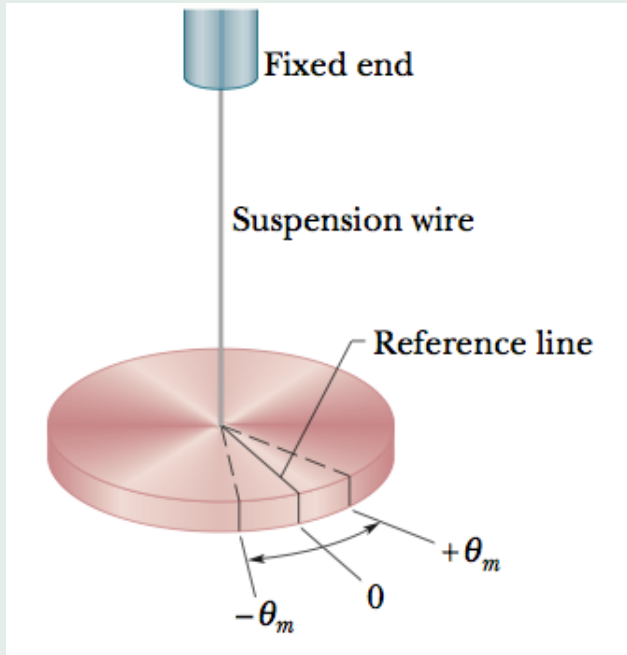
$$K + U = \frac{m\omega^2 x_m^2}{2} = \text{constant}$$

[Home Page](#)
[Title Page](#)
[<<](#)
[>>](#)
[<](#)
[>](#)

Page 15 of 31

[Go Back](#)
[Full Screen](#)
[Close](#)
[Quit](#)

- **An angular SHM: torsion pendulum**



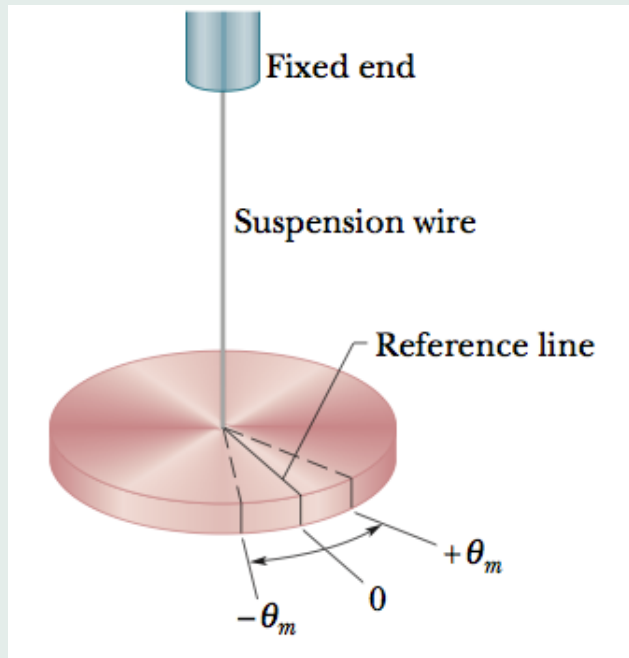
- **Resoring torque:** from twisting of the suspension wire

$$\tau = -\kappa\theta$$

θ = angular displacement

- **Equation of motion:**

$$I \frac{d^2\theta}{dt^2} = -\kappa\theta$$



- **Analogy with linear SHM:**

$$\theta \leftrightarrow x$$

$$I \leftrightarrow m$$

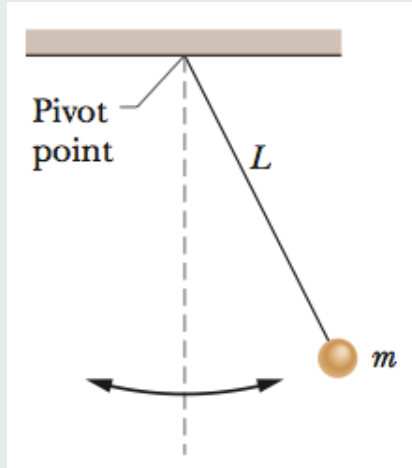
$$\kappa \leftrightarrow k = m\omega^2$$

$$I \frac{d^2\theta}{dt^2} = -\kappa\theta \leftrightarrow m \frac{d^2x}{dt^2} = -kx$$

$$\theta = \theta_m \cos(\omega t + \phi), \quad \omega = \sqrt{\frac{\kappa}{I}}$$

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{\kappa}}$$

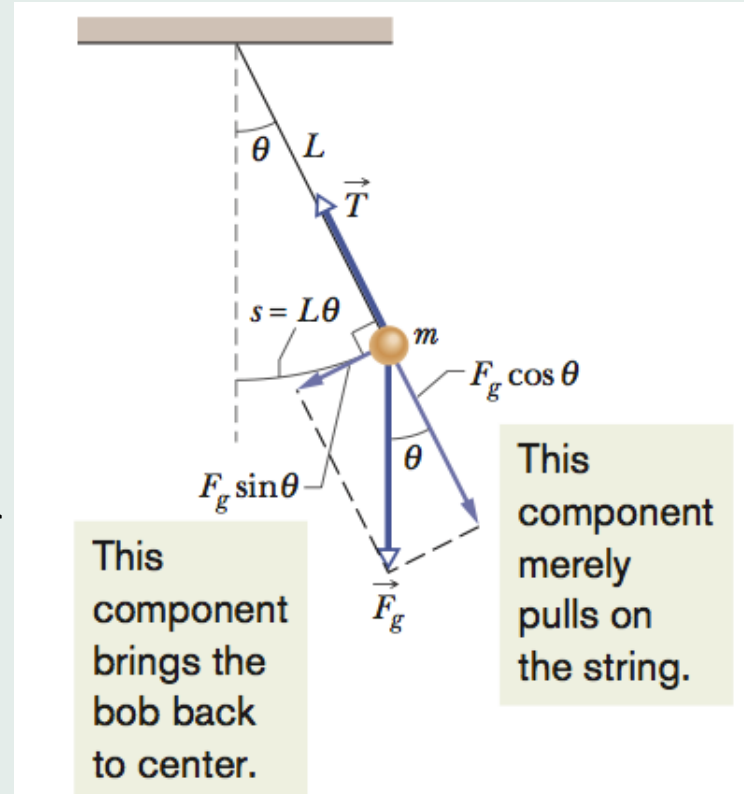
- The simple pendulum



- **Restoring force** = tangential component of gravitational force

$$F_g \sin \theta$$

Not linear in θ !



- Small angle approximation:

$$\sin\theta \simeq \theta$$

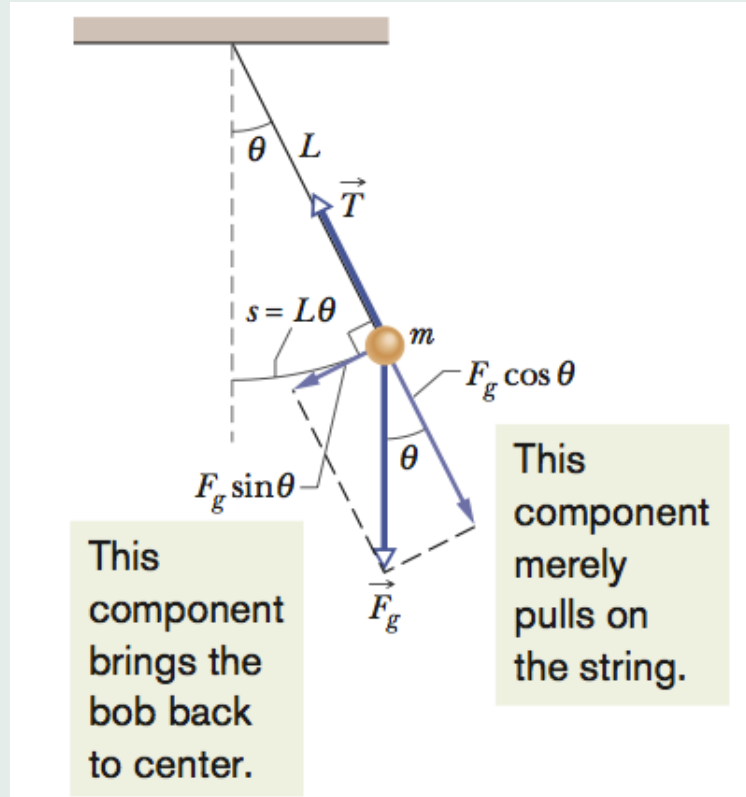
Good for $\theta \leq 10^\circ = \pi/18 \text{ rad}$

$$\sin(10^\circ) = 0.1736481777\dots$$

$$10^\circ = 0.1745329252\dots \text{ rad}$$

$$\left| \frac{\sin(10^\circ) - \pi/18}{\sin(10^\circ)} \right| \simeq 0.0051\dots$$

$$< 1\%$$



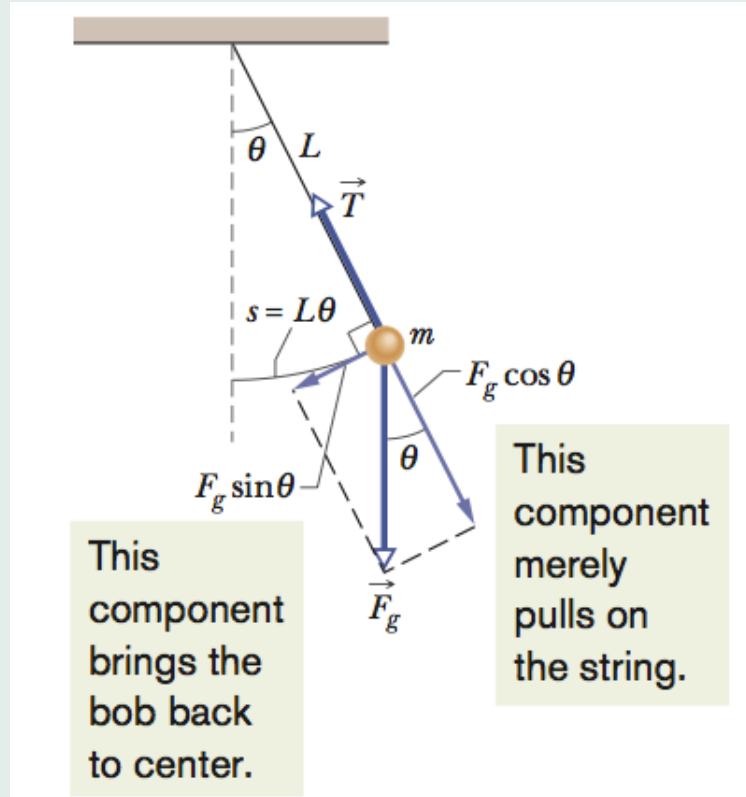
- Equation of motion for small θ :

$$\begin{aligned} I \frac{d^2\theta}{dt^2} &= -\tau_{F_g} \\ &= -mgL\sin\theta \\ &\simeq -mgL\theta \end{aligned}$$

$$I \frac{d^2\theta}{dt^2} = -mgL\theta$$

- I = rotational inertia about an axis \perp to plane of motion passign through O .

- τ_{F_g} = torque of gravitational force about the same axis.



- Analogy with linear SHM:

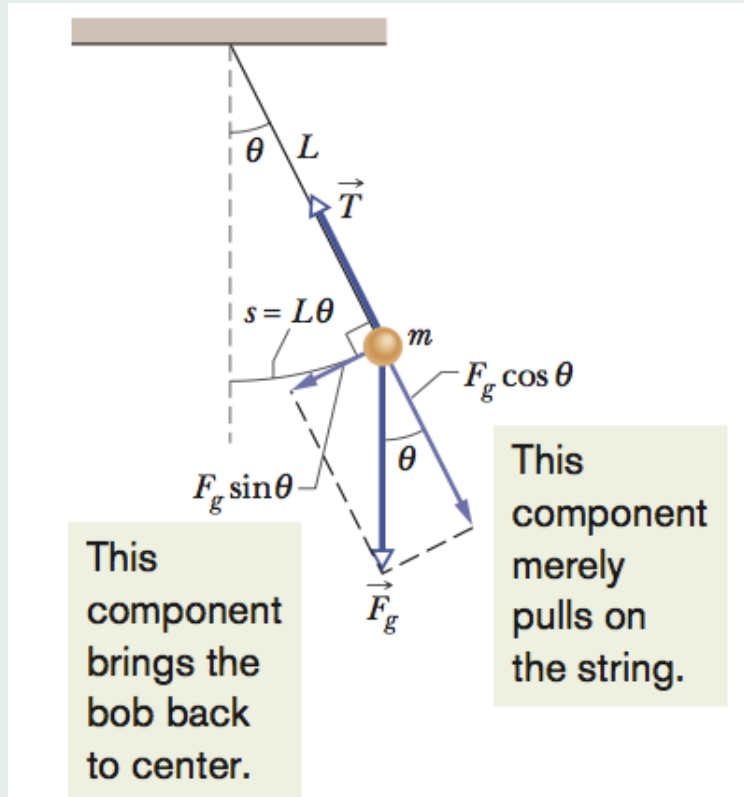
$$I \frac{d^2\theta}{dt^2} = -mgL\theta$$

\Downarrow

$$\theta = \theta_m \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{mgL}{I}}$$

$$T = 2\pi \sqrt{\frac{I}{mgL}}$$



[Home Page](#)

[Title Page](#)

[◀](#)

[▶](#)

[◀](#)

[▶](#)

Page 21 of 31

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

Pointlike mass, ideal cord: $I = mL^2$

$$\omega = \sqrt{\frac{g}{L}} \quad T = 2\pi\sqrt{\frac{L}{g}}$$

[Home Page](#)

[Title Page](#)



Page 22 of 31

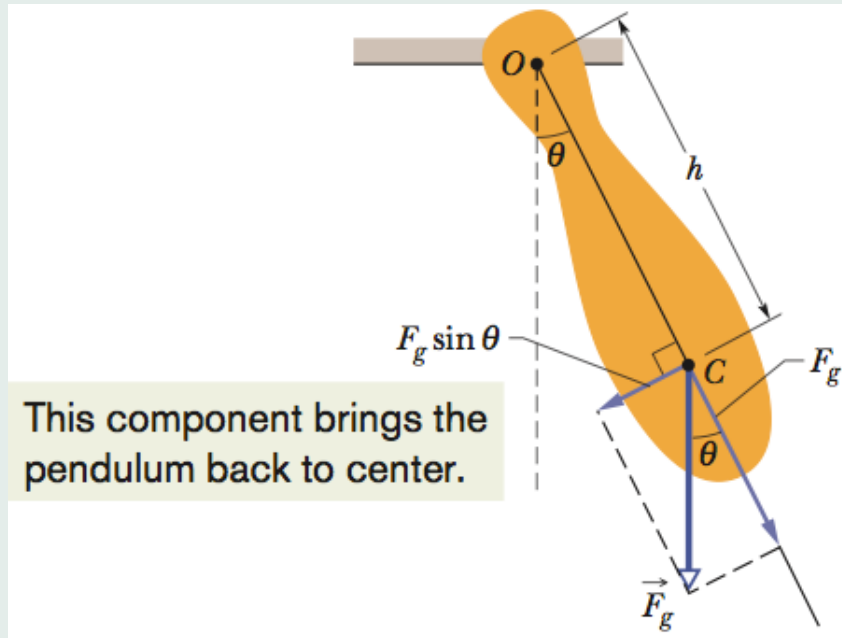
[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

- Physical Pendulum



- Same reasoning

$$I \frac{d^2 \theta}{dt^2} = -mgh \theta$$

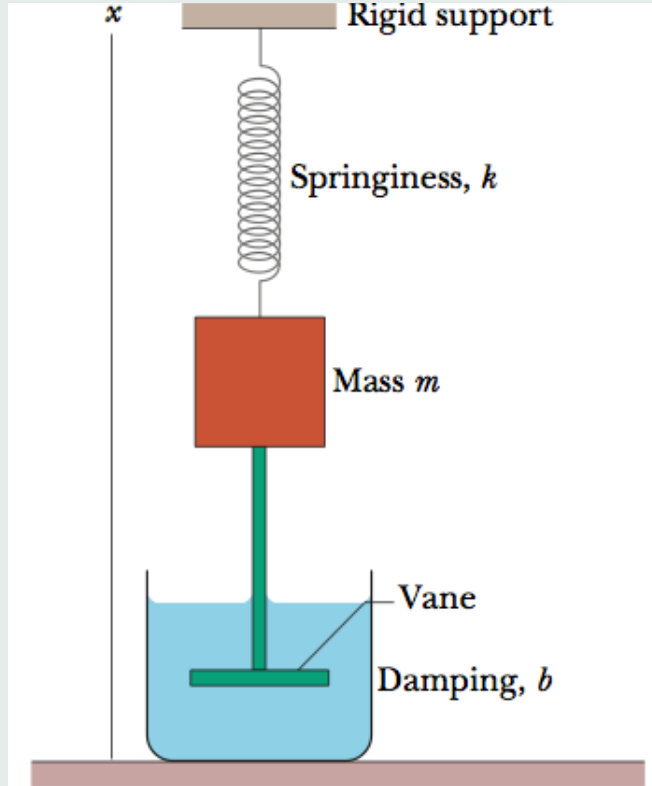


$$\theta = \theta_m \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{mgh}{I}}$$

$$T = 2\pi \sqrt{\frac{I}{mgh}}$$

- **Damped oscillations**



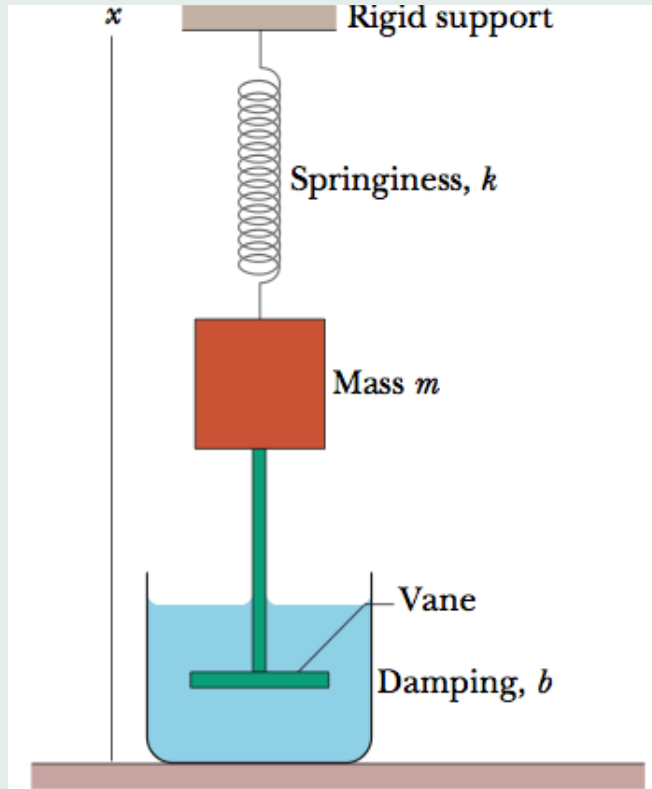
- Restoring spring force

$$F_s = -ky$$

- Damping force

$$F_d = -bv_y$$

- Assume gravitational force very small compared to first two.



- Equation of motion along y axis:

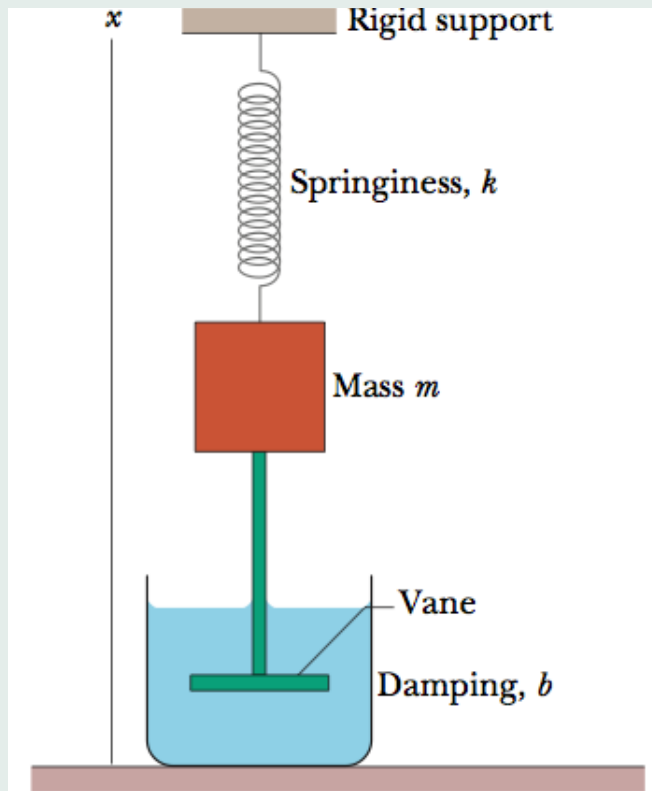
$$m \frac{d^2 y}{dt^2} = -ky - b \frac{dy}{dt}$$

$$m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + ky = 0$$

$$y(t) = y_m e^{-bt/2m} \cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

(assume $k > b^2/4m$)

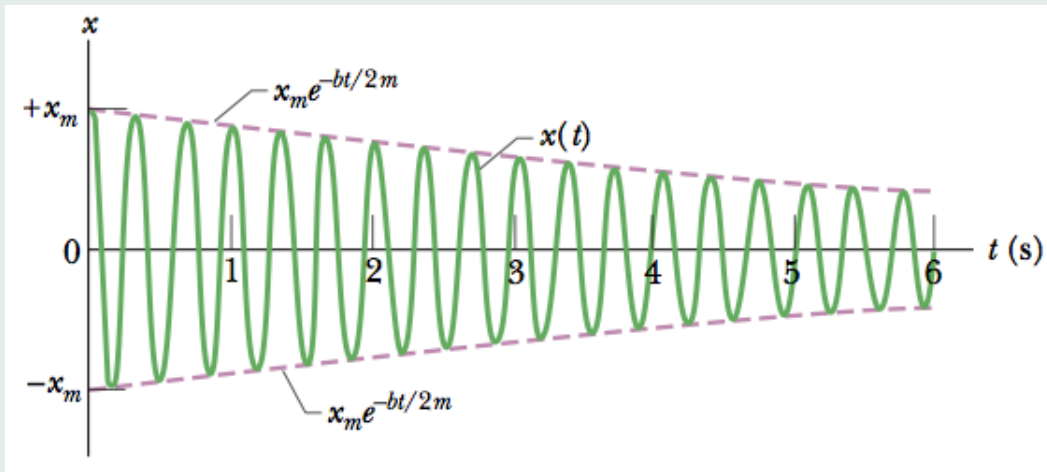


$$y(t) = y_m e^{-bt/2m} \cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

- $\sim \cos$ function with amplitude decreasing in time
- Mechanical energy also decreasing in time:

$$E(t) \simeq \frac{1}{2} k x_m^2 e^{-bt/m}$$



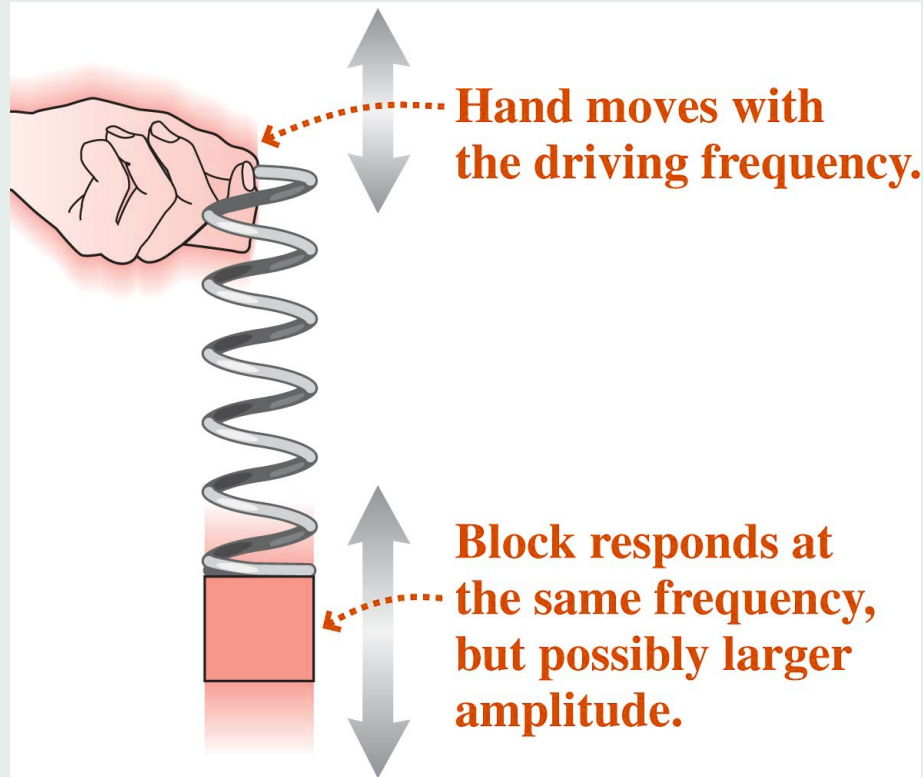
$$y(t) = y_m e^{-bt/2m} \cos(\omega' t + \phi)$$

[Home Page](#)
[Title Page](#)
[<<](#)
[>>](#)
[<](#)
[>](#)

Page 27 of 31

[Go Back](#)
[Full Screen](#)
[Close](#)
[Quit](#)

- **Forced oscillations and resonance**



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- Suppose the hand moves with a given frequency ω_d (**driving frequency**)

- Assume gravity is negligible.

- Equation of motion:

$$m \frac{d^2 y}{dt^2} = -ky - b \frac{dy}{dt} + F_0 \cos(\omega_d t)$$

(note the additional external force)

[Home Page](#)

[Title Page](#)

[◀](#)

[▶](#)

[◀](#)

[▶](#)

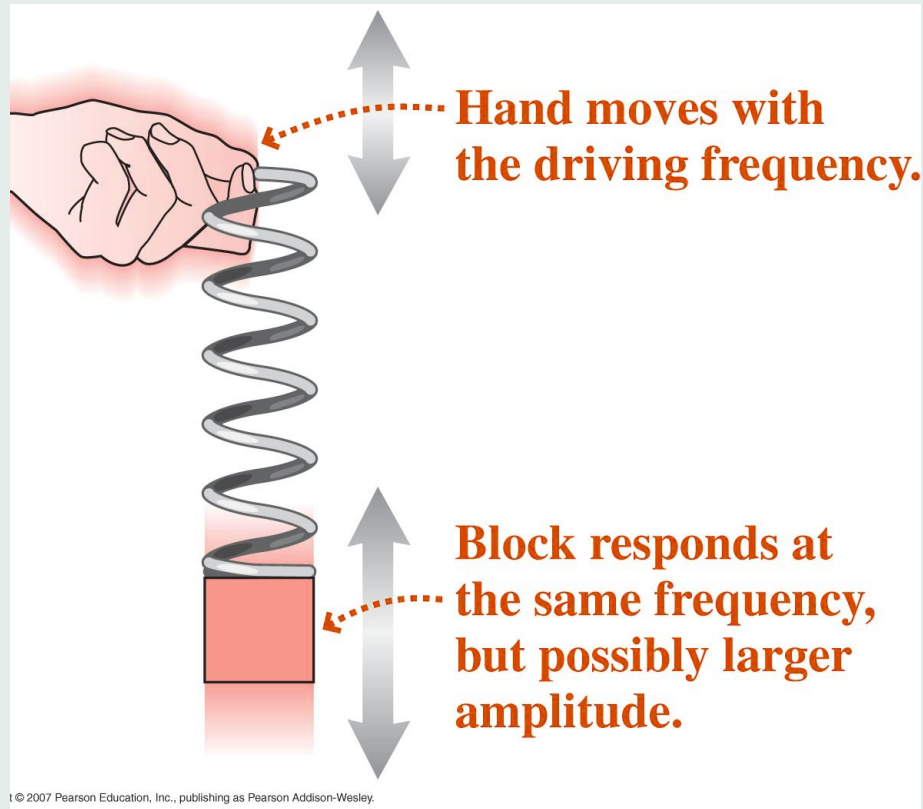
Page 28 of 31

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)



$$m \frac{d^2 y}{dt^2} + ky + b \frac{dy}{dt} = F_0 \cos(\omega_d t)$$

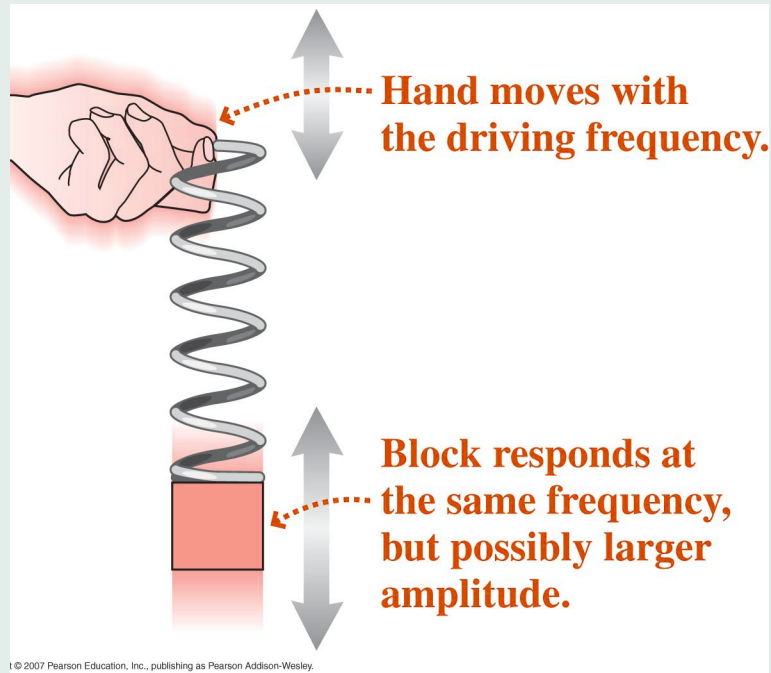
$$y = y_m \cos(\omega_d t + \phi)$$

Note: The oscillation frequency equals the frequency of the external force.

$$y_m = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + b^2 \omega_d^2 / m^2}}$$

$$\omega_0 = \sqrt{k/m}$$

[Home Page](#)
[Title Page](#)
[<<](#)
[>>](#)
[<](#)
[>](#)
[Page 29 of 31](#)
[Go Back](#)
[Full Screen](#)
[Close](#)
[Quit](#)



$$y = y_m \cos(\omega_d t + \phi)$$

$$y_m = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + b^2\omega_d^2/m^2}}$$

The amplitude depends on ω, ω_0

y_m is **maximal** when

$$\omega_d = \omega_0$$

Resonance: $\omega_d = \omega_0 \Rightarrow$ maximal energy transfer between external force and oscillating system.

Home Page

Title Page

◀

▶

◀

▶

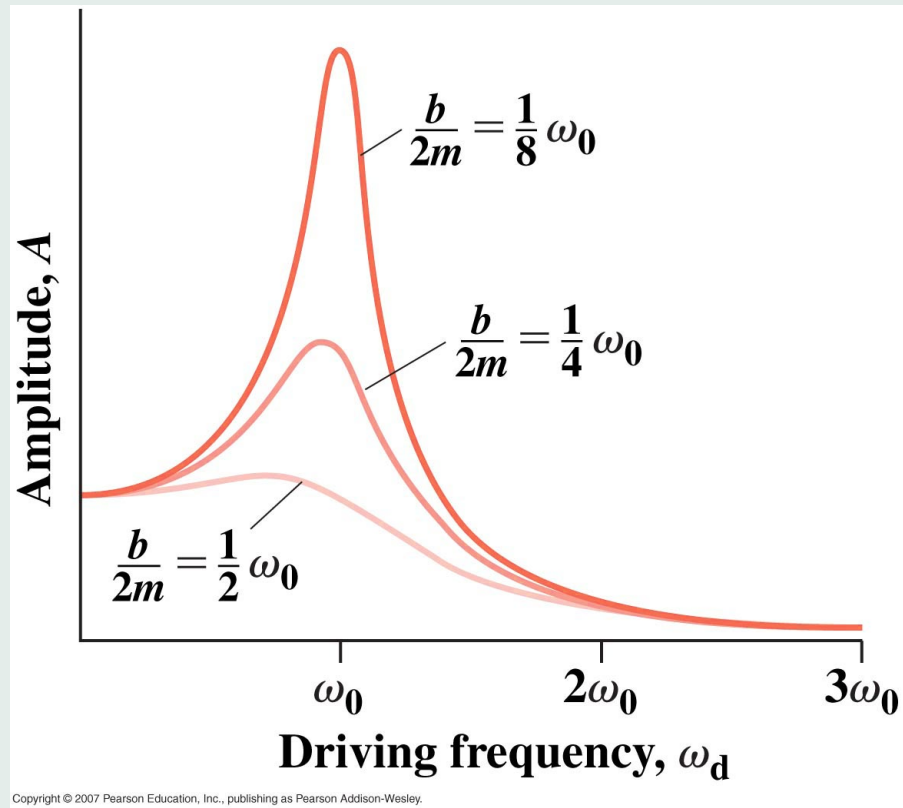
Page 30 of 31

Go Back

Full Screen

Close

Quit

[Home Page](#)[Title Page](#)

Page 31 of 31

[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

Have a Good Thanksgiving!

Home Page

Title Page



Page 32 of 31

Go Back

Full Screen

Close

Quit