Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 20

1. **No** quizzes during Thanksgiving week. There will be recitation according to the regular schedule using the $Th \to T$, $Fr \to Wed$ switch.

2. **No** lecture on Wednesday Nov 23rd (Friday classes)

Midterm II

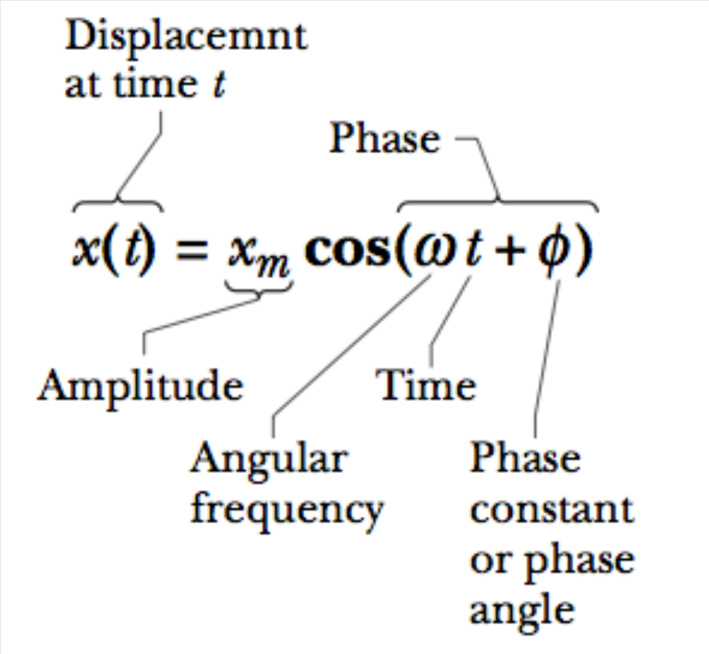
100	90	80	70	60	50	40	30	20	10
26	44	53	29	18	7	0	0	0	0

Class average: 80.6

- **Simple harmonic motion (SHM)**

One dimensional motion of a point particle given by

$$x(t) = x_m \cos(\omega t + \phi).$$



- x_m **amplitude** = maximum displacement
- t time
- ω **angular frequency**
- ϕ **phase constant** or **phase angle**

- **Period:**

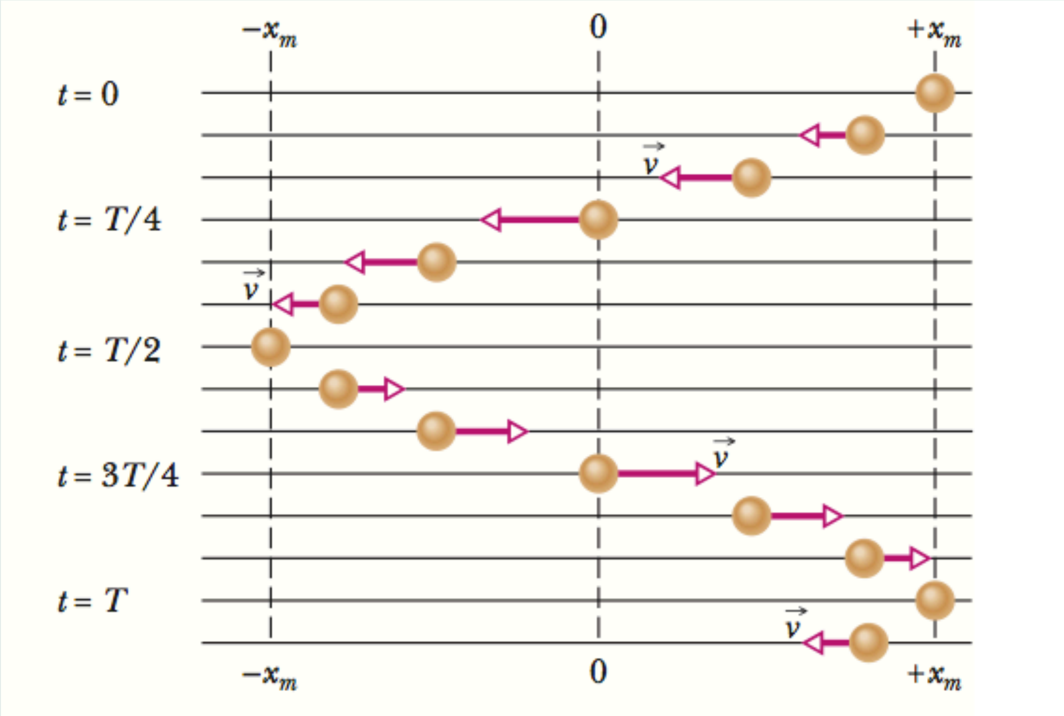
T= time needed to complete one oscillation

$$T = \frac{2\pi}{\omega}$$

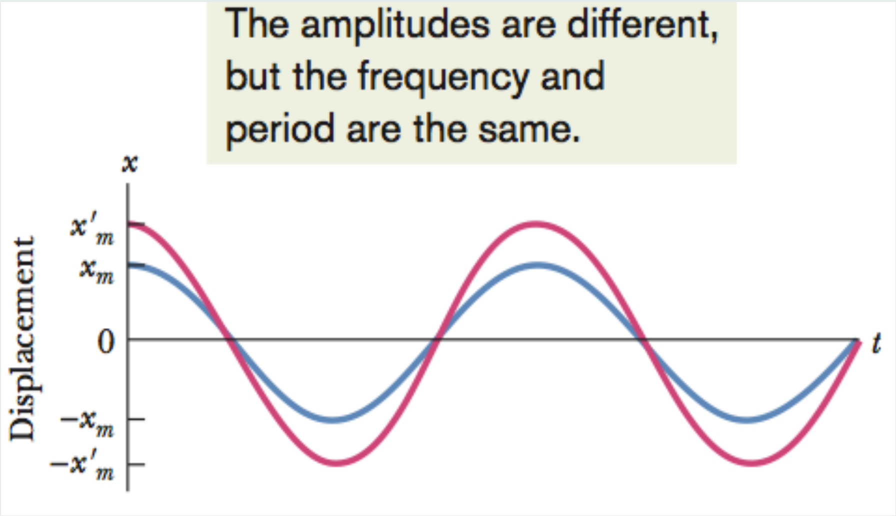
- **Frequency:**

f = number of oscillations per unit time.

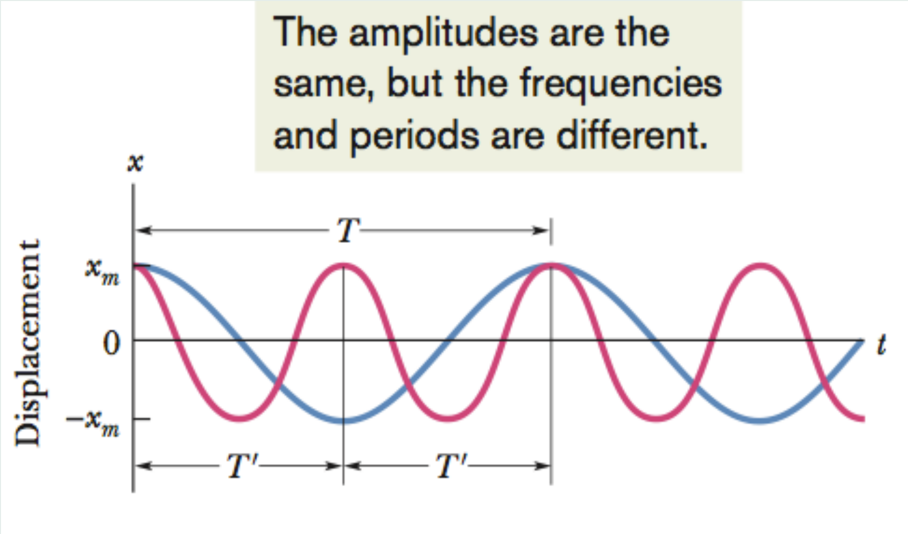
$$f = \frac{1}{T} = \frac{\omega}{2\pi}.$$



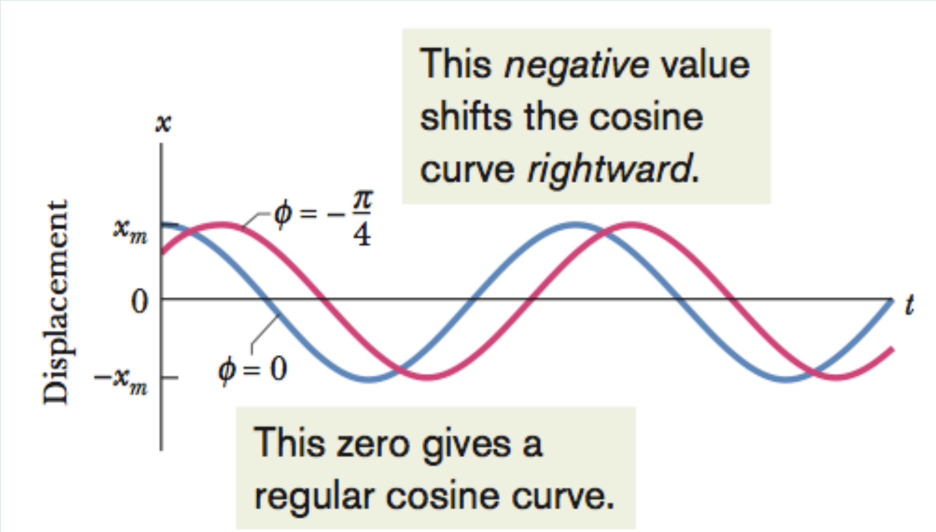
- back and forth from $+x_m$ to $-x_m$
- $\vec{v} = 0$ when $x(t) = \pm x_m$.
- $v = v_{\text{max}}$ when $x(t) = 0$.



- same ω, ϕ
- $x'_m > x_m$



- same x_m, ϕ
- $T = 2T' \Leftrightarrow \omega' = 2\omega$



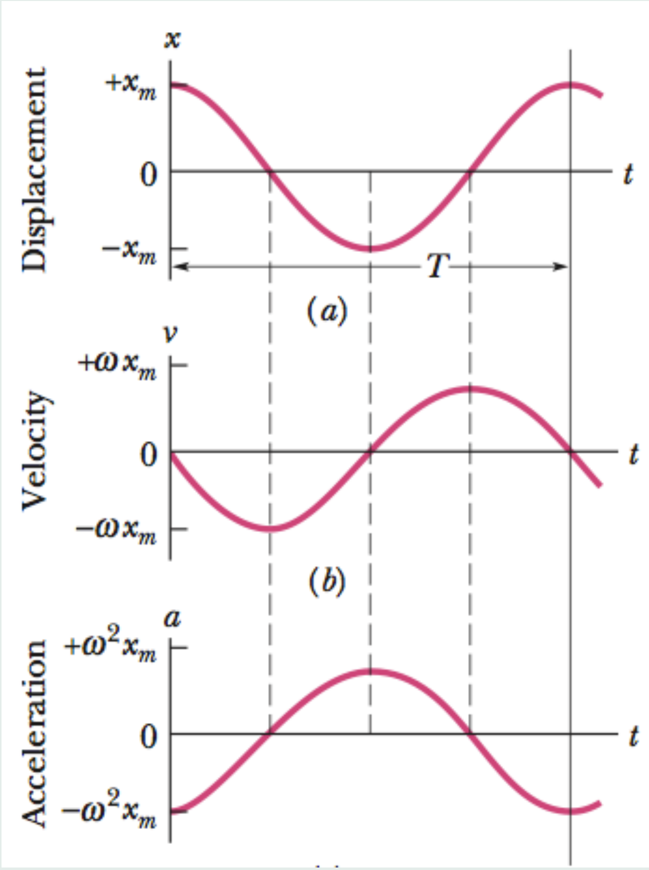
- same x_m, ω
- $\phi = 0$, $\phi' = -\pi/4$
- $\phi < 0 \Rightarrow$ **right** shift
- $\phi > 0 \Rightarrow$ **left** shift

The value of ϕ depends on the displacement and the velocity at time $t = 0$:

$$x_m \cos\phi = x(0) \qquad -\omega x_m \sin\phi = v_x(0)$$

• Velocity and acceleration

$$x(t) = x_m \cos(\omega t + \phi)$$



$$v_x(t) = \frac{dx}{dt} = -\omega x_m \sin(\omega t + \phi)$$

$$a_x(t) = \frac{dv_x}{dt} = -\omega^2 x_m \cos(\omega t + \phi)$$
$$= -\omega^2 x(t)$$

$$\boxed{a_x(t) = -\omega^2 x(t)}$$

- **Force law in SHM**

Newton's 2nd law:

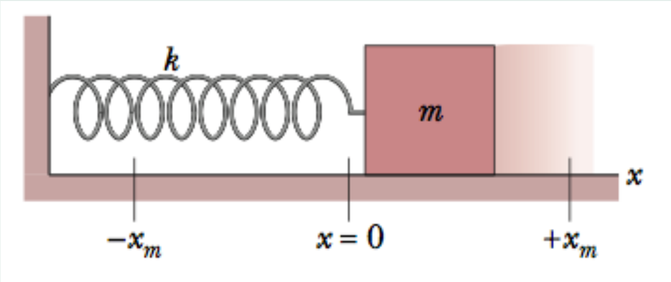
$$F_x = ma_x$$

$$\Downarrow$$

$$\boxed{F_x = -m\omega^2 x}$$

Spring force!

SHM is the motion executed by a system subject to a force that is proportional to the displacement of the system but opposite in sign.



$$F_x = -kx \;\Rightarrow\; k = m\omega^2$$

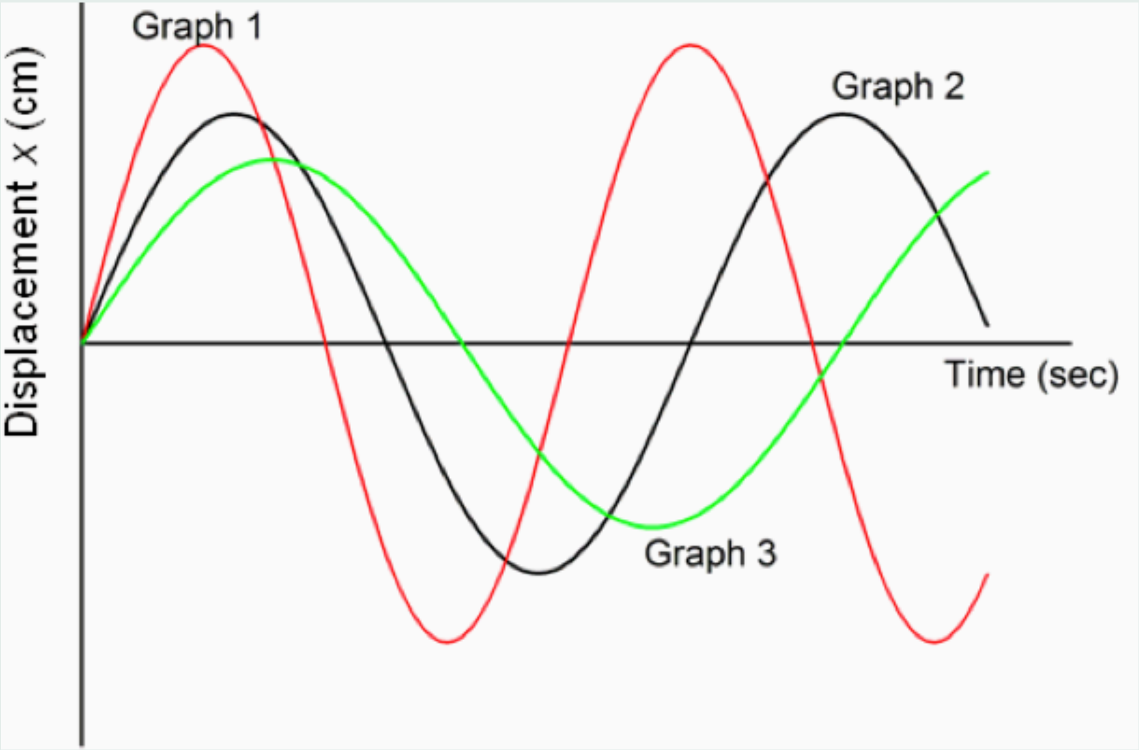
$$\omega = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Note:

$$m\frac{d^2x}{dt^2} = -kx \;\Rightarrow\; x = x_m\cos(\omega t + \phi)$$

$$x_0 = x(0) = x_m\cos\phi \qquad v_{x0} = v_x(0) = -\omega x_m\sin\phi$$



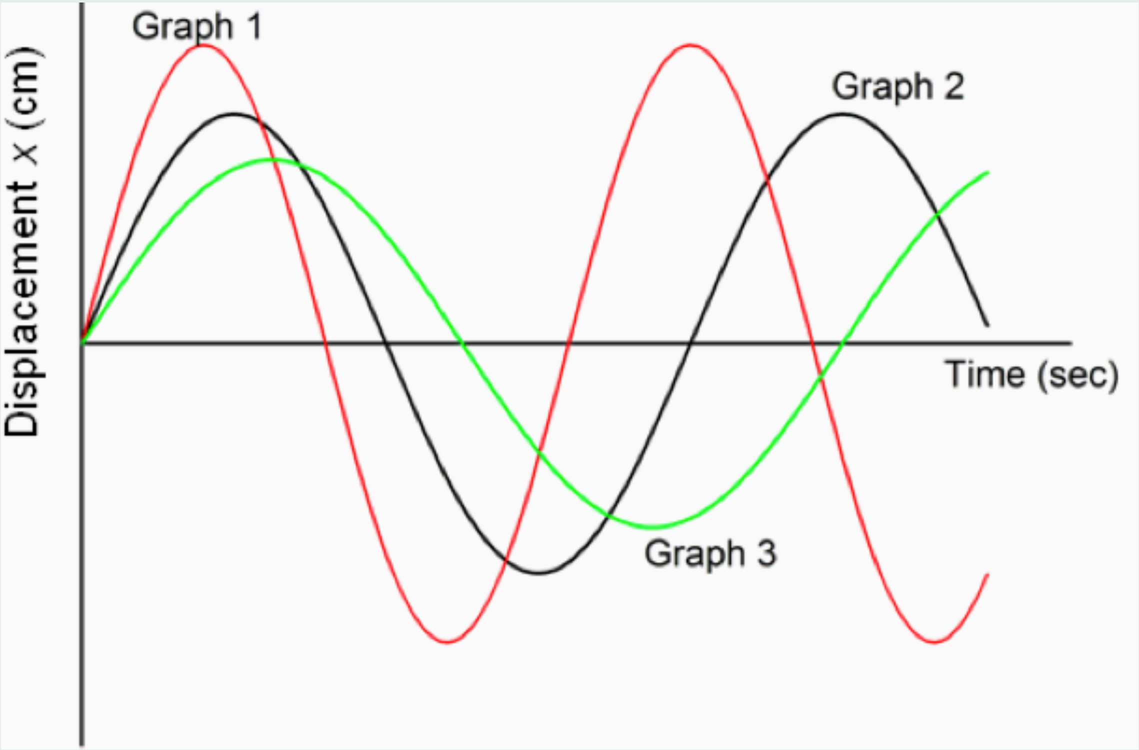
- 3 different springs, identical masses
- which one has the smallest k?

$A)$ Graph 1

$B)$ Graph 2

$C)$ Graph 3

$D)$ All have the same k



- 3 different springs, identical masses
- which one has the smallest k?

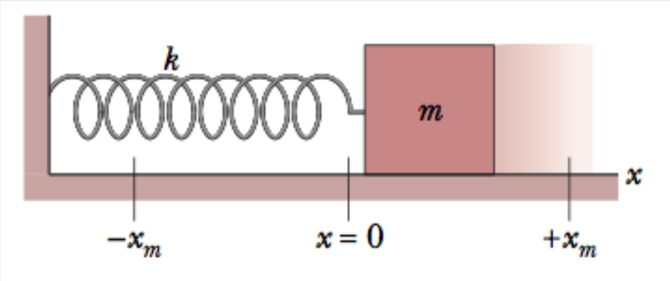
$A)$ Graph 1

$B)$ Graph 2

$C)$ Graph 3 $\quad (T = 2\pi\sqrt{m/k})$

$D)$ All have the same k

- **Energy in SHM**

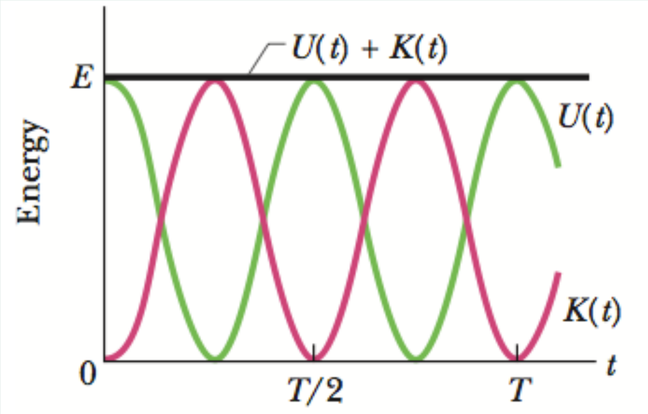


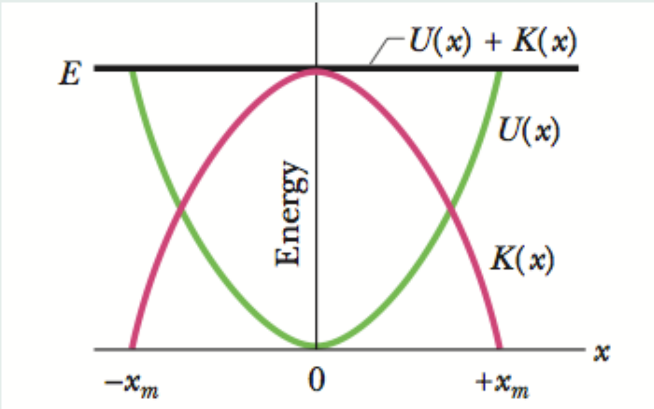
- **Kinetic Energy**

$$K = \frac{1}{2}mv^2$$
$$= \frac{m\omega^2 x_m^2}{2}\sin^2(\omega t + \phi)$$

- **Potential energy**

$$U = \frac{1}{2}kx^2$$
$$= \frac{kx_m^2}{2}\cos^2(\omega t + \phi)$$
$$= \frac{m\omega^2 x_m^2}{2}\cos^2(\omega t + \phi)$$



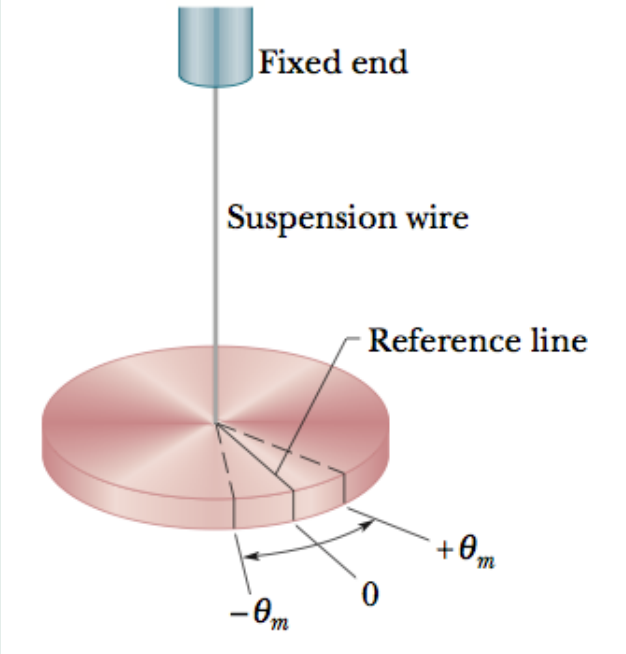


$$K = \frac{m\omega^2 x_m^2}{2}\sin^2(\omega t + \phi)$$

$$U = \frac{m\omega^2 x_m^2}{2}\cos^2(\omega t + \phi)$$

$$K + U = \frac{m\omega^2 x_m^2}{2} = \textbf{constant}$$

- **An angular SHM: torsion pendulum**



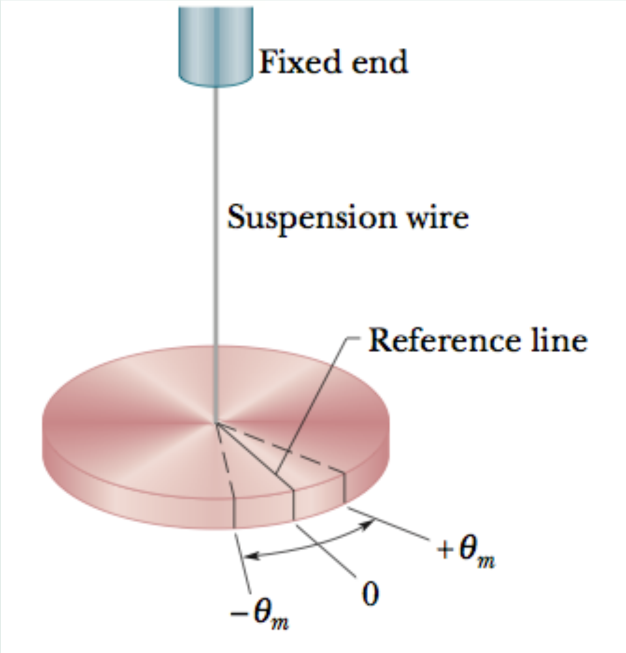
- **Resoring torque:** from twisting of the suspension wire

$$\tau = -\kappa\theta$$

$\theta =$ angular displacement

- **Equation of motion:**

$$I\frac{d^2\theta}{dt^2} = -\kappa\theta$$



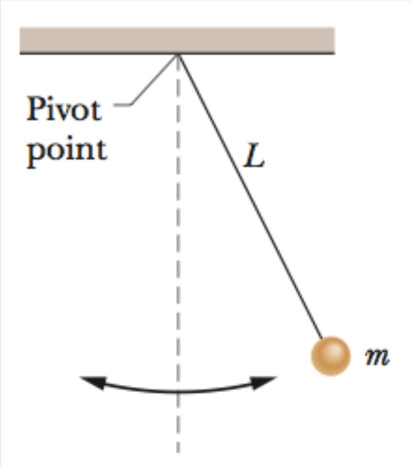
- **Analogy with linear SHM:**

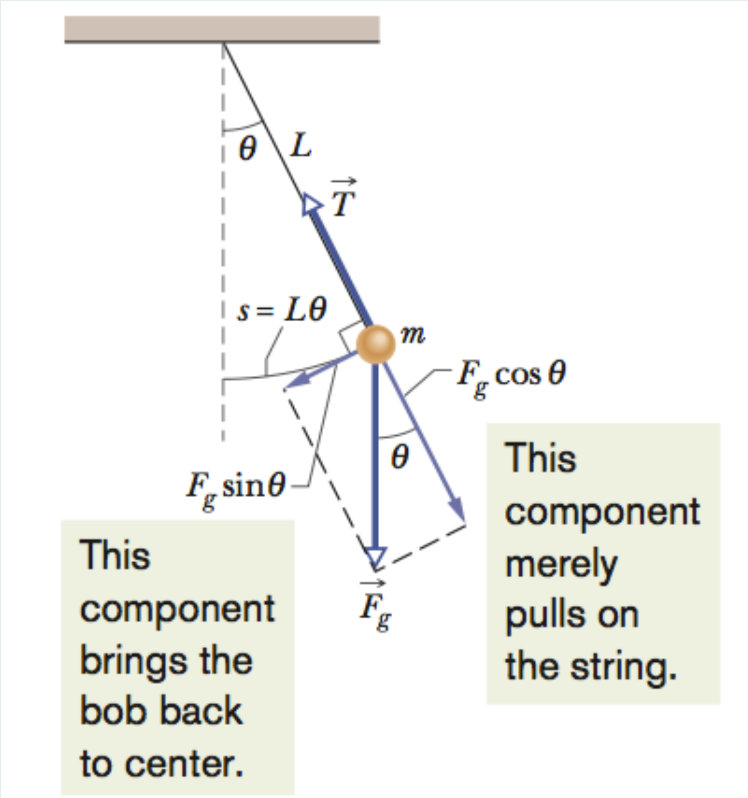
$$\theta \leftrightarrow x$$
$$I \leftrightarrow m$$
$$\kappa \leftrightarrow k = m\omega^2$$
$$I\frac{d^2\theta}{dt^2} = -\kappa\theta \leftrightarrow m\frac{d^2x}{dt^2} = -kx$$

$$\theta = \theta_m \cos(\omega t + \phi), \qquad \omega = \sqrt{\frac{\kappa}{I}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$$

- **The simple pendulum**

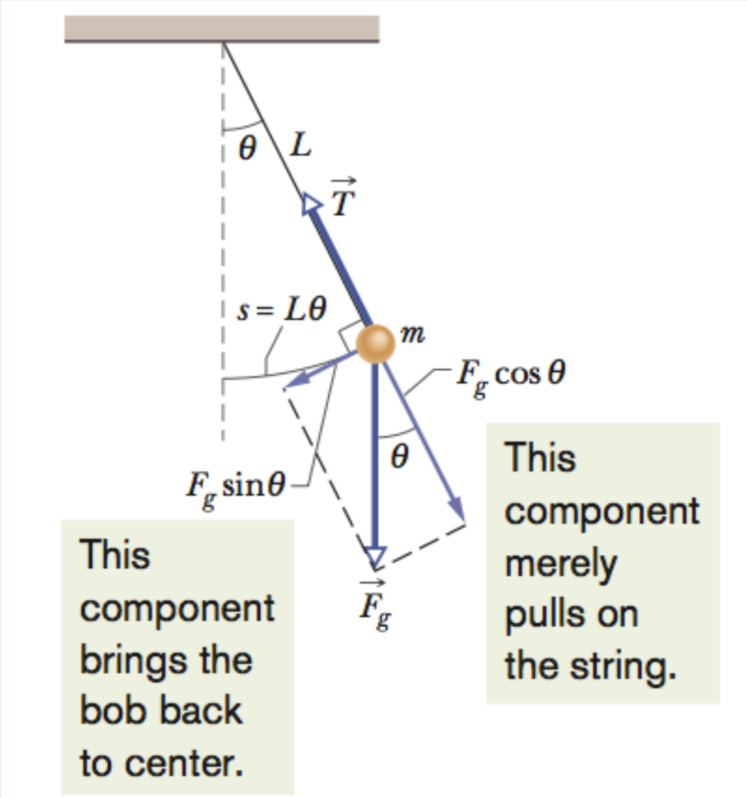




- **Restoring force** = tangential component of gravitational force

$$F_g \sin\theta$$

Not linear in θ!



- Small angle approximation:

$$\sin\theta \simeq \theta$$

Good for $\theta \leq 10^\circ = \pi/18\,\text{rad}$

$$\sin(10^\circ) = 0.1736481777...$$

$$10^\circ = 0.1745329252\ldots\ \text{rad}$$

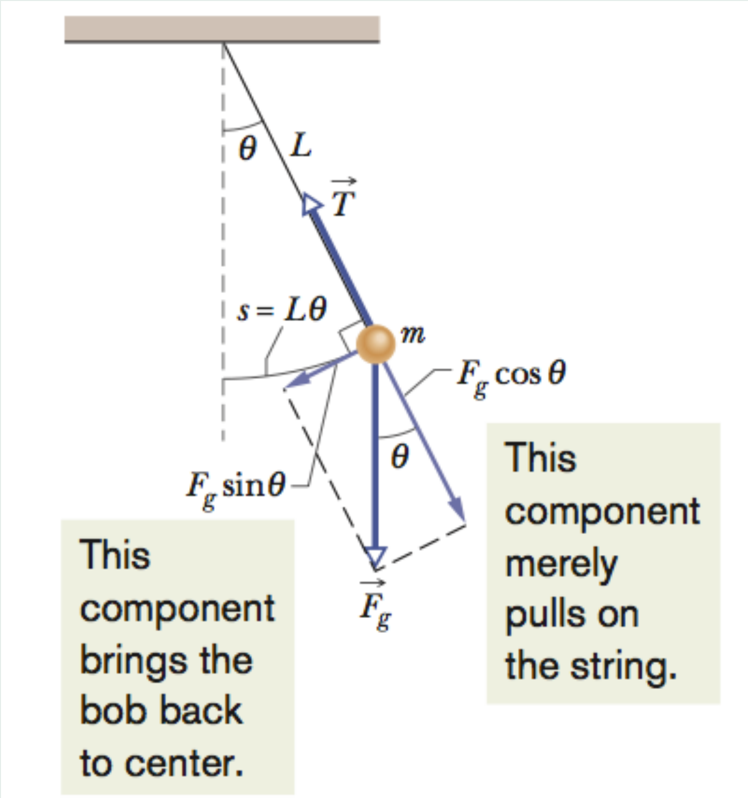
$$\left|\frac{\sin(10^\circ) - \pi/18}{\sin(10^\circ)}\right| \simeq 0.0051\ldots$$

$$< 1\%$$

- Equation of motion for small θ:

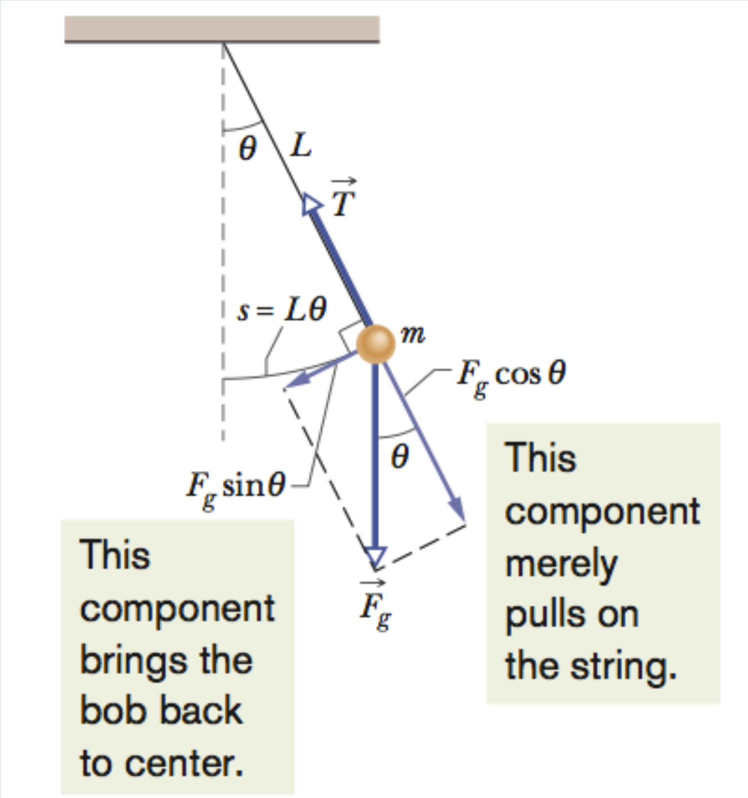
$$I\frac{d^2\theta}{dt^2} = -\tau_{F_g}$$
$$= -mgL\sin\theta$$
$$\simeq -mgL\theta$$

$$I\frac{d^2\theta}{dt^2} = -mgL\theta$$



- I = rotational inertia about an axis $\perp$ to plane of motion passign through O.

- τ_{F_g} = torque of gravitational force about the same axis.



- Analogy with linear SHM:

$$I\frac{d^2\theta}{dt^2} = -mgL\theta$$

$$\Downarrow$$

$$\theta = \theta_m \cos(\omega t + \phi)$$

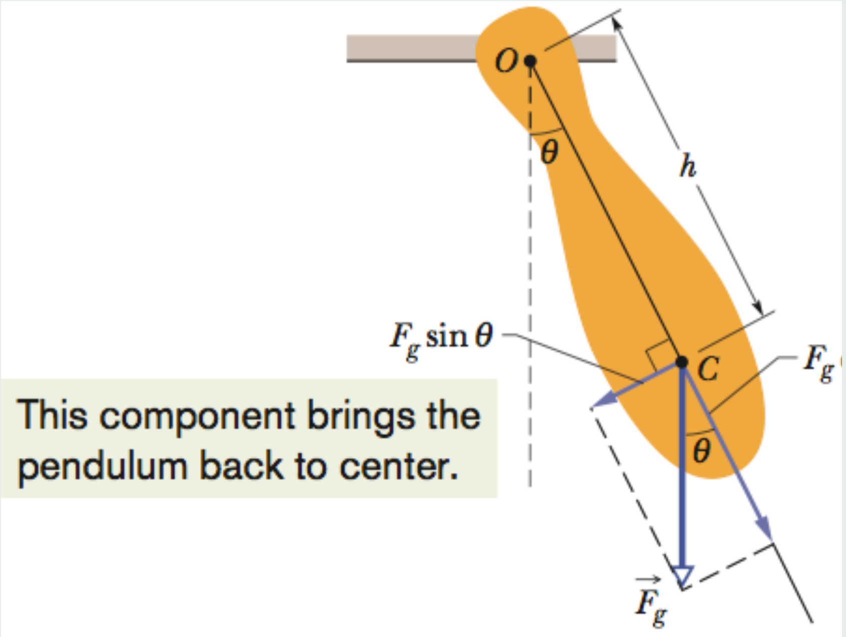
$$\omega = \sqrt{\frac{mgL}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

Pointlike mass, ideal cord: $I = mL^2$

$$\omega = \sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

- **Physical Pendulum**



- Same reasoning

$$I\frac{d^2\theta}{dt^2} = -mgh\theta$$

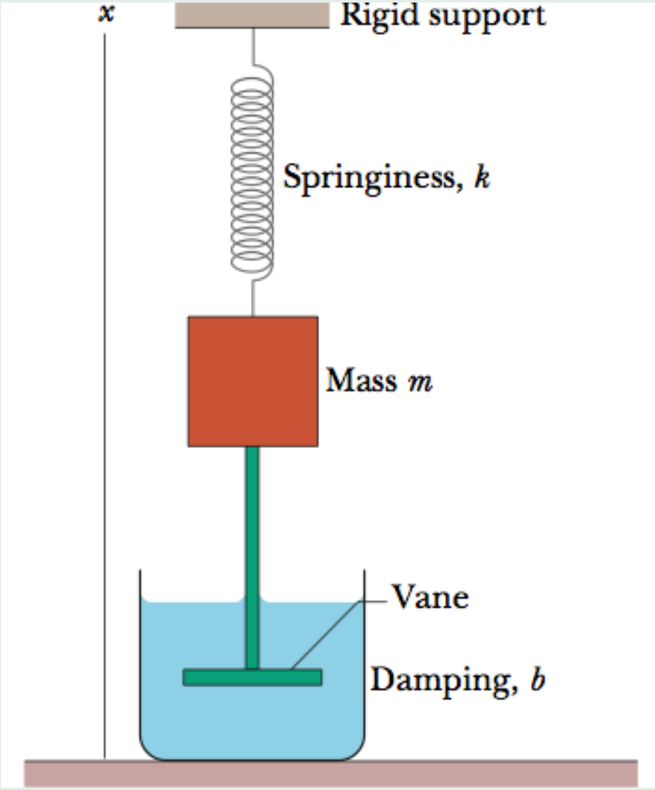
$$\Downarrow$$

$$\theta = \theta_m \cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{mgh}{I}}$$

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

- **Damped oscillations**



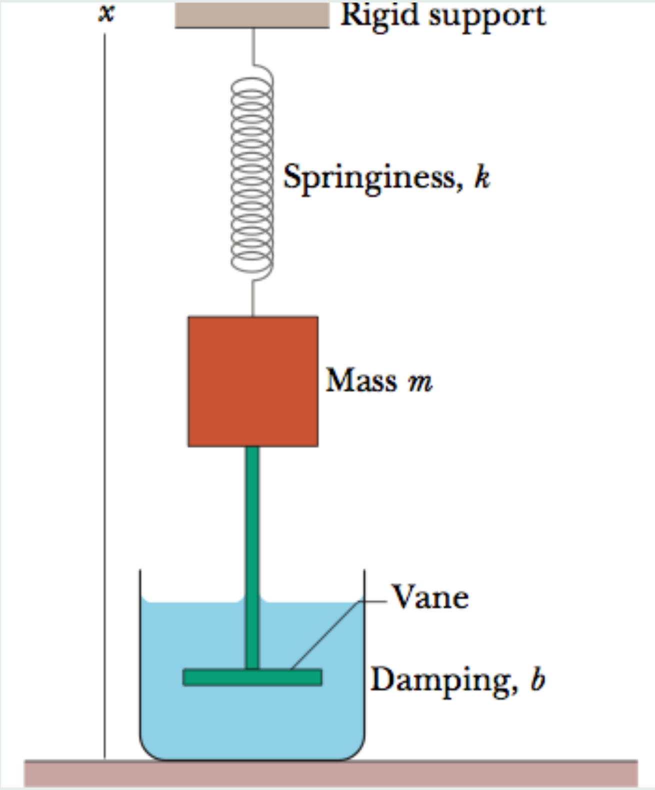
- Restoring spring force

$$F_s = -ky$$

- Damping force

$$F_d = -bv_y$$

- Assume gravitational force very small compared to first two.



- Equation of motion along y axis:

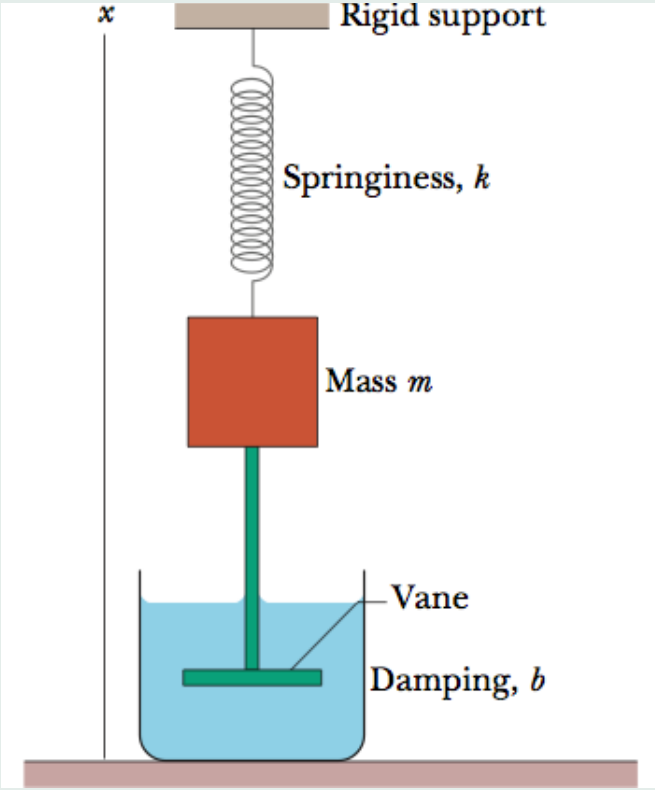
$$m\frac{d^2y}{dt^2} = -ky - b\frac{dy}{dt}$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$$

$$y(t) = y_m e^{-bt/2m}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

(assume $k > b^2/4m$)



$$y(t) = y_m e^{-bt/2m} \cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

- $\sim$ cos function with amplitude decreasing in time

- Mechanical energy also decreasing in time:

$$E(t) \simeq \frac{1}{2} k x_m^2 e^{-bt/m}$$

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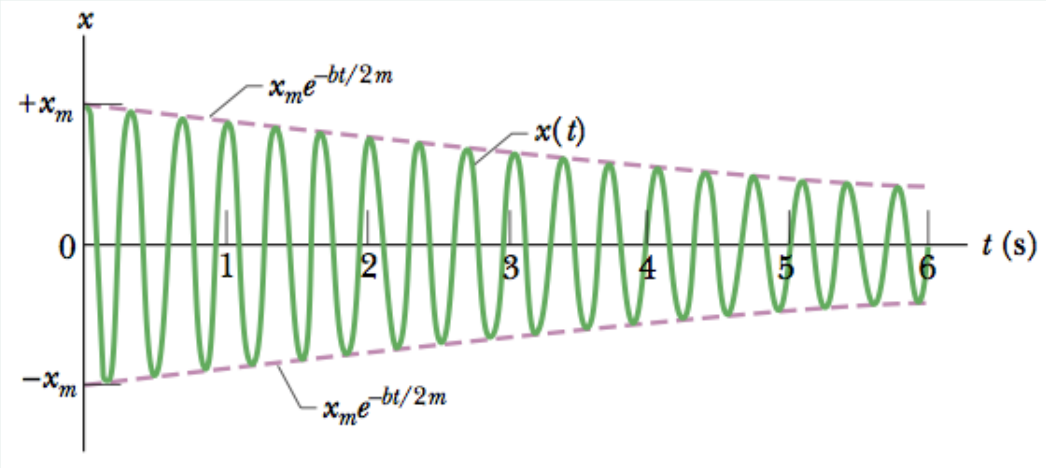
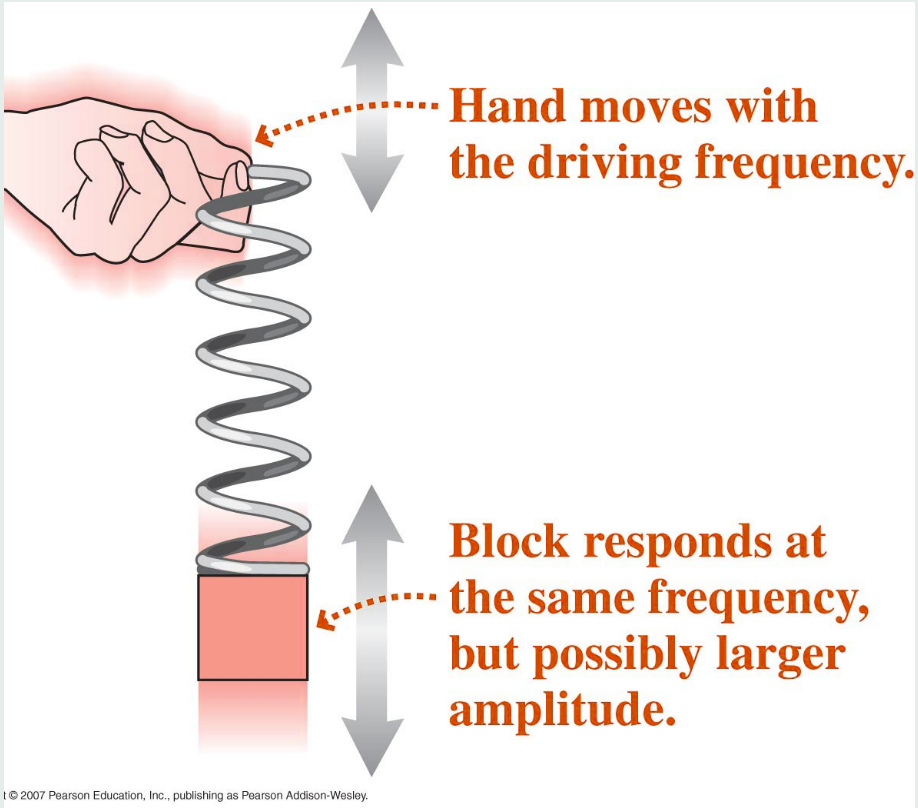


$$y(t) = y_m e^{-bt/2m} \cos(\omega' t + \phi)$$

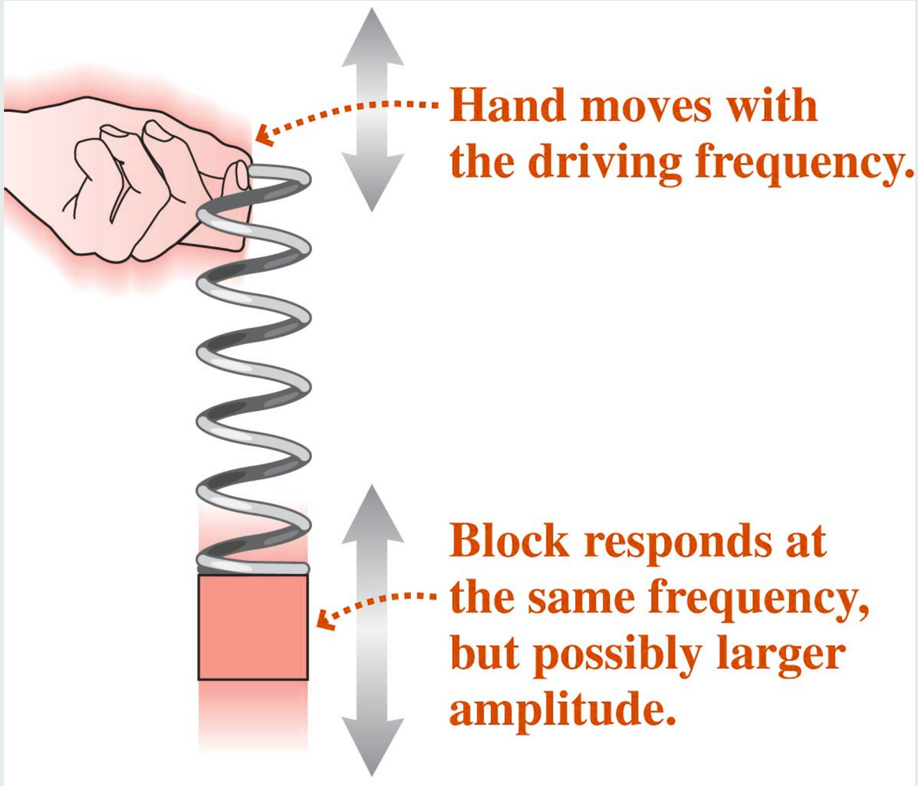
- **Forced oscillations and resonance**



- Suppose the hand moves with a given frequency ω_d (**driving frequency**)
- Assume gravity is negligible.
- Equation of motion:

$$m\frac{d^2y}{dt^2} = -ky - b\frac{dy}{dt} + F_0\cos(\omega_d t)$$

(note the additional external force)



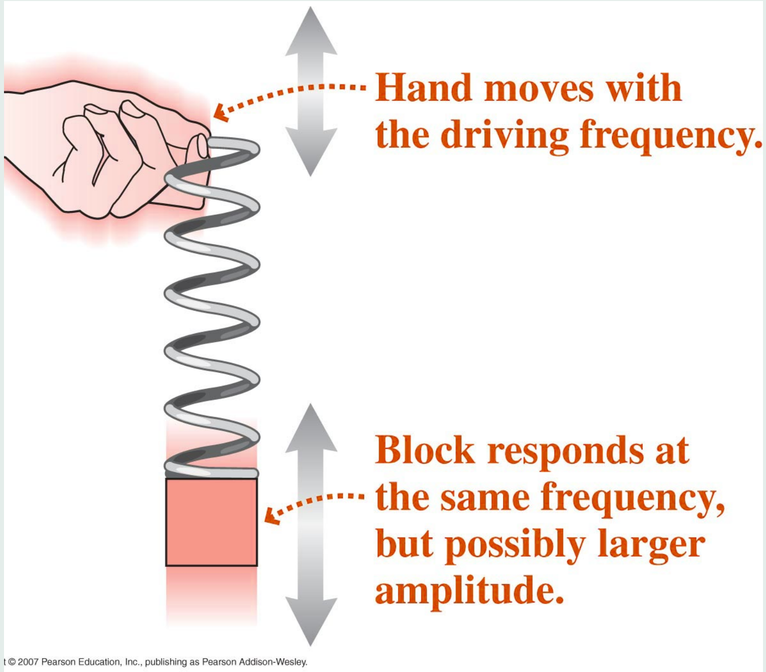
$$m\frac{d^2y}{dt^2} + ky + b\frac{dy}{dt} = F_0\cos(\omega_d t)$$

$$y = y_m\cos(\omega_d t + \phi)$$

Note: The oscillation frequency equals the frequency of the external force.

$$y_m = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + b^2\omega_d^2/m^2}}$$

$$\omega_0 = \sqrt{k/m}$$



$$y = y_m \cos(\omega_d t + \phi)$$

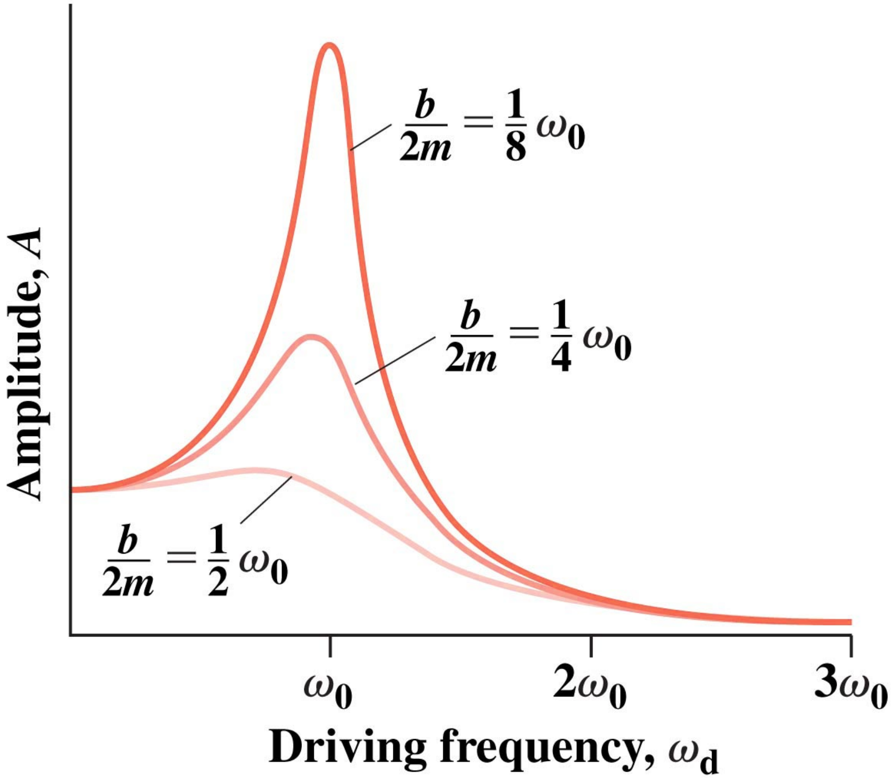
$$y_m = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + b^2\omega_d^2/m^2}}$$

The amplitude depends on ω, ω_0

y_m is **maximal** when

$$\omega_d = \omega_0$$

Resonance: $\omega_d = \omega_0 \Rightarrow$ maximal energy transfer between external force and oscillating system.



Amplitude, A
$\frac{b}{2m} = \frac{1}{8}\omega_0$
$\frac{b}{2m} = \frac{1}{4}\omega_0$
$\frac{b}{2m} = \frac{1}{2}\omega_0$
ω_0
$2\omega_0$
$3\omega_0$
Driving frequency, ω_d

Have a Good Thanksgiving!