Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I

Lecture 2

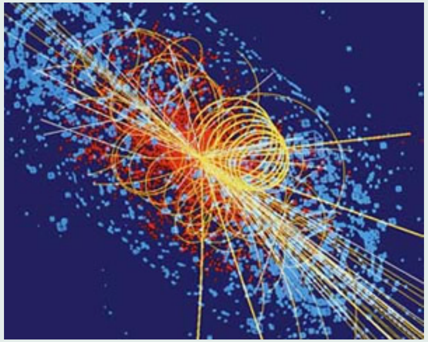
2. Motion along a straight line

Goals:

- To introduce position and displacement in one dimension.
- To define and differentiate average and instantaneous linear velocity.
- To define and differentiate average and instantaneous linear acceleration.
- To explore some applications of one dimensional motion with constant acceleration.
- To examine freely falling bodies.

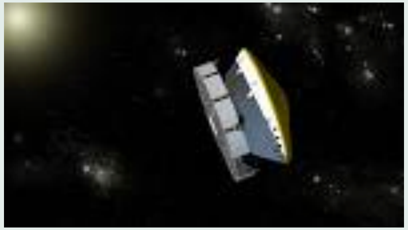
One dimensional motion.

- Motion is along a straight line only.
- Will study the characteristics of motion i.e kinematics, not its cause (dynamics.)
- Physical objects will be assumed pointlike.



Elementary particles.

Good model



Spaceship between Earth and Mars.

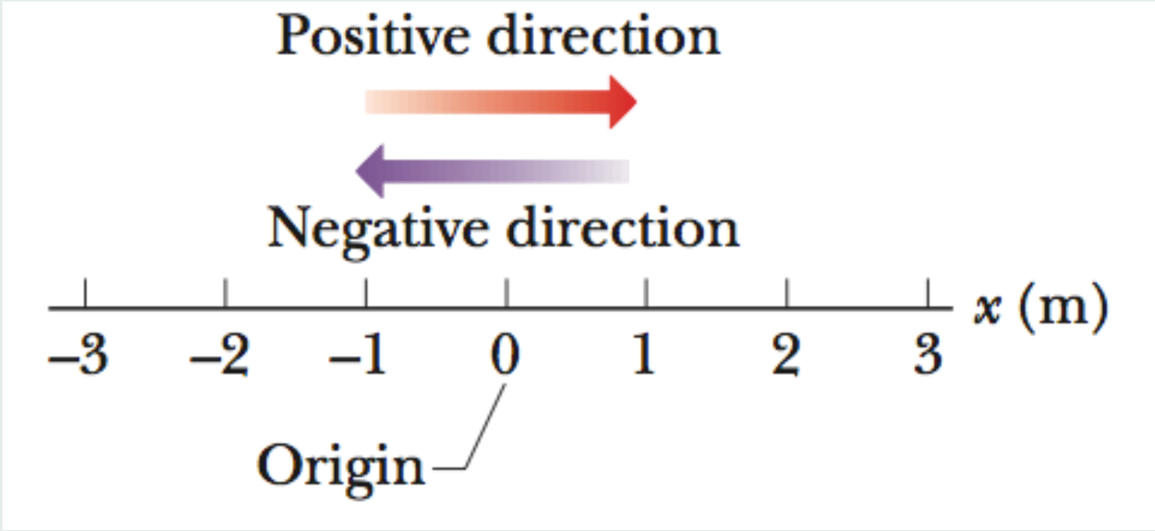
Car in parking lot.

Bad
model

Spaceship in asteroid cloud.

Note: scale is very important in physics!

Position and displacement.



- Fix origin $\Rightarrow$ position determined by one number x
- Positive direction $x \nearrow$
- Negative direction $x \searrow$

- **Displacement:** change from position x_1 to x_2

$$\Delta x = x_2 - x_1$$

- Displacement is a vector quantity $\begin{cases} \text{magnitude} \\ \text{direction} \end{cases}$

$$x_1 = 3\text{m}, \quad x_2 = 8\text{m} \quad \Rightarrow \quad \Delta x = 5\text{m} > 0$$

Positive direction

$$x_1 = 8\text{m}, \quad x_2 = 3\text{m} \quad \Rightarrow \quad \Delta x = -5\text{m} < 0$$

Negative direction

$$\Delta x = (\text{Sign})|\Delta x|, \qquad |\Delta x| > 0.$$

$|\Delta x|$ = Magnitude: distance covered from initial to from initial to final position.

Sign: direction of motion from initial to final position.

$+ \leftrightarrow$ positive direction.

$- \leftrightarrow$ negative direction.

Note: displacement depends only on the initial and final position.

i-Clicker:

Suppose a particle moves from $x = 2\,\mathrm{m}$ out to $x = 5\,\mathrm{m}$ and back to $x = 2\,\mathrm{m}$. Then the displacement is:

(A) $\Delta x = 3\,\mathrm{m}$

(B) $\Delta x = -3\,\mathrm{m}$

(C) $\Delta x = 0\,\mathrm{m}$

(D) $\Delta x = 5\,\mathrm{m}$

(E) $\Delta x = 2\,\mathrm{m}$

Answer

Suppose a particle moves from $x = 2\,\mathrm{m}$ out to $x = 5\,\mathrm{m}$ and back to $x = 2\,\mathrm{m}$. Then the displacement is:

$(A)\ \Delta x = 3\,\mathrm{m}$

$(B)\ \Delta x = -3\,\mathrm{m}$

$(C)\ \Delta x = 0\,\mathrm{m}$

$(D)\ \Delta x = 5\,\mathrm{m}$

$(E)\ \Delta x = 2\,\mathrm{m}$

Initial position:

$$x_1 = 2\,\mathrm{m}$$

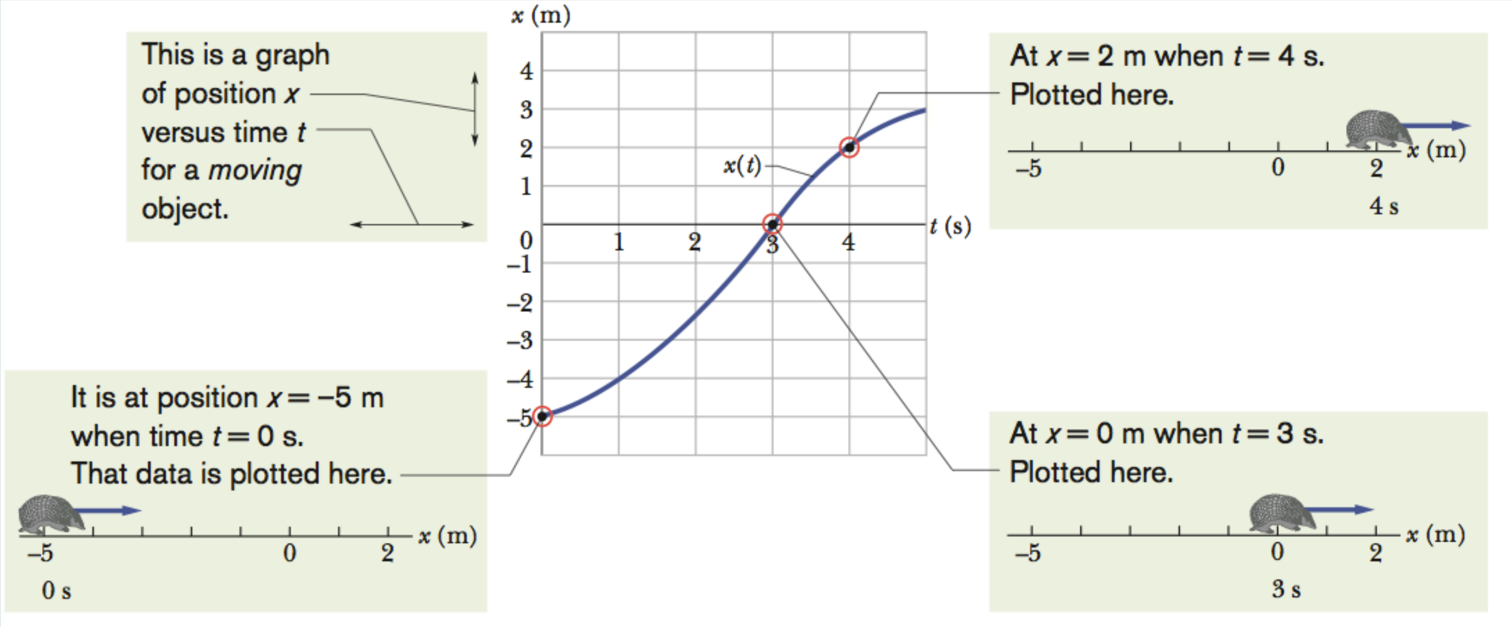
Final position:

$$x_2 = 2\,\mathrm{m}$$

$$\Delta x = 2\,\mathrm{m} - 2\,\mathrm{m} = 0\,\mathrm{m}$$

Average velocity and average speed

- One dimensional motion $\leftrightarrow$ graph of the position x as function of time t



- **Average velocity:** rate of change of position

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$x_1 = x(t_1) \text{ position at time } t_1$$

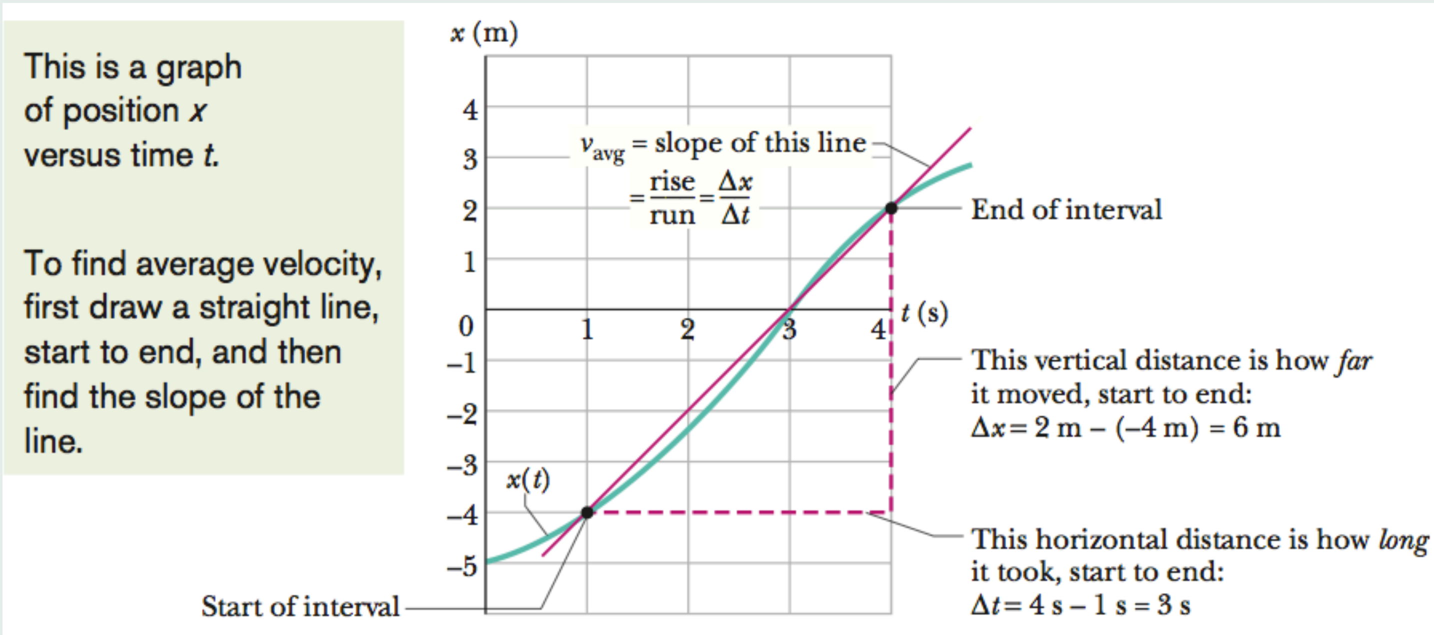
$$x_2 = x(t_2) \text{ position at time } t_2 > t_1$$

v_{avg} vector quantity: same sign as Δx since

$$t_2 - t_1 > 0$$

Units for v_{avg} : $\text{meter/second} = \text{m/s}.$

Geometric interpretation:



v_{avg} = slope of straight line connecting the points (t_1, x_1), (t_2, x_2). Above $v_{\text{avg}} = 6/3\,\text{m/s} = 2\,\text{m/s}$.

- **Average speed**

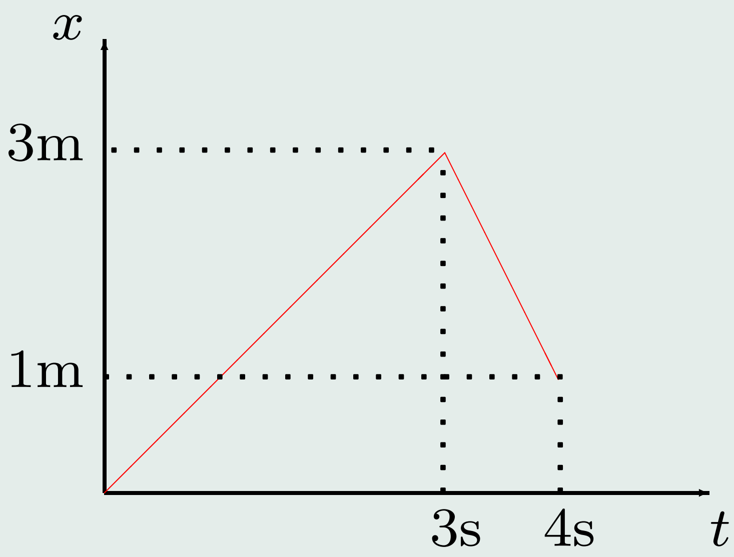
$$s_{\mathsf{avg}} = \frac{\text{total distance travelled in time interval } \Delta t}{\Delta t}$$

s_{avg} scalar quantity; no sign, no direction.

Units for s_{avg} : $\text{meter/second} = \text{m/s}.$

i-Clicker

A particle moves from $x = 0\,\mathrm{m}$ to $x = 3\,\mathrm{m}$ and then from $x = 3\,\mathrm{m}$ to $x = 1\,\mathrm{m}$ as shown in the graph below. Then v_{avg} is:



(A) $v_{\text{avg}} = 1\mathrm{m/s}$

(B) $v_{\text{avg}} = 0.75\mathrm{m/s}$

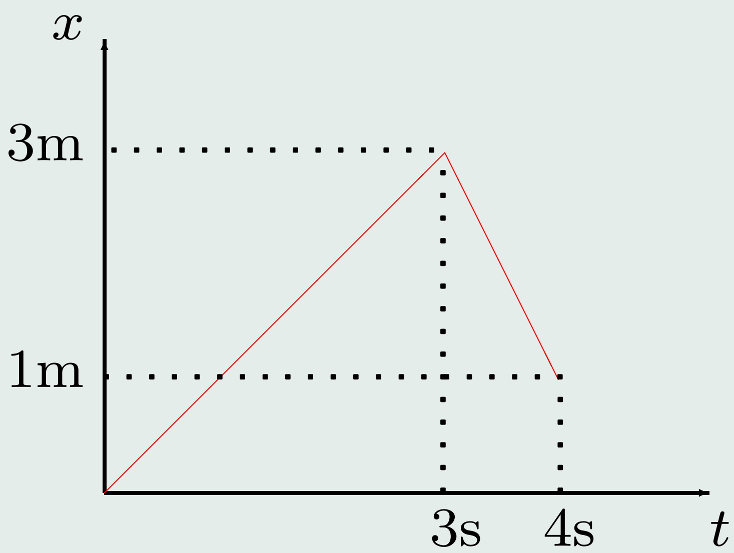
(C) $v_{\text{avg}} = 1.25\mathrm{m/s}$

(D) $v_{\text{avg}} = 0.25\mathrm{m/s}$

(E) none of the above

Answer

A particle moves from $x = 0\,\mathrm{m}$ to $x = 3\,\mathrm{m}$ and then from $x = 3\,\mathrm{m}$ to $x = 1\,\mathrm{m}$ as shown in the graph below. Then v_{avg} is:



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(C) $v_{\text{avg}} = 1.25\mathrm{m/s}$

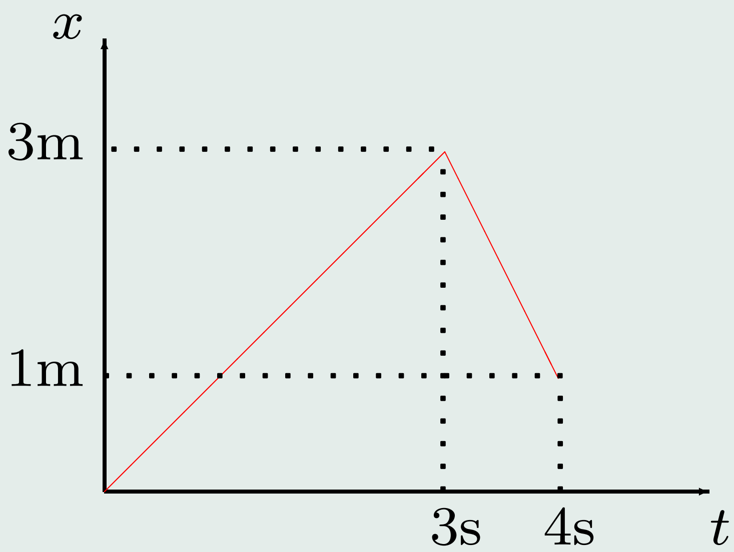
(D) $v_{\text{avg}} = 0.25\mathrm{m/s}$

(E) none of the above

$$\Delta x = 1\,\mathrm{m} - 0\,\mathrm{m} = 1\,\mathrm{m},\ \Delta t = 4s,\ v_{\text{avg}} = \tfrac{1}{4}\,\mathrm{m/s} = 0.25\,\mathrm{m/s}$$

i-Clicker

For the same graph s_{avg} is:



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Answer

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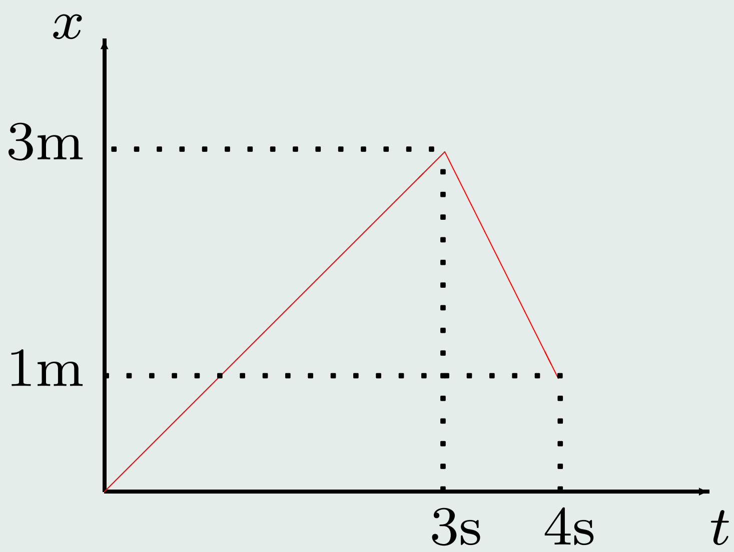

(A) $s_{\text{avg}} = 1\text{m/s}$

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(C) $s_{\text{avg}} = 0.25\text{m/s}$

(D) $s_{\text{avg}} = 0.75\text{m/s}$

(E) none of the above.

Total distance $3\,\text{m} + 2\,\text{m} = 5\,\text{m}$, $\Delta t = 4\text{s}$,

$$s_{\text{avg}} = \frac{5}{4}\text{m/s} = 1.25\text{m/s}.$$

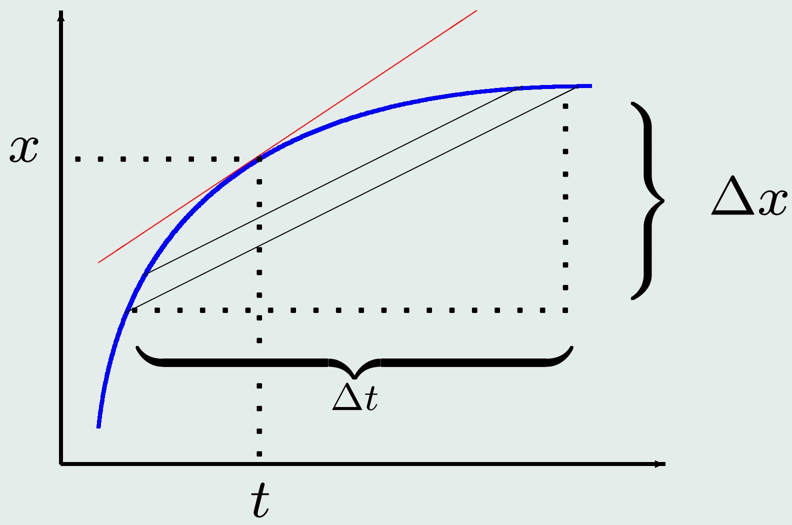
Instantaneous velocity and speed

- Instantaneous velocity: velocity of a particle at a given moment in time.

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$v =$ limit of v_{avg} over smaller and smaller time intervals Δt centered at a current point (x, t)

Geometric interpretation



$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$= \text{slope of the tangent line to motion graph at current point}$

$$= \frac{dx}{dt} \text{ (}\textbf{derivative}\text{)}$$

Note: v is a vector quantity $\begin{cases} \text{direction} \\ \text{magnitude} \end{cases}$

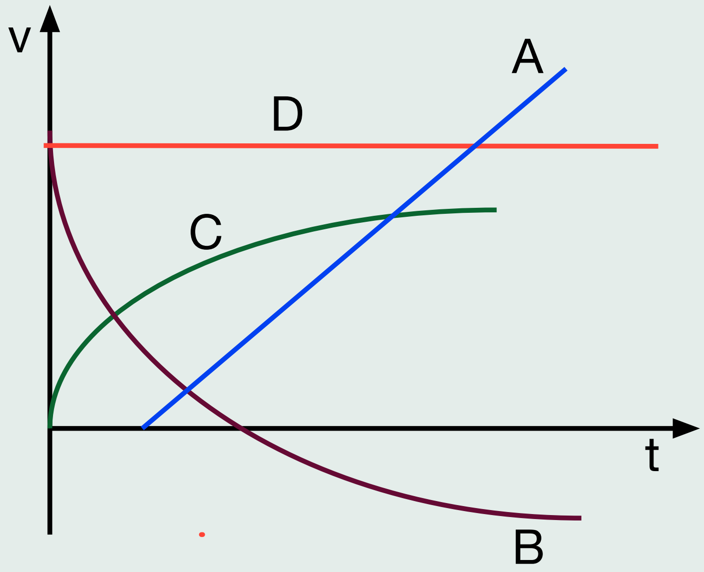
- **Instantaneous speed:** magnitude of v

$$s = |v| = \left| \frac{dx}{dt} \right|$$

Units for v, s: m/s.

i-Clicker

Which of the following velocity graphs represents a car initially moving forward and then reversing direction?



(A)

(B)

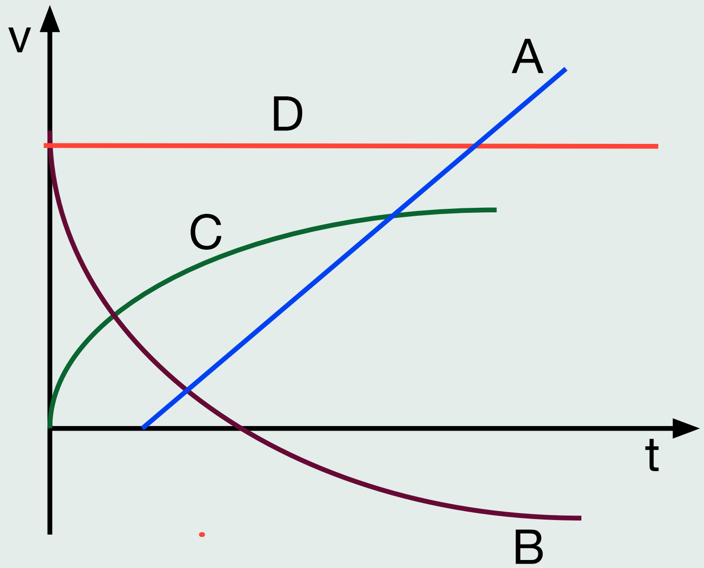
(C)

(D)

(E) none of the above

Answer

Which of the following velocity graphs represents a car initially moving forward and then reversing direction?



(A)

(B)

(C)

(D)

(E) none of the above

The velocity must be $v = 0$ at some instant in time.

Acceleration

- **Acceleration:** the rate of change of velocity.
- **Average acceleration**

$$a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

$v_1 = v(t_1)$ instantaneous velocity at time t_1

$v_2 = v(t_2)$ instantaneous velocity at time $t_2 > t_1$

a_{avg} vector quantity: same sign as Δv since

$$t_2 - t_1 > 0$$

- **Instantaneous acceleration**

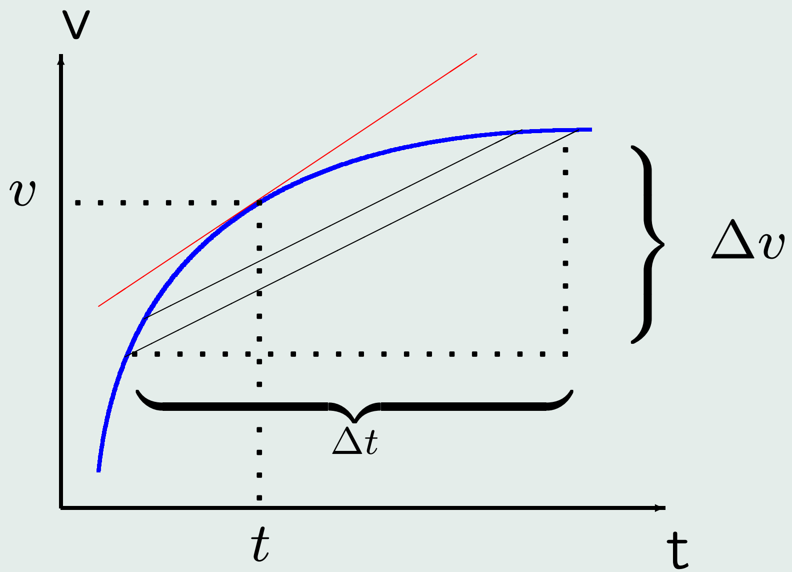
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Note: a is a vector quantity $\begin{cases} \text{direction} \\ \text{magnitude} \end{cases}$

Units for a_{avg}, a :

$$(\text{meter}/\text{second})/\text{second} = \text{m}/\text{s}^2.$$

Geometric interpretation: velocity graph



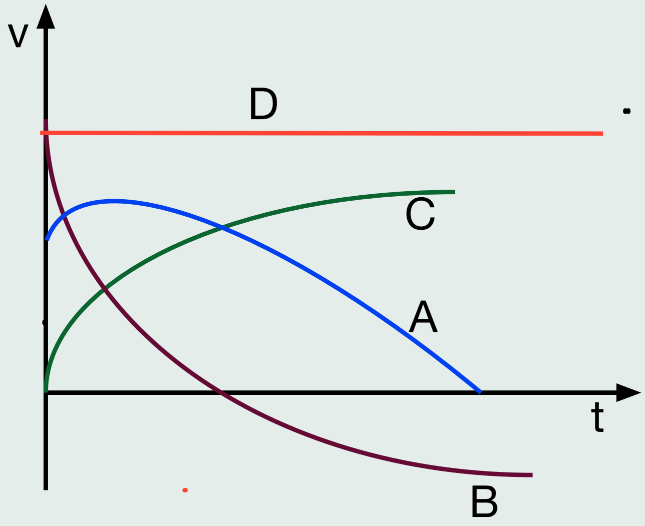
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$
$$= \text{slope of the tangent line to motion graph at current point}$$
$$= \frac{dv}{dt}$$

Alternative formula:

$$a = \frac{d^2x}{dt^2} \quad (\textbf{second derivative})$$

i-Clicker

Which of the following velocity graphs represents a car moving with $a > 0$ for a finite time interval and then switching to $a < 0$?



(A)

(B)

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Answer

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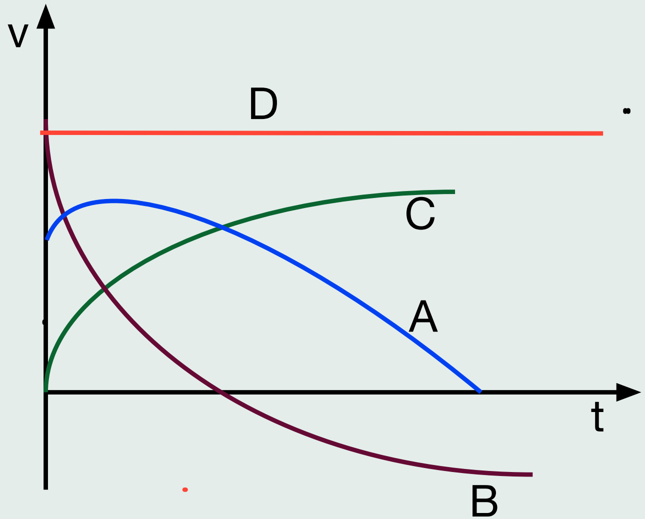


(A)

(B)

(C)

(D)

(E) none of the above

$a > 0 \Rightarrow v \nearrow$ $\qquad$ $a < 0 \Rightarrow v \searrow$

Constant acceleration

What if a is constant, independent of time?

Time dependence of velocity v and position x?

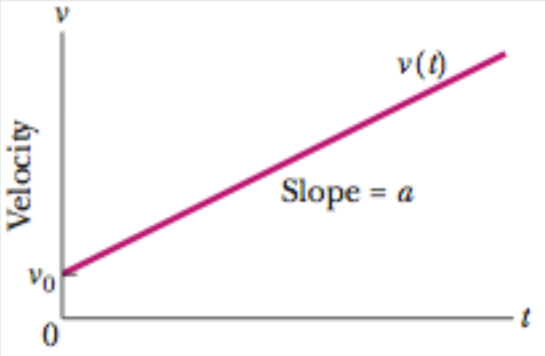
$$a = \text{constant} \quad \Rightarrow \quad a = a_{\text{avg}}$$

a_{avg} = average acceleration from $t = 0$ to time $t > 0$:

$$a_{\text{avg}} = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t}, \qquad v_0 = \text{velocity at } t = 0$$

$$\boxed{v = v_0 + at}$$

Linear!



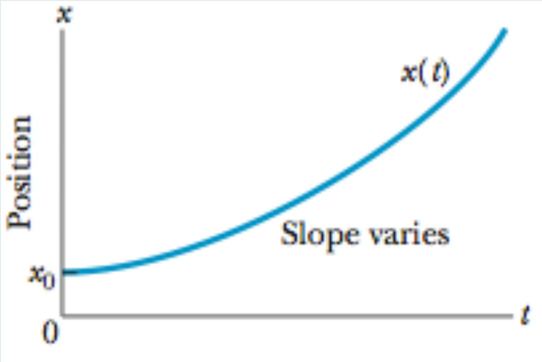
Average velocity from $t = 0$ to time t:

$$v \quad \textbf{Linear} \quad \Rightarrow \quad v_{\text{avg}} = \frac{v + v_0}{2} = v_0 + \frac{at}{2}$$

By definition

$$v_{\text{avg}} = \frac{x - x_0}{t - 0} = \frac{x - x_0}{t}, \qquad x_0 = \text{position at } t = 0.$$

$$\boxed{x = x_0 + v_0 t + \frac{at^2}{2}}$$



Third equation $v \leftrightarrow x$

$$v = v_0 + at \quad \Rightarrow \quad at = v - v_0$$

$$x = x_0 + v_0 t + at^2/2 \quad \Rightarrow \quad ax = ax_0 + v_0(at) + (at)^2/2$$

Substitution:

$$ax = ax_0 + v_0(v - v_0) + (v - v_0)^2/2$$ No t here!

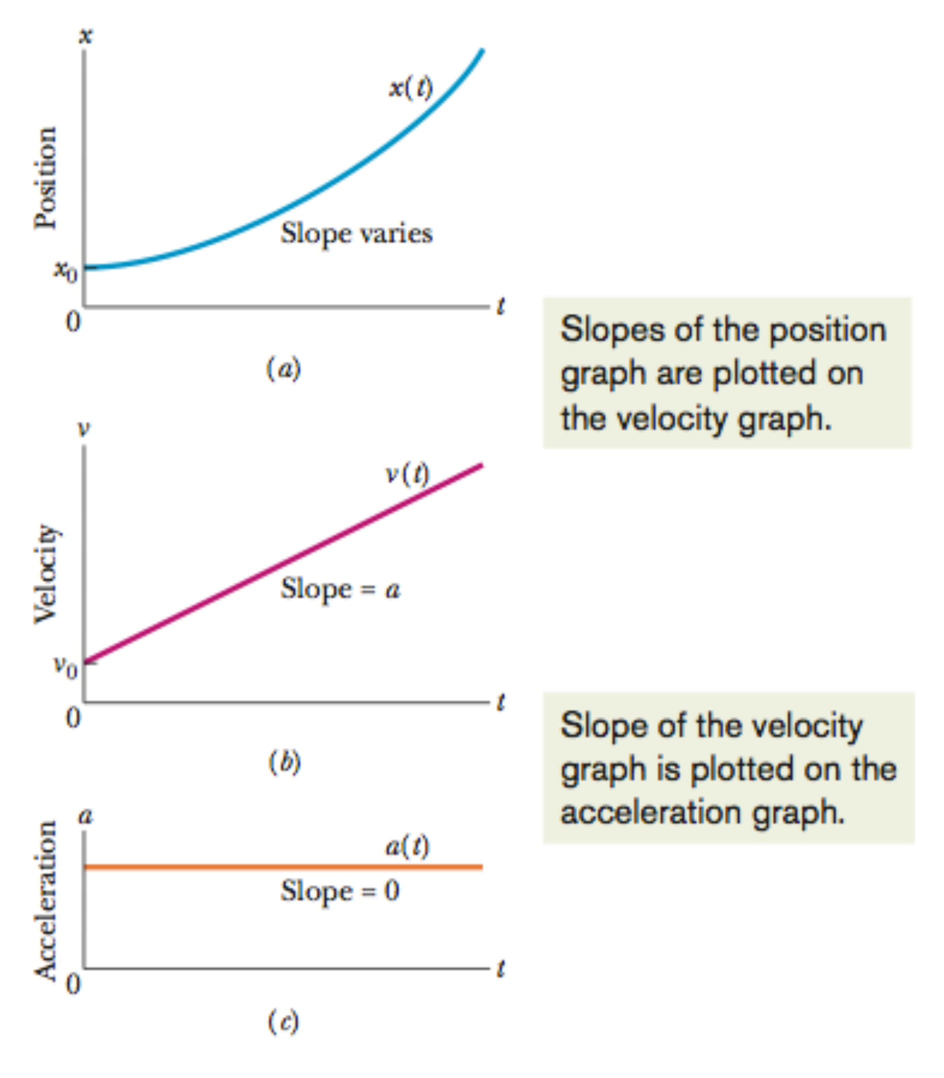
Algebra: $(v - v_0)^2 = v^2 + v_0^2 - 2vv_0$. Then

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

$$x = x_0 + v_0 t + \frac{at^2}{2}$$

$$v = v_0 + at$$

$$a = \text{constant}$$



Another useful formula:

$$v^2 = v_0^2 + 2a(x - x_0)$$

Warning!

The above equations are **not** valid if $a \neq$ **constant**.

For general motion with $a \neq$ constant:

$$v = \int a(t)dt + c, \qquad x = \int v(t)dt + c'$$

$\int f(t)dt =$ integral (anti-derivative) of $f(t)$

c, c' constants determined from initial conditions

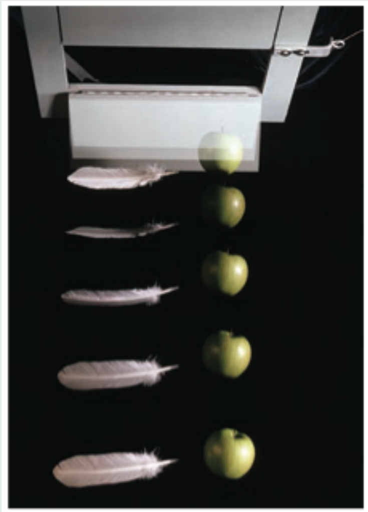
$$v(0) = v_0, \qquad x(0) = x_0.$$

Free-fall acceleration

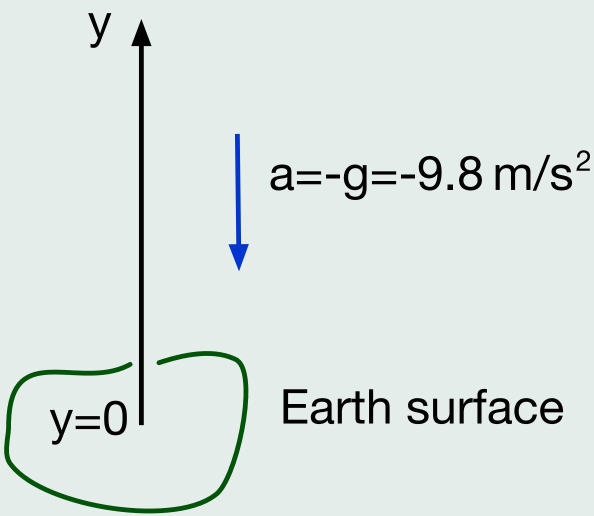
- **Free-fall:** motion of objects close to Earth's surface in absence of all external forces except for their weight.

- In vacuum all objects accelerate downwards at the same constant rate.

$$a_{\text{apple}} = a_{\text{feather}}$$



- Constant acceleration model



- $y =$ height with respect to Earth's surface.
- $a = -g = -9.8\,\mathrm{m/s^2}$ for all objects if air resistance is negligible.
- $g = 9.8\,\mathrm{m/s^2}$ magnitude of acceleration.

$$v = v_0 - gt \qquad y = y_0 + v_0 t - gt^2/2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

i-Clicker

An object is dropped from rest at time $t = 0$ and it falls freely with constant acceleration $a = -9.8\,\mathrm{m/s^2}$. This implies that:

(A) It falls 9.8 m during each second.

(B) It falls 9.8 m only during the first second.

(C) Its speed increases by $9.8\,\mathrm{m/s}$ during each second.

(D) Its speed increases by $9.8\,\mathrm{m/s}$ only during the first second.

(E) Its speed does not change.

Answer

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(E) Its speed does not change.

$a =$ constant $\Rightarrow \Delta v = a\Delta t$ for any time interval.

i-Clicker

Ignoring air resistance, if you drop an object, it accelerates downward at $9.8\,\mathrm{m/s^2}$. What will its acceleration be if instead you throw it down.

(A) $9.8\,\mathrm{m/s^2}$

(B) More than $9.8\,\mathrm{m/s^2}$.

(C) Less than $9.8\,\mathrm{m/s^2}$

(D) 0

(E) not constant

Answer

Ignoring air resistance, if you drop an object, it accelerates downward at $9.8\,\mathrm{m/s^2}$. What will its acceleration be if instead you throw it down.

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(C) Less than $9.8\,\mathrm{m/s^2}$

(D) 0

(E) not constant

The acceleration is independent of initial velocity.

Next class: Wednesday September 9th

Vectors

9th edition: Ch. 3.1-3.8

10th edition: Ch 3.1-3.3