Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 16

Midterm II

- Monday, November 14th, 1:55-2:50pm, Physics Lecture Hall

- Chapter 6: Force and Motion II (including friction)

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Chapter 9: COM. Linear momentum.
(**No rotation**)

- sample test and practice problems posted at

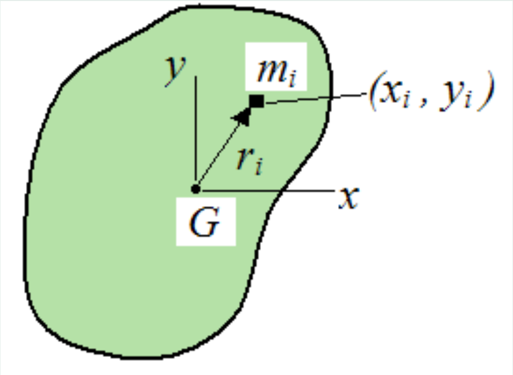
www.physics.rutgers.edu/ugrad/271/exams.html

Final Exam

- Wednesday, December 21st, 8-11am.

- **Rotational Inertia:**

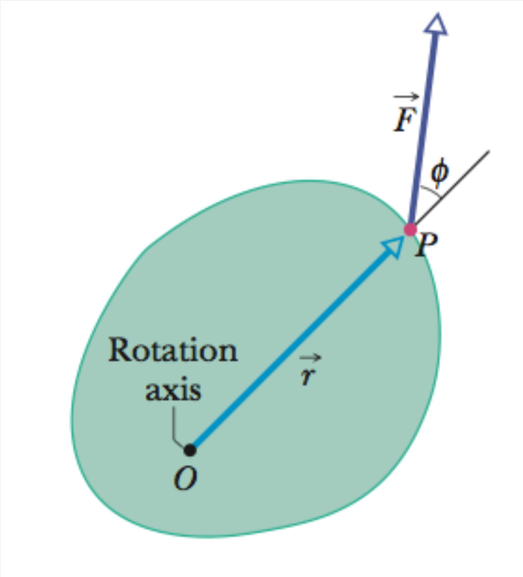
$$I = \sum_i m_i r_i^2 \qquad I = \int r^2 dm$$



- $\vec{r}_i$ defined relative to the **pivot** point where the rotation axis intersects the transverse section.

- **Kinetic energy of rotation:**

$$K = \frac{1}{2} I \omega^2$$



The torque due to this force causes rotation around this axis (which extends out toward you).

- **Newton's law for rotation:**

$$\tau_{\text{net}} = I\alpha$$

$$\vec{\tau}_{\text{net}} = \sum_i \vec{\tau}_i = \sum_i \vec{r}_i \times \vec{F}_i$$

$$\vec{\tau}_{\text{net}} = I\vec{\alpha}$$

- **Work-kinetic energy theorem for rotation**

$$\boxed{\Delta K = K_f - K_i = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2 = W}$$

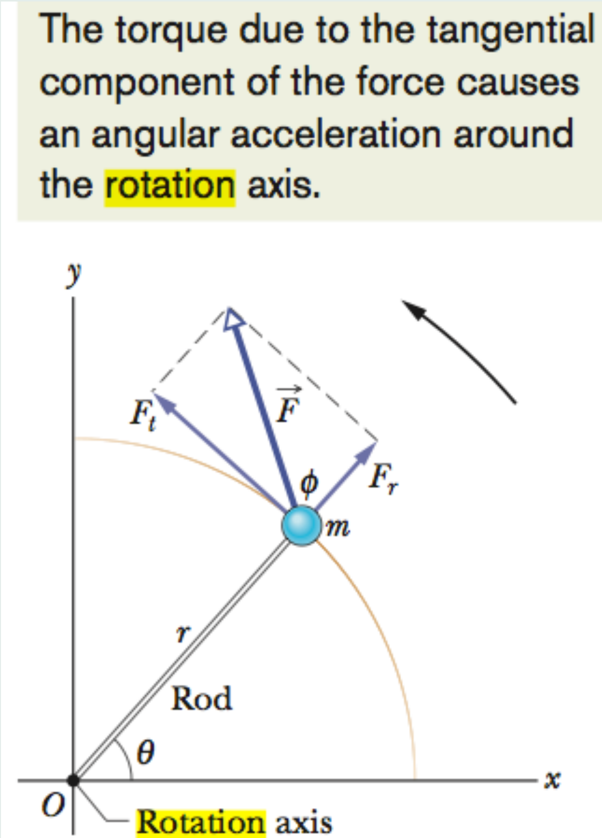
where

$$W = \int_{\theta_i}^{\theta_f} \tau_{\text{net}} d\theta$$

is the work done by the torque τ_{net}.

For **constant** τ_{net}

$$W = \tau_{\text{net}} \Delta\theta$$



- **Derivation:** assume the rod massless

$$dW = \vec{F} \cdot d\vec{s} = F_t ds$$

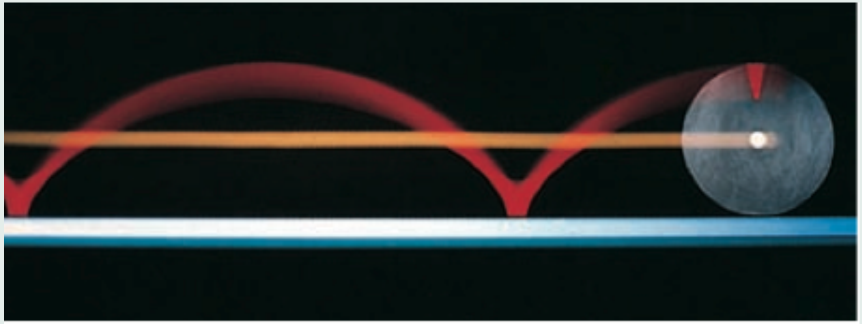
$$dW = rF_t d\theta = \tau d\theta$$

- **Power:**

$$P = \frac{dW}{dt} = \tau\omega$$

11. Rolling, torque and angular momentum

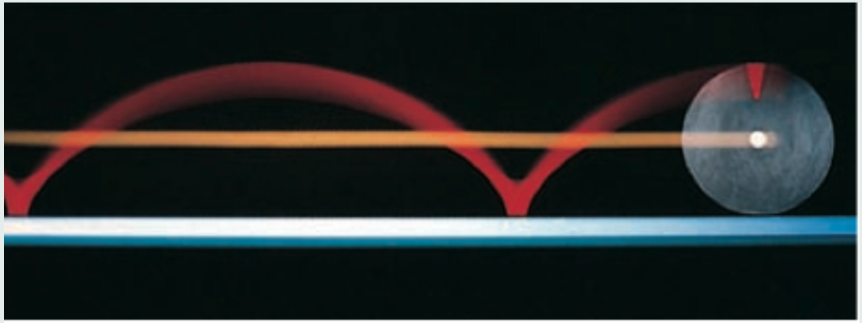
• Rolling as translation and rotation combined



• disk rolling smoothly along a horizontal surface (no sliding, bouncing)

• center of mass moves along a straight horizontal line

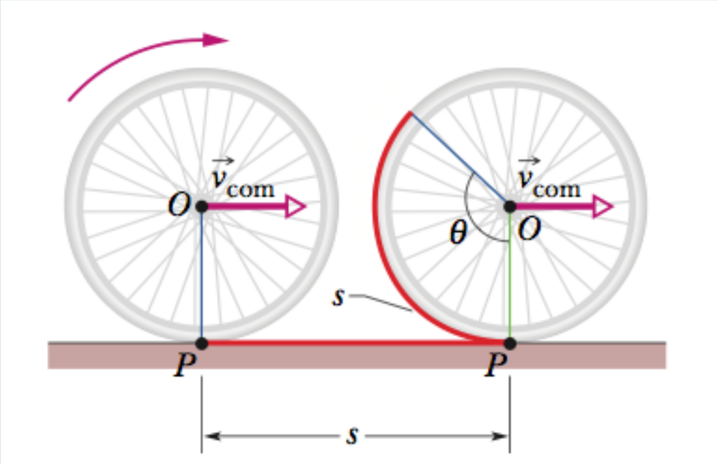
• points on the rim have a more complicated trajectory



The motion has two components:

COM translation	+	**Rotation around COM**

• **Smooth rolling** (**without sliding**)



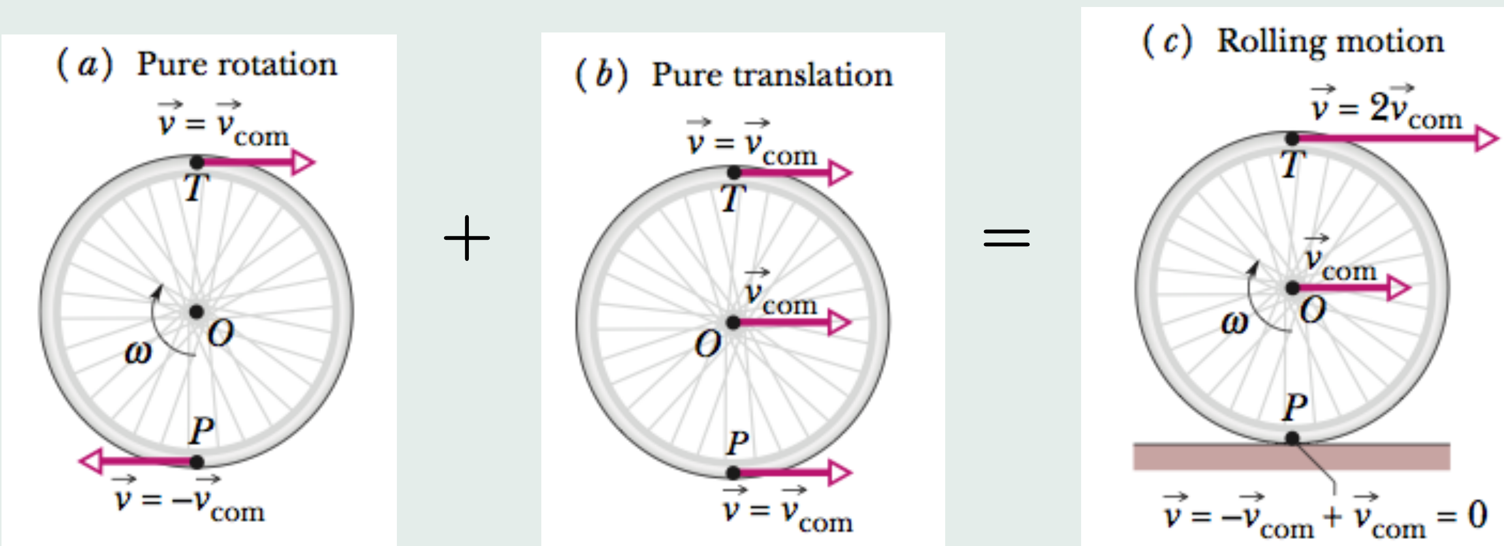
The center of mass O moves a distance s at velocity $\vec{v}_{\text{com}}$ while the wheel rotates through angle θ. The point of contact P also moves a distance s.

$$s = R\theta \Rightarrow \frac{ds}{dt} = R\frac{d\theta}{dt}$$

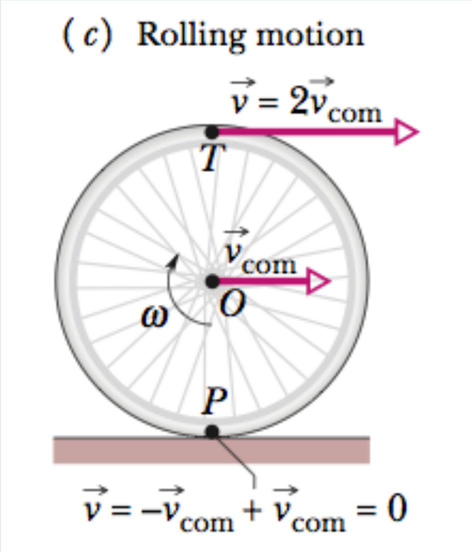
$$\boxed{v_{\text{com}} = R\omega}$$

Note : relation for **magnitudes**

COM translation + **Rotation around COM**



- **Rolling as pure rotation**



- At any time t rolling $=$ rotation about a horizontal axis passing through P, perpendicular to the plane of the wheel.

- This axis is called **instantaneous axis of rotation** since it **moves** with P.

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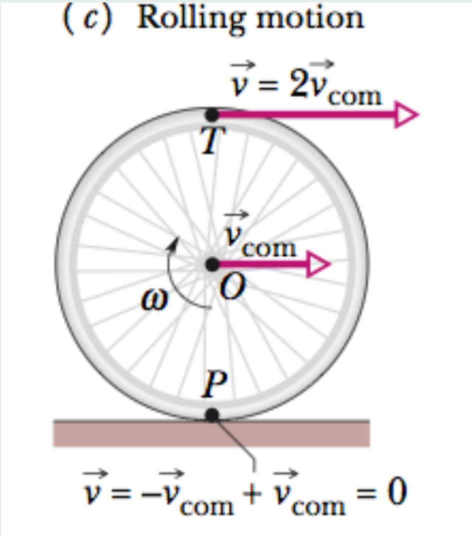

What is the angular speed of rotation about the instantaneous axis through P?

$A)$ ω

$B)$ 2ω

$C)$ $\omega/2$

$D)$ none of the above

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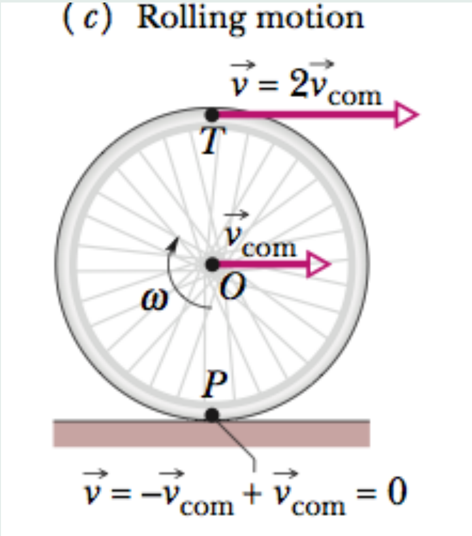


What is the angular speed of rotation about the instantaneous axis through P?

$A)\ \omega$

$B)\ 2\omega$

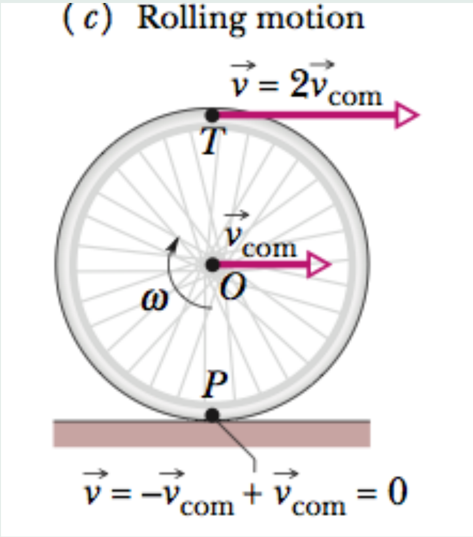
$C)\ \omega/2$

$D)$ none of the above

Note that the COM moves with speed v_{com} on a circle of radius R relative to P. Hence

$$\omega_{\text{com,P}} = \frac{v_{\text{com}}}{R} = \omega$$

- **Kinetic energy of rolling**



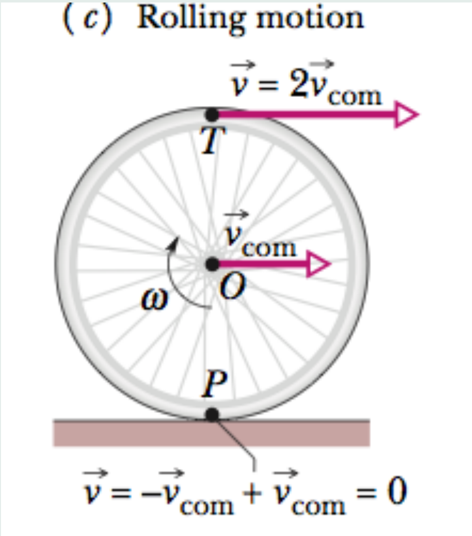
- **Method 1**: rolling = translation + rotation about COM

$$K = K_{\text{translation}} + K_{\text{rotation}}$$

$$K_{\text{translation}} = \frac{1}{2} M v_{\text{com}}^2$$

$$K_{\text{rotation}} = \frac{1}{2} I_{\text{com}} \omega^2$$

$$\boxed{K = \frac{1}{2} M v_{\text{com}}^2 + \frac{1}{2} I_{\text{com}} \omega^2}$$



- **Method 2**: rolling = instantaneous rotation about P

$$K = K_{\text{rotation,P}}$$

$$K_{\text{rotation,P}} = \frac{1}{2} I_P \omega^2$$

Parallel axis theorem:

$$I_P = I_{\text{com}} + MR^2$$

$$K_{\text{rotation,P}} = \frac{1}{2}(I_{\text{com}} + MR^2)\omega^2$$

$$\boxed{K = \frac{1}{2} M v_{\text{com}}^2 + \frac{1}{2} I_{\text{com}} \omega^2}$$

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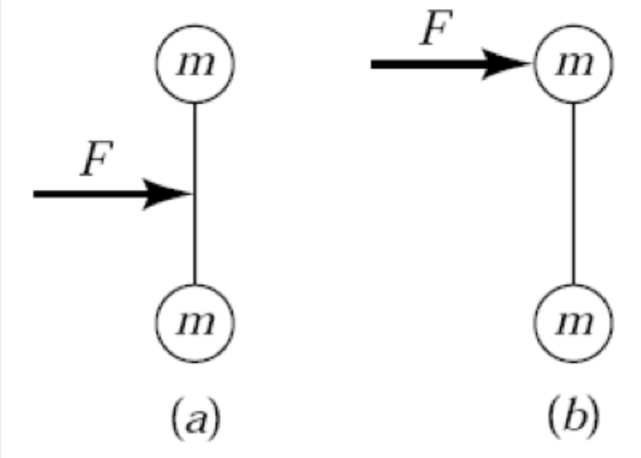

A force F is applied to a dumbbell for a time interval Δt, first as in (a) and then as in (b). In which case does the dumbbell acquire the greater energy?

$A)$ (a)

$B)$ (b)

$C)$ no difference

$D)$ depends on the inertia I_{dumbell}.

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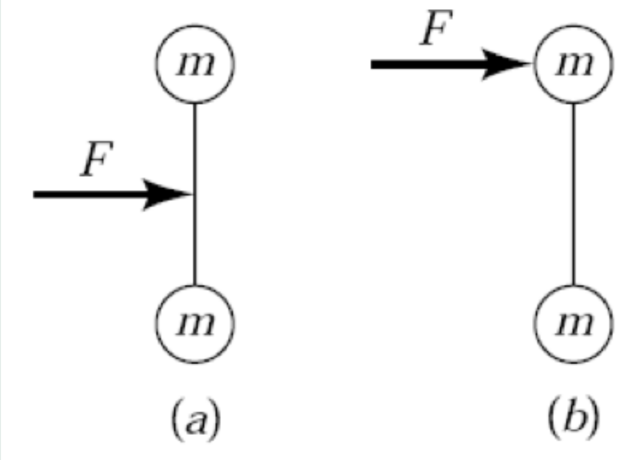

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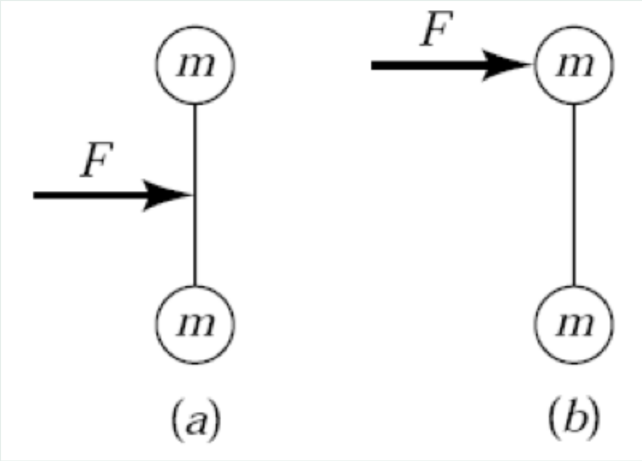
In both cases



$$F\Delta t = 2mv_{\text{com}}$$

$$v_{\text{com}} = \frac{F\Delta t}{2m}$$

$$K^{(a)}_{\text{translation}} = K^{(b)}_{\text{translation}}$$

$$K^{(a)}_{\text{rotation}} = 0$$

$$K^{(b)}_{\text{rotation}} > 0$$

$$K^{(a)}_{\text{total}} < K^{(b)}_{\text{total}}$$

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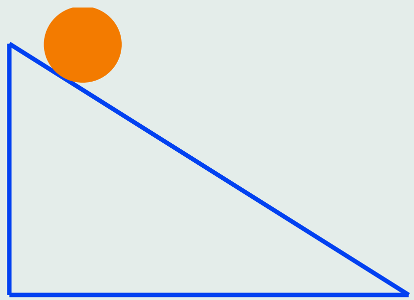

A cylinder and a sphere of the same mass and radius roll smoothly down an inclined plane starting from the same height with the same initial velocity of the center of mass. Which one will have a higher speed at the bottom of the slope?

$$I_{\text{cyl}} = \frac{1}{2}MR^2 \qquad I_{\text{sphere}} = \frac{2}{5}MR^2$$

$A)$ cylinder

$B)$ sphere

$C)$ same speed

$D)$ cannot be determined from the data

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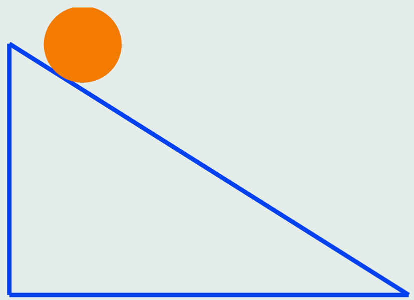

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No sliding $\Rightarrow$ mechanical energy conserved

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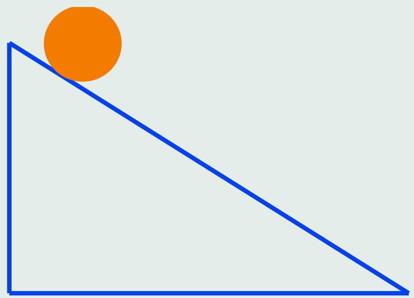


$$Mgh = \frac{1}{2} I_P \omega^2 \;\Rightarrow\; \omega = \sqrt{\frac{2Mgh}{I_P}}$$

$$I_P = I + MR^2$$

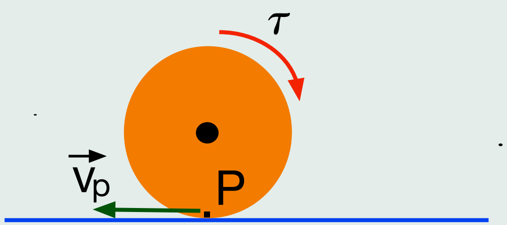
$$\omega_{\text{cylinder}} = \frac{2}{R}\sqrt{\frac{gh}{3}}$$

$$\omega_{\text{sphere}} = \frac{1}{R}\sqrt{\frac{10gh}{7}}$$

$$v_{\text{cylinder}} = R\omega_{\text{cylinder}} \qquad v_{\text{sphere}} = R\omega_{\text{sphere}}$$

$$\frac{v_{\text{cylinder}}}{v_{\text{sphere}}} = 2\sqrt{7/30} < 1$$

- **The forces of rolling. Friction and rolling**



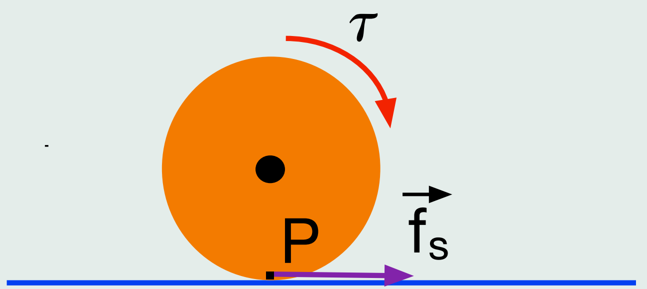
- An external torque is applied to a stationary wheel on a smooth surface

- Will it start rolling?

$$I\frac{d\omega}{dt} = \tau \;\Rightarrow\; \alpha = \frac{\tau}{I} \qquad F = M\frac{dv_{\text{com}}}{dt} = 0 \;\Rightarrow\; a_{\text{com}} = 0$$

No rolling! Rotation about COM!

Relative motion between the point P and the surface.



What opposes the motion of P relative to the surface?

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Friction

- Supose **No sliding** $\Rightarrow$ **Static friction force** pushing the wheel **forward**

$$v_{\rm com} = \omega R \;\Rightarrow\; a_{\rm com} = \alpha R$$

$$m a_{\rm com} = f_s, \qquad \tau - f_s R = I_{\rm com}\alpha$$

$$a_{\rm com} = \frac{\tau R}{I_{\rm com} + mR^2}, \qquad f_s = \frac{m\tau R}{I_{\rm com} + mR^2}$$

$$f_s \le \mu_s m g \;\Rightarrow\; \tau \le \mu_s g \frac{I_{\rm com} + mR^2}{R}$$

- What if

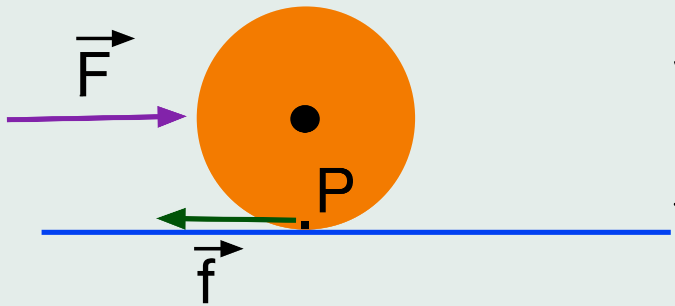
$$\tau > \mu_s g \frac{I_{\text{com}} + mR^2}{R}?$$

Then **Sliding** $\Rightarrow$ **Kinetic friction force**

$$a_{\text{com}} \neq R\alpha$$

$$ma_{\text{com}} = \mu_k mg \;\Rightarrow\; a_{\text{com}} = \mu_k g.$$

$$\tau - \mu_k mgR = I_{\text{com}}\alpha \;\Rightarrow\; \alpha = \frac{\tau - \mu_k mgR}{I_{\text{com}}}$$



What happens if we act with a horizontal force aligned with the center of mass?

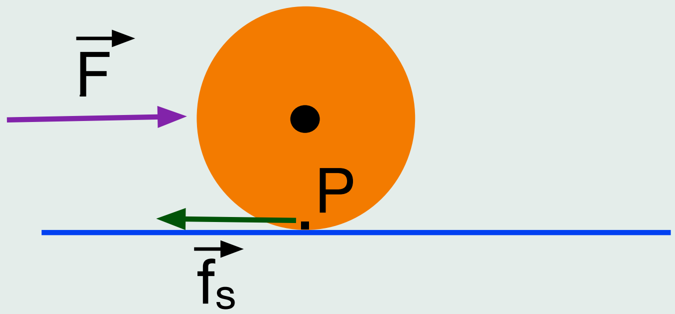
- **No friction** $\mu_s = 0 \Rightarrow a_{\text{com}} = F/M$

  $\tau_{\text{com}} = 0 \;\Rightarrow\; \alpha = 0$ **No rolling. Sliding only.**

- Suppose $\mu_s \neq 0 \Rightarrow$ static friction $\vec{f_s}$ opposes sliding. Will the wheel roll? Suppose it does.

$$Ma_{\text{com}} = F - f_s \qquad FR = I_P\alpha \qquad a_{\text{com}} = \alpha R$$

$$f_s = F\left(1 - \frac{MR^2}{I_P}\right) = F\left(1 - \frac{MR^2}{I_{\text{com}} + MR^2}\right)$$



$$f_s = F\frac{I_{com}}{I_{com} + MR^2}$$

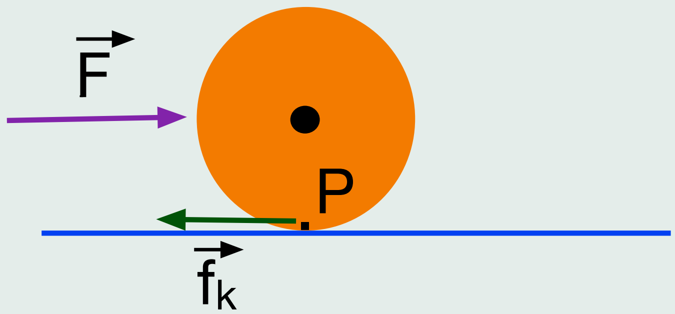
$$f_s \le \mu_s mg$$

$$\mu_s \ge \frac{F}{mg}\frac{I_{com}}{I_{com} + MR^2}$$

$$\mu_s \ge \frac{F}{mg}\frac{I_{com}}{I_{com} + MR^2} \quad \Rightarrow \quad \textbf{rolling}$$

$$a_{com} = \frac{R^2}{I_{com} + MR^2}F \qquad \alpha = \frac{a_{com}}{R}$$

- **Note:** α the magnitude of angular acceleration



$$\mu_s < \frac{F}{mg} \frac{I_{\text{com}}}{I_{\text{com}} + MR^2} \Rightarrow$$

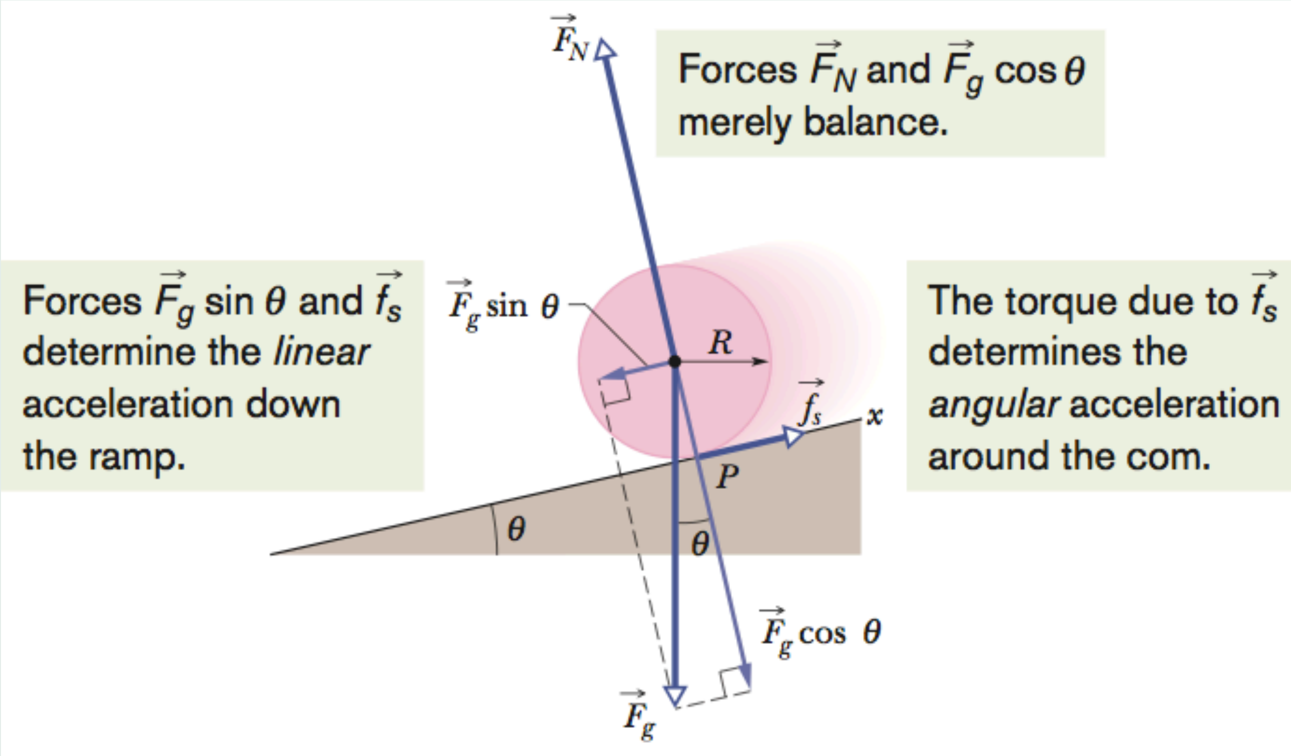
sliding at contact surface

friction must be **kinetic** friction

$$a_{\text{com}} = \frac{F - \mu_k Mg}{M}$$

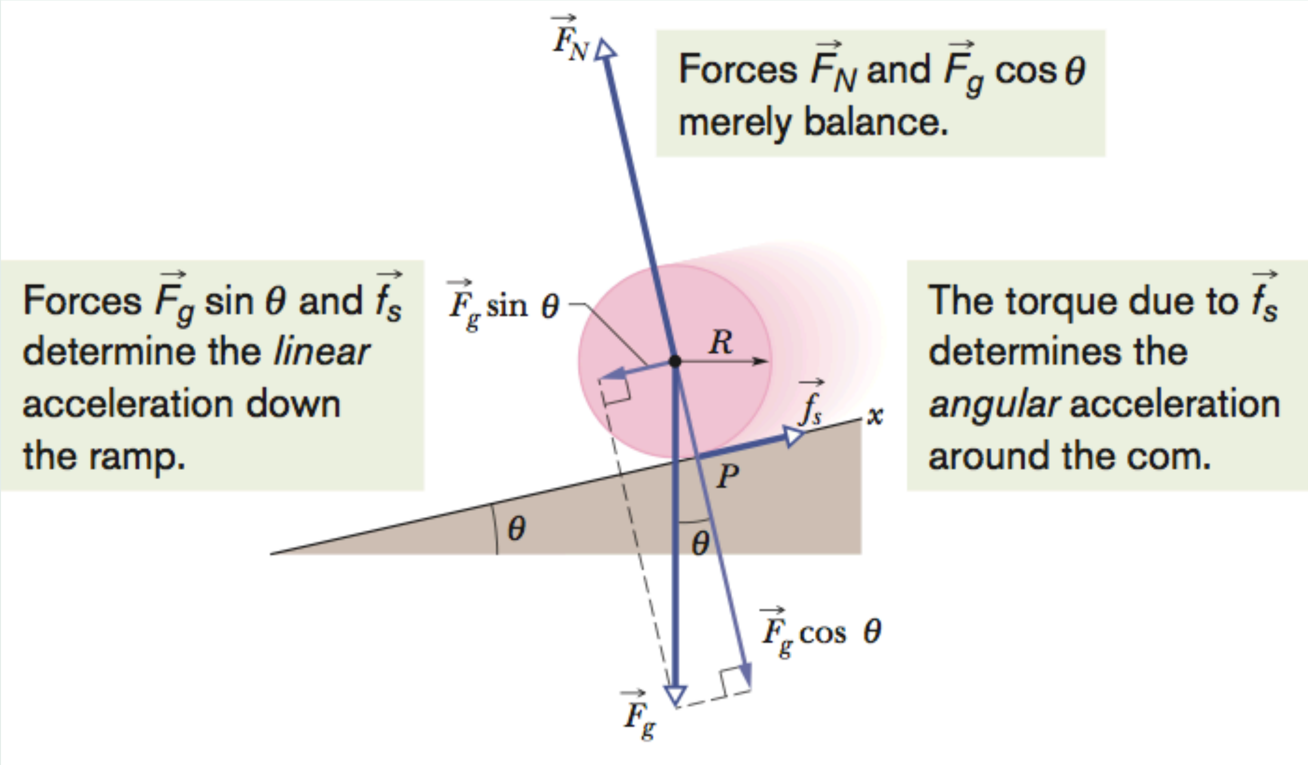
$$\tau_{f_k} = I_{\text{com}}\alpha \;\Rightarrow\; \alpha = \frac{\mu_k MgR}{I_{\text{com}}}$$

$$a_{\text{com}} \neq \alpha R$$



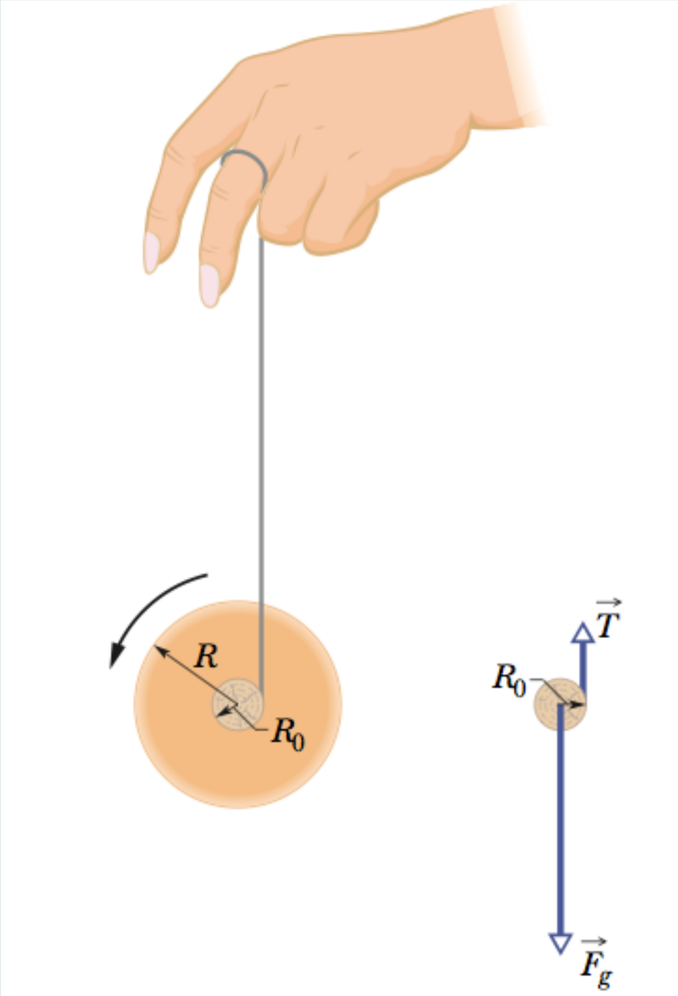
$$Ma_{\text{com,x}} = f_s - Mg\sin\theta \qquad I_{\text{com}}\alpha = \tau_{f_s} = f_s R$$

$$a_{\text{com,x}} = -\alpha R$$



$$-MR\alpha = \frac{I_{\text{com}}}{R}\alpha - Mg\sin\theta \;\Rightarrow\; \alpha = \frac{MgR\sin\theta}{I_{\text{com}} + MR^2}$$

$$a_{\text{com,x}} = -\alpha R$$

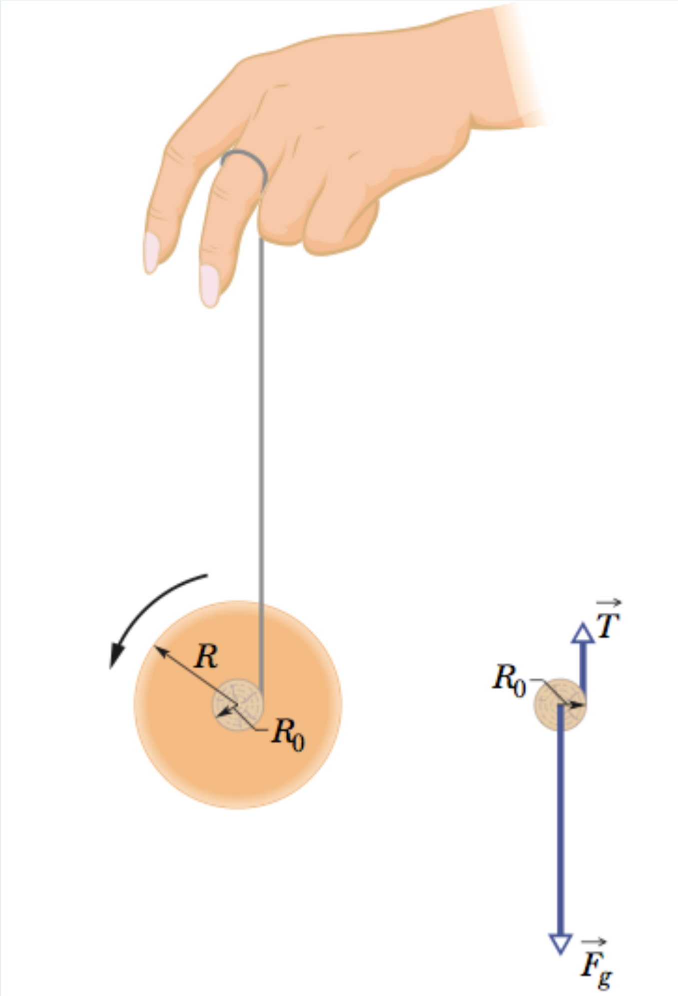


Yo-yo

1. Instead of rolling down a ramp, the yo-yo rolls down a string at angle $\theta = 90^\circ$ with the horizontal.

2. Instead of rolling on its outer surface at radius R, the yo-yo rolls on an axle of radius R_0.

3. Instead of being slowed by frictional force f_s, the yo-yo is slowed by the tension $\vec{T}$ in the string



Ramp

$$a_{\text{com},x} = -\frac{MgR^2\sin\theta}{I_{\text{com}} + MR^2}$$

$$\Downarrow$$

Yo-yo

$$a_{\text{com},y} = -\frac{MgR_0^2}{I_{\text{com}} + MgR_0^2}$$