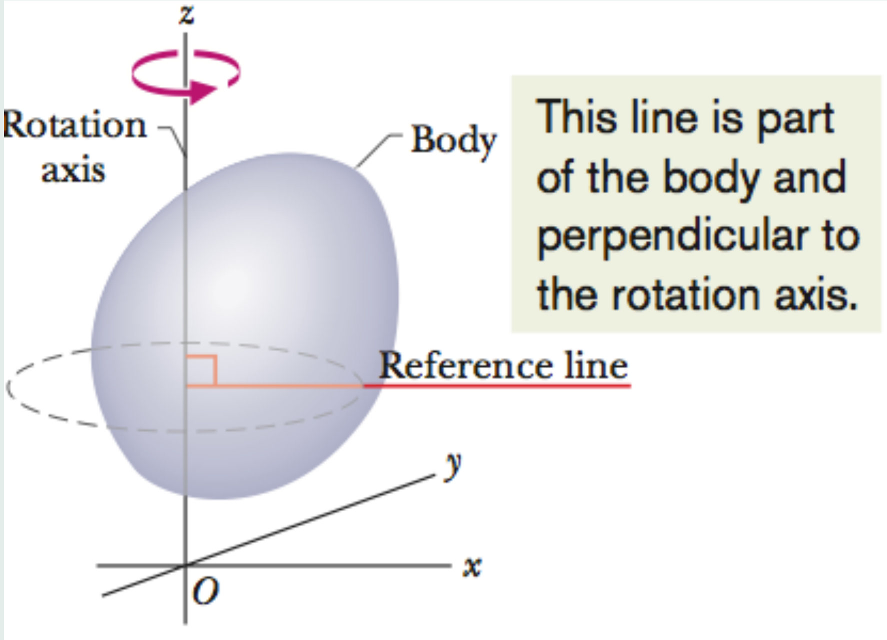
Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 15

10. Rotation

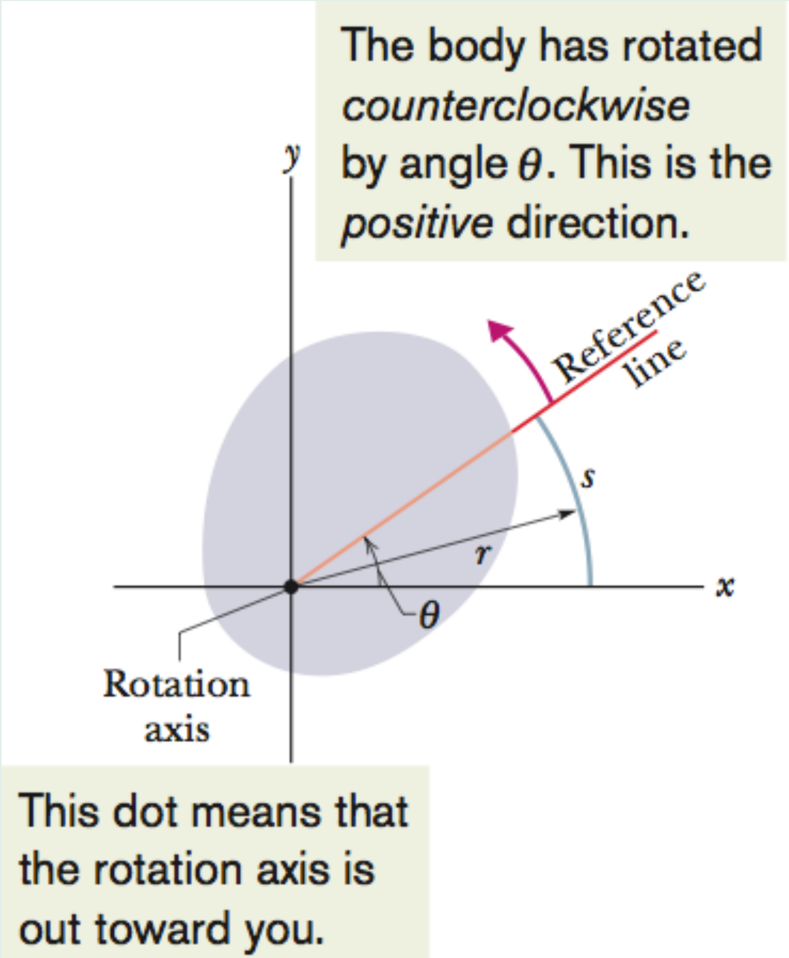
- **Rotation variables**



- **rotation axis**: **fixed** axis
- **reference line:**

(a) **perpendicular** to rotation axis.

(b) **rotates with the body**



- **Angular position**:

$$\theta = \frac{s}{r}$$

s length of circular arc

r radius of circle

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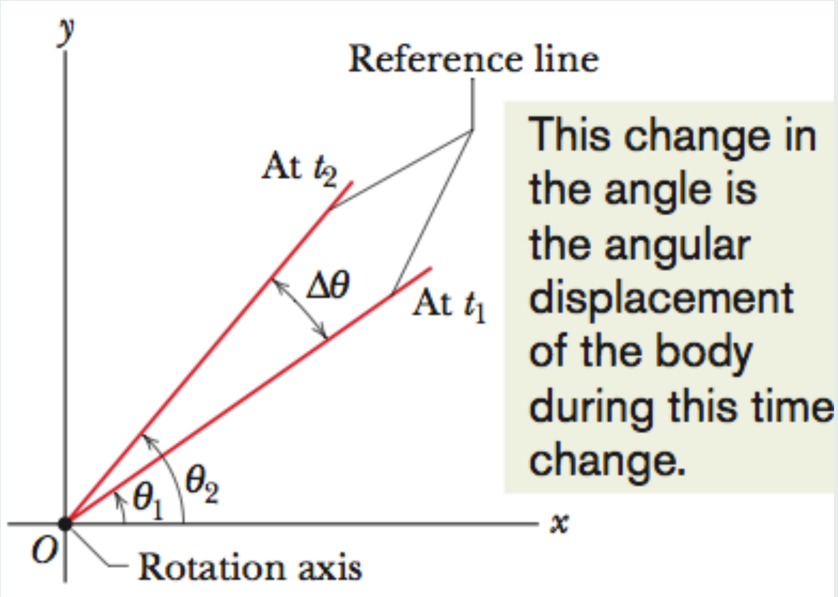
- **Angular velocity and acceleration**



- **Angular velocity**

$$\omega = \frac{d\theta}{dt}$$

- **Angular acceleration**

$$\alpha = \frac{d\omega}{dt}$$

- **Constant angular acceleration**

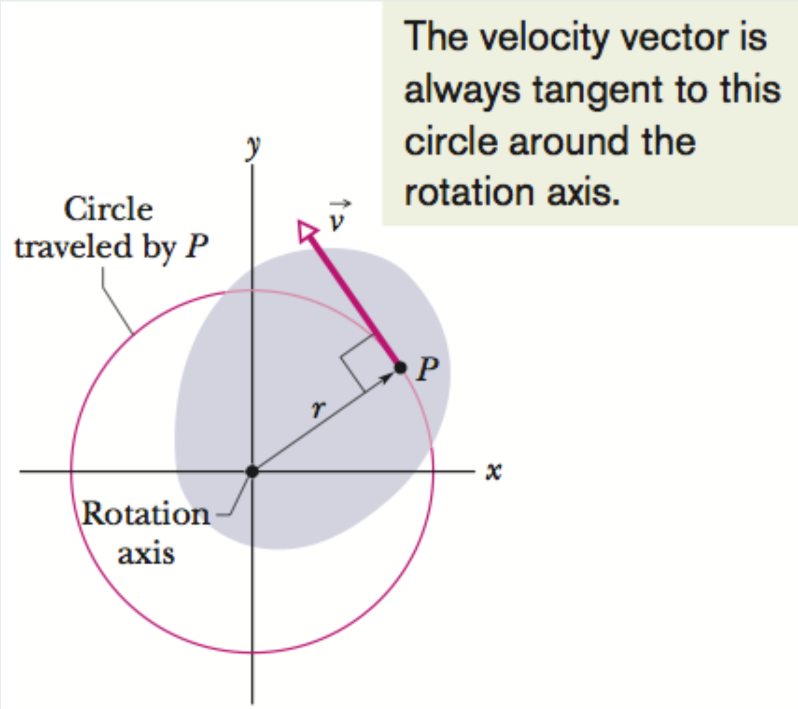
Table 10-1

Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable		Angular Equation
(2-11)	$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
(2-15)	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
(2-18)	$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Constant **angular** acceleration $\leftrightarrow$ Constant **linear** acceleration

- **Relating linear and angular variables**



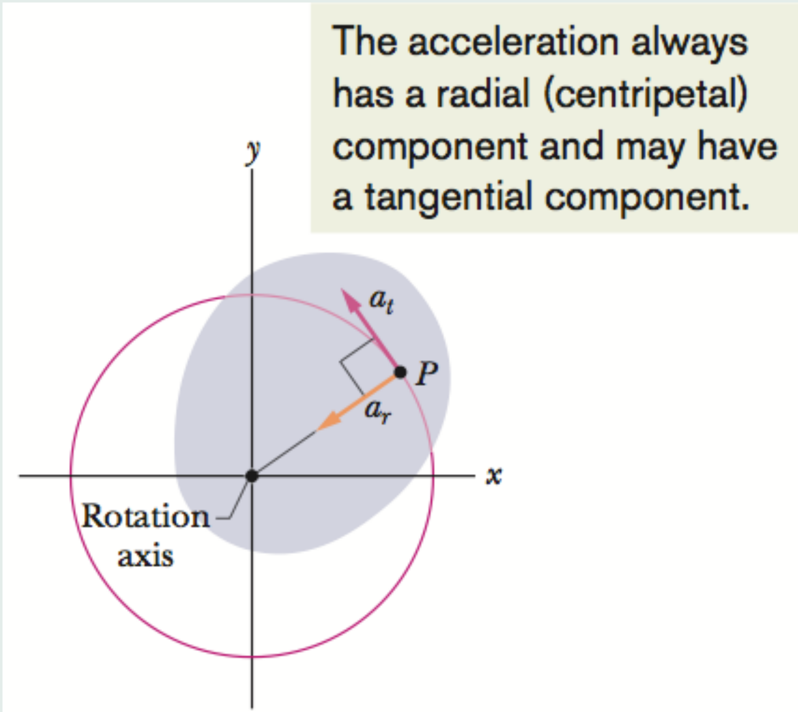
- Each point P moves on a circular trajectory of radius r in a plane perpendicular to the rotation axis.
- Distance along circular arc:

$$s = r\theta \qquad (\text{radians!})$$

- Speed:

$$v = r\omega$$

- Period of revolution: $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$



- **Acceleration**

$$\vec{a} = \frac{d\vec{v}}{dt}$$

- **Tangential acceleration:**

$$a_t = \alpha r$$

- **Centripetal acceleration**

$$a_r = -\frac{v^2}{r} = -\omega^2 r$$

- $\vec{a}_t$ tangent to trajectory, encodes the rate of variation of the magnitude of $\vec{v}$
- $\vec{a}_r$ perpendicular to trajectory, encodes the rate of variation of the direction of $\vec{v}$

- **Kinetic energy of rotation**

Treating the body as a collection of particles with different speeds, we have

$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \cdots = \sum_i \frac{1}{2}m_i v_i^2$$

$$K = \sum_i \frac{1}{2}m_i \omega^2 r_i^2 = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

- **Rotational Inertia** (moment of inertia)

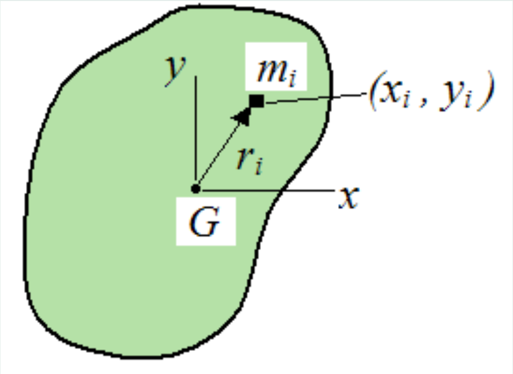
$$I = \sum_i m_i r_i^2 \;\Rightarrow\; K = \frac{1}{2}I\omega^2$$

- **Continuous mass distribution**

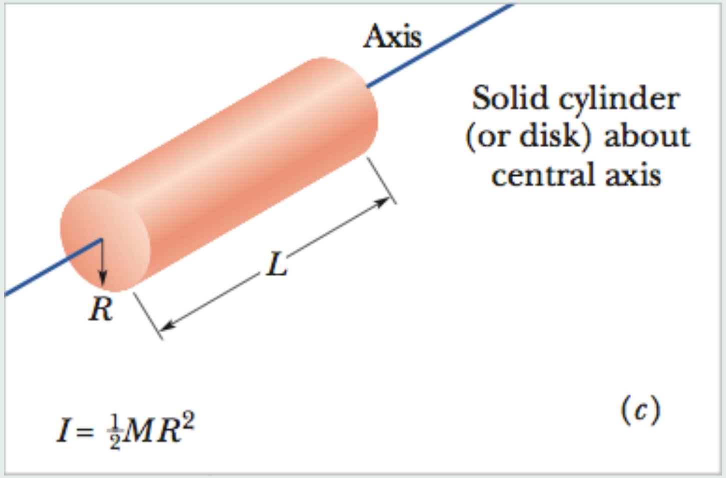
$$I = \int r^2 dm = \int r^2 \rho dv = \int r^2 \rho dxdydz$$

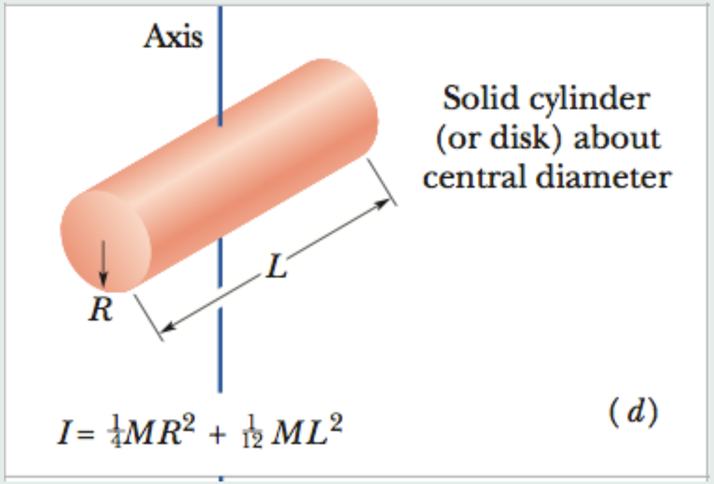
r = distance between the volume element dv and rotation axis. If the rotation axis is the z-axis, then

$$r = \sqrt{x^2 + y^2}$$



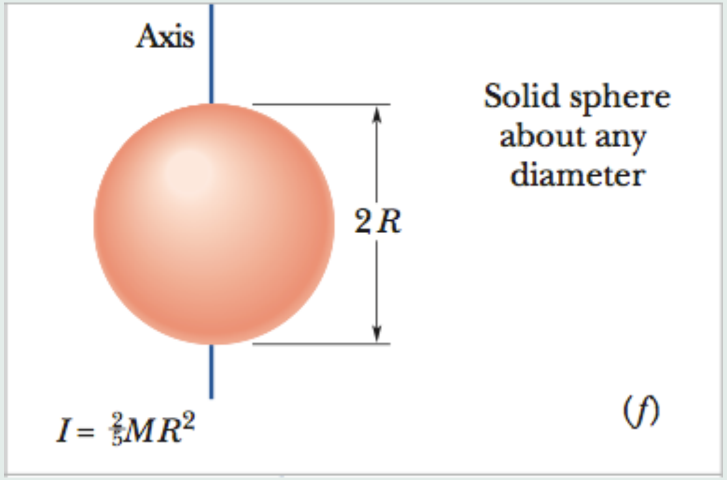
Examples:

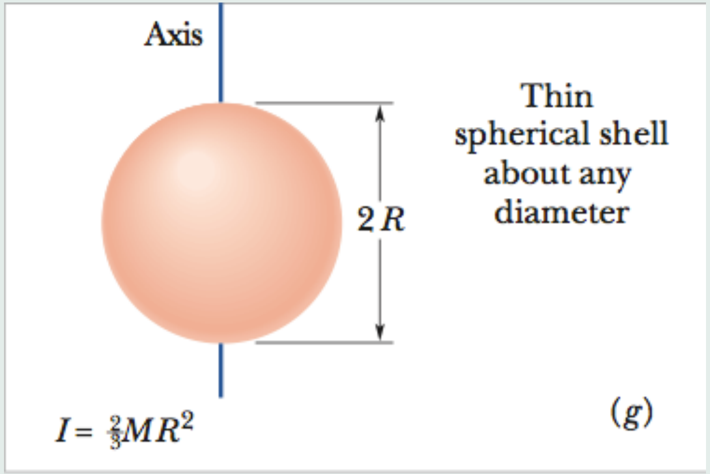




Note:

- I depends on the axis of rotation!
- $I \sim$ measures how hard it is to spin an object


Axis
Solid sphere
about any
diameter
2R
I = ⅖MR²
(f)


Axis
Thin
spherical shell
about any
diameter
2R
I = ⅔MR²
(g)

i-Clicker

To decrease the rotational inertia of an object about an axis, you must:

A) decrease ω

B) decrease α

C) deform the object such that more mass is placed closer to the axis

D) deform the object such that more mass is placed further from the axis

Answer

To decrease the rotational inertia of an object about an axis, you must:

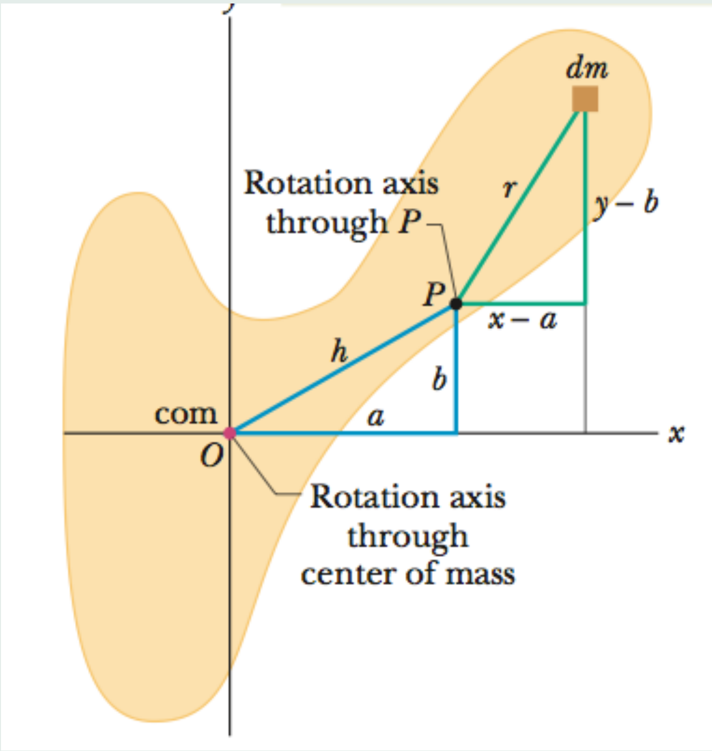
A) decrease ω

B) decrease α

C) deform the object such that more mass is placed closer to the axis

D) deform the object such that more mass is placed further from the axis

Parallel Axis Theorem



$$\boxed{I_P = I_{\text{com}} + Mh^2}$$

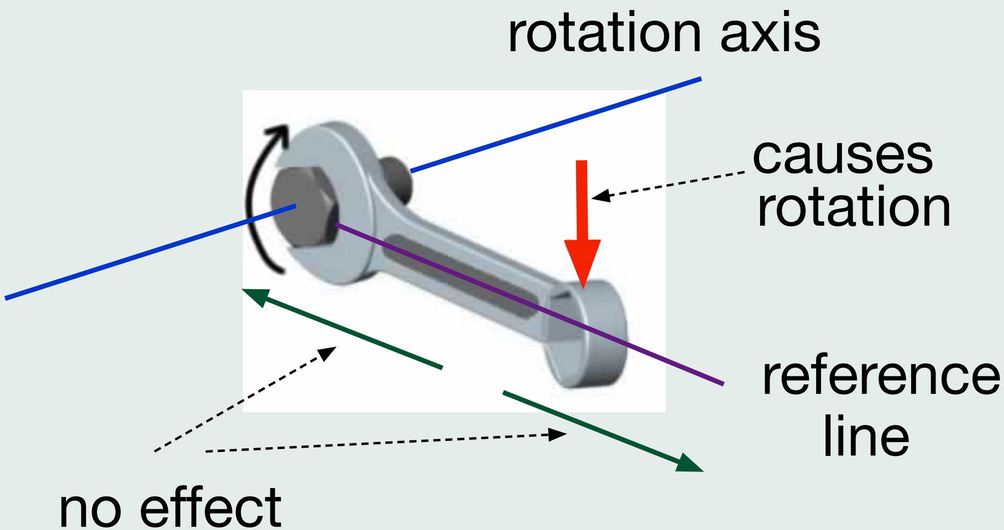
I_P = rotational inertia about axis through P

I_{com} = rotational inertia about parallel axis through COM

h = distance between axis through P and parallel axis through COM.

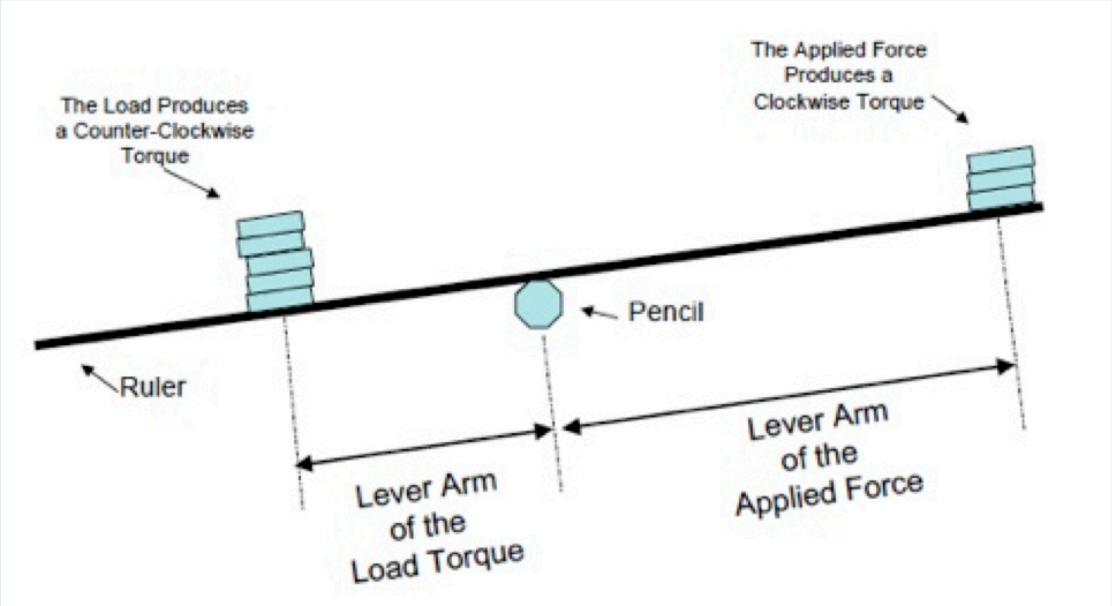
M = total mass of the object

What causes rotation?



In order to cause rotation the applied force must be **perpendicular** to the plane formed by the **rotation axis** and the **reference line**.

Who wins?

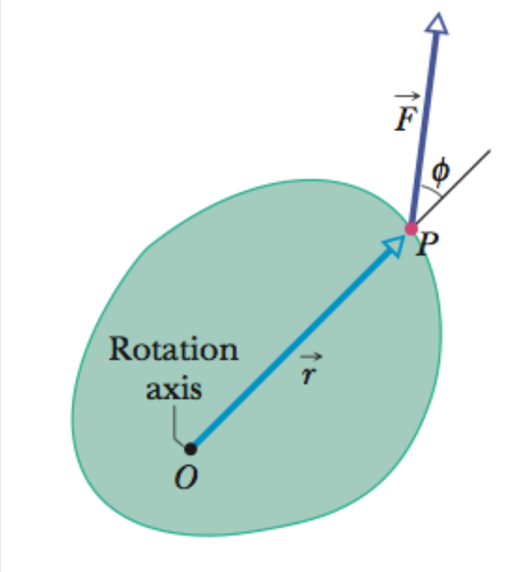


The winner is the side with the larger product

$$\textbf{weight} \times \textbf{length of lever arm}$$

not the larger **weight.**

- **Torque**

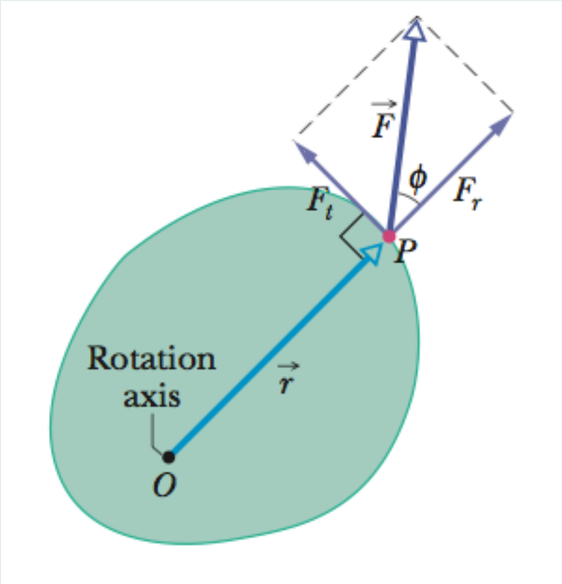


The torque due to this force causes rotation around this axis (which extends out toward you).

- horizontal section through a rotating body
- rotation axis vertical, $\perp$ to page
- applied force is contained in the horizontal plane

$$\vec{F} \cdot \vec{k} = 0$$

- **Pivot**: the point O of intersection of the rotation axis with the horizontal plane.



But actually only the *tangential* component of the force causes the rotation.

- **Torque:**

$$\tau = rF_t = rF\sin\phi$$

- $F_t = F\sin\phi$ the **tangential** component of F **causes rotation**

- $F_r = F\cos\phi$ the **radial** component of F has **no rotation effect.**

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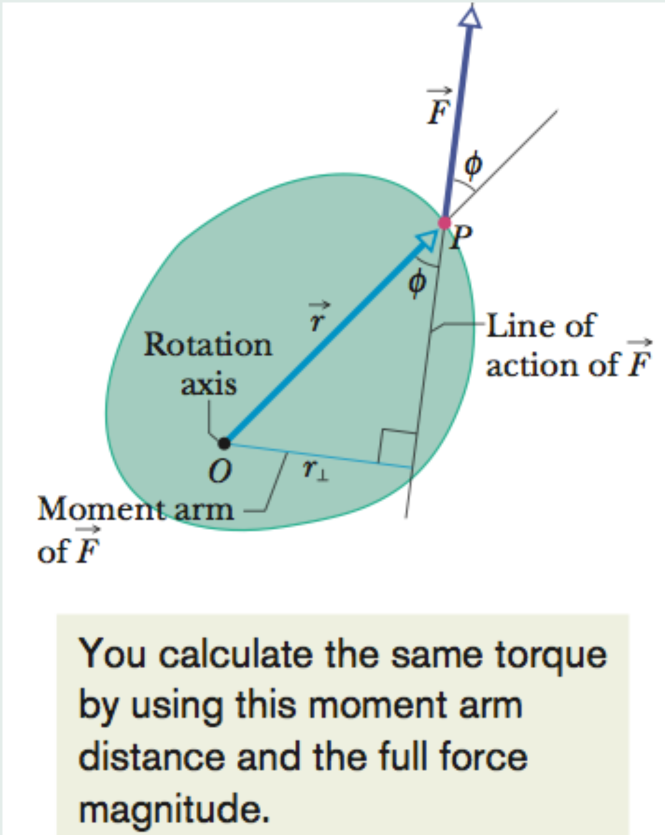


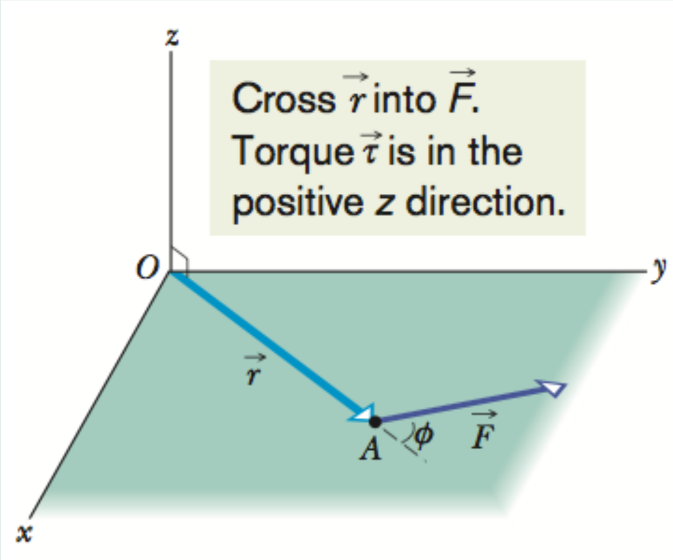
- **line action** of $\vec{F}$: the line in the horizontal plane containing $\vec{F}$.

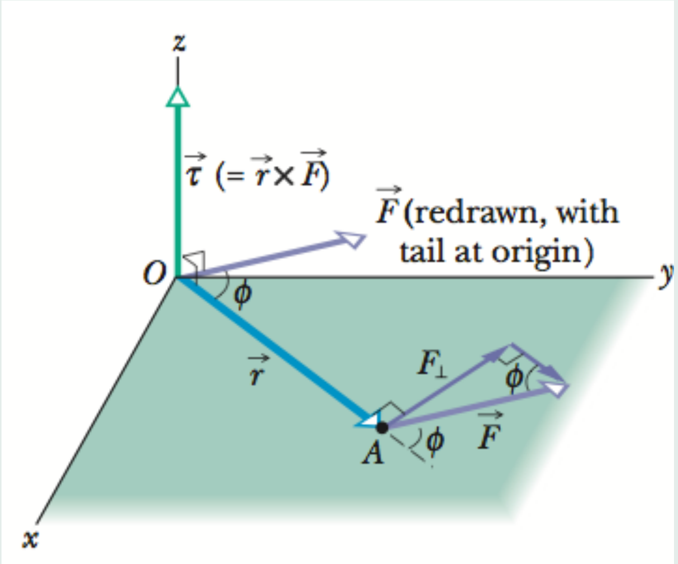
- **moment arm** of $\vec{F}$: $r_\perp =$ the distance from the pivot to the **line action**

- $\tau = rF_t = rF\sin\phi = r_\perp F$

- **Note:** $\tau > 0$ if $\vec{F}$ rotates the body **counterclockwise** and $\tau < 0$ if $\vec{F}$ rotates the body **clockwise**.

- **Torque as a vector**





$$\vec{\tau} = \vec{r} \times \vec{F} \qquad |\vec{\tau}| = rF\sin\phi$$

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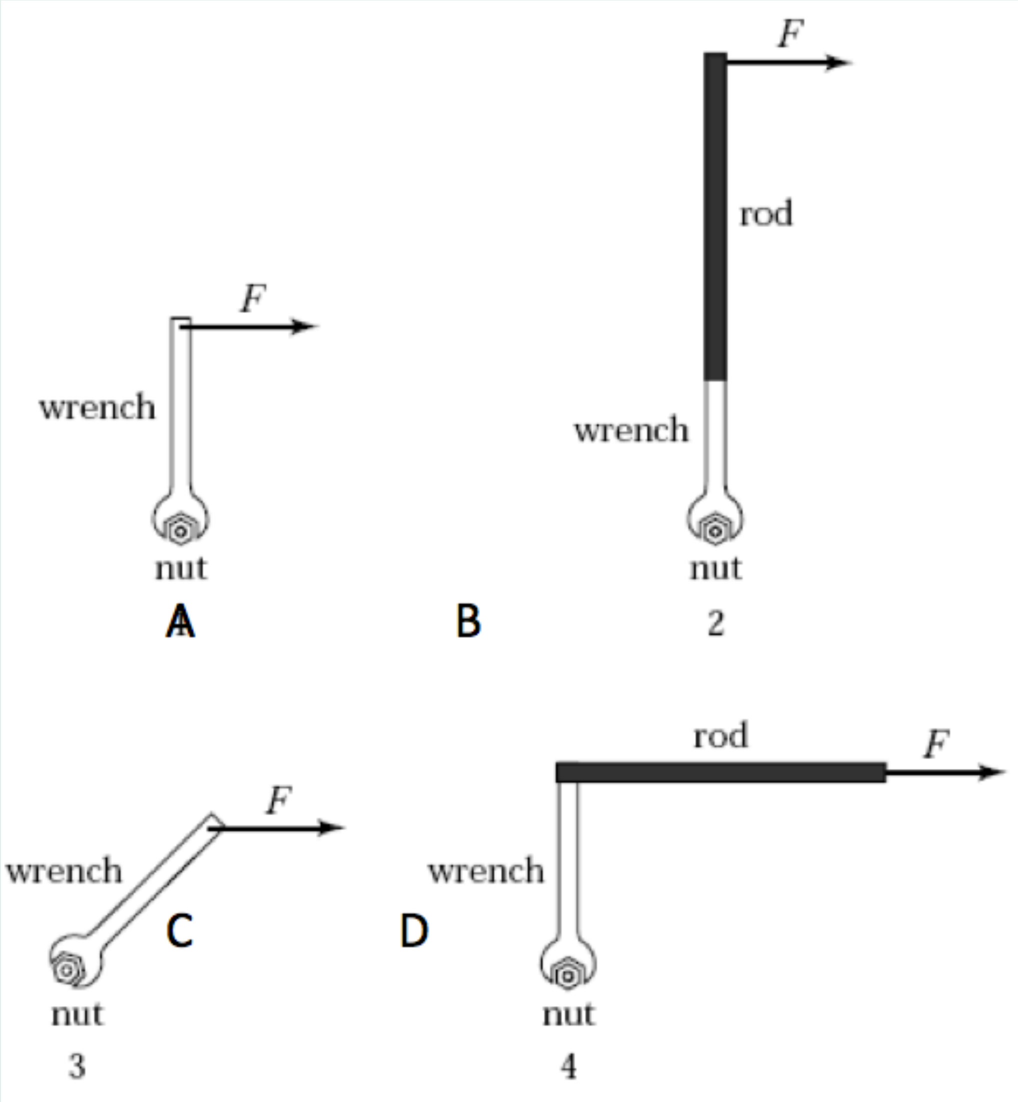
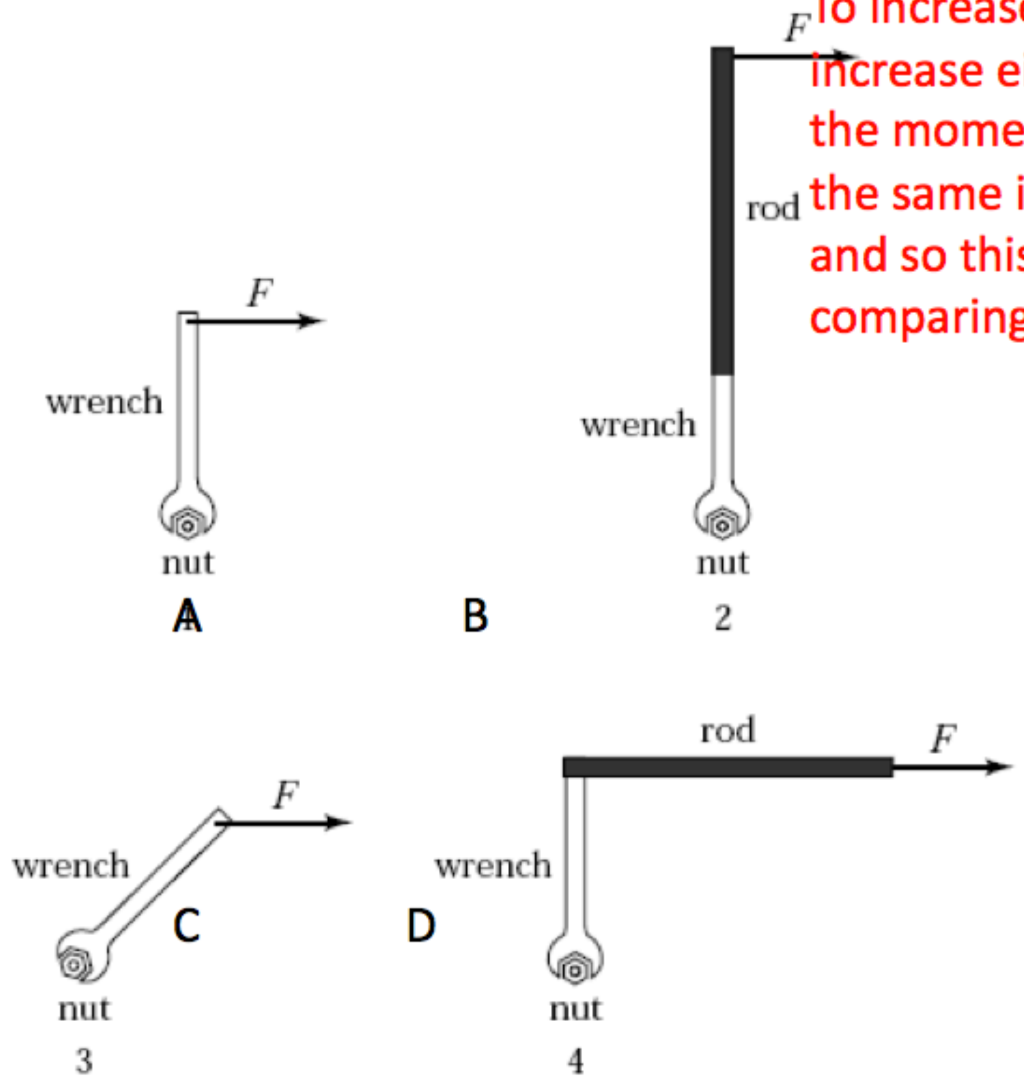
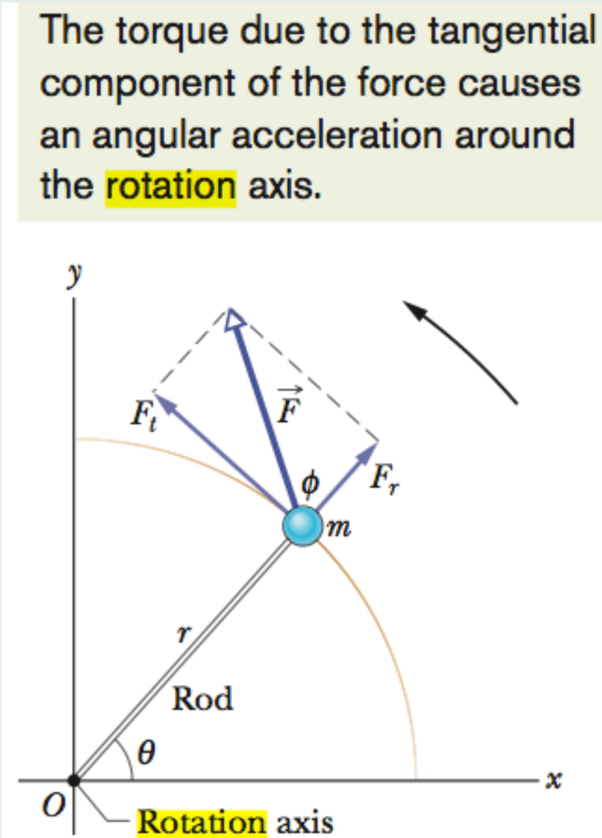
You are using a wrench to loosen a rusty nut. Which arrangement is the most effective, assuming the same force.



To increase a torque, you can increase either the applied for the moment arm. Here the for the same in all four situations, and so this question boils dow comparing moment arms.



- **Newton's Law of Rotation**



$$\boxed{\tau_{\text{net}} = I\alpha}$$

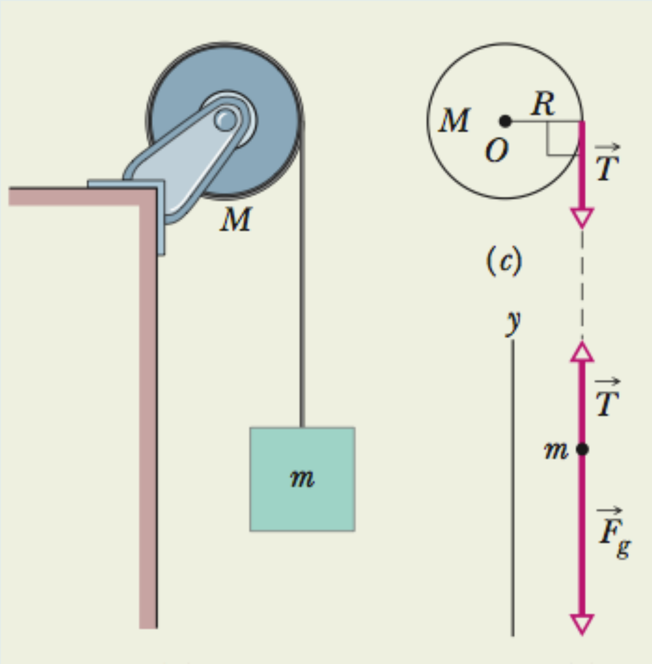
$$\tau_{\text{net}} = \sum_i \tau_i$$

Derivation: assume the rod massless

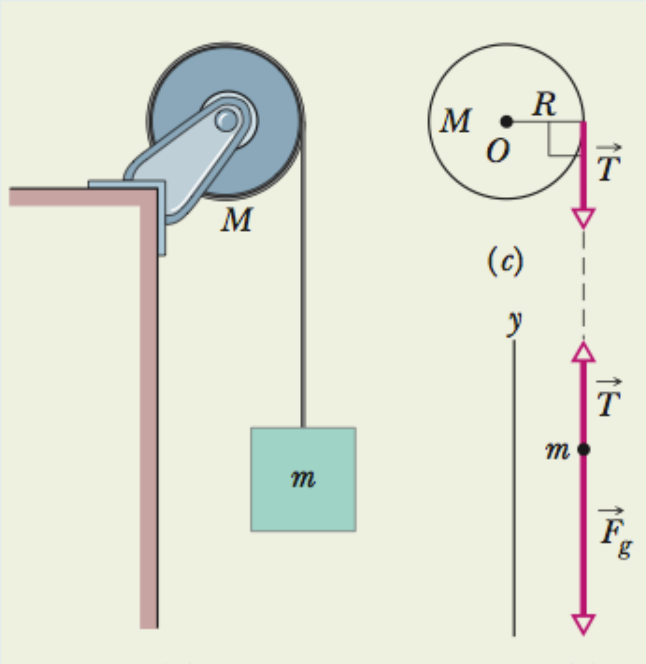
$$F_t = ma_t = mr\alpha$$

$$rF_t = mr^2\alpha = I\alpha$$

- **Example:**



- A uniform disk, with mass $M = 2.5\,\mathrm{kg}$ and radius $R = 20\,\mathrm{cm}$ is mounted on a fixed horizontal axle.
- A block with mass $m = 1.2\,\mathrm{kg}$ hangs from a massless cord that is wrapped around the rim of the disk.
- Find the acceleration of the falling block, the angular acceleration of the disk and the tension in the cord.



- Newton's 2nd law for block:

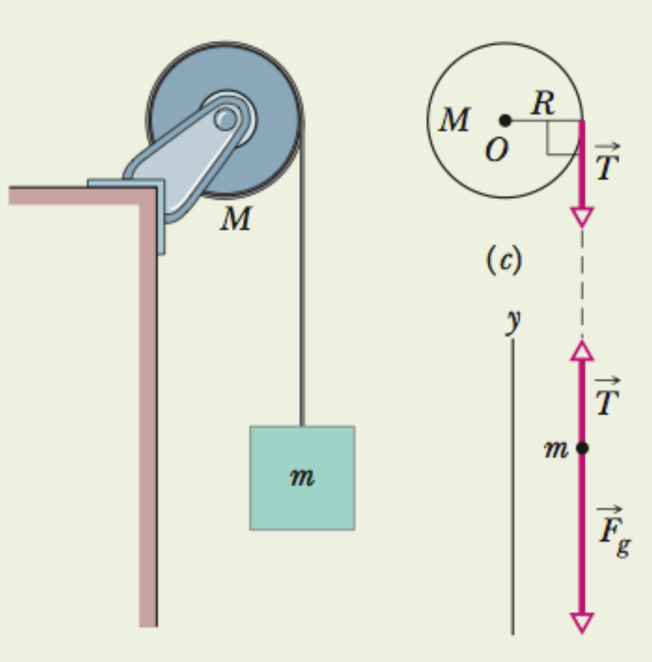
$$-m|a_y| = -mg + T$$

- Newton's rotation law for disk

$$-I|\alpha| = \tau = -TR$$

- **Note:** $\tau < 0$, $\alpha < 0$ since $\vec{T}$ rotates the disk in clockwise direction; $a_y < 0$ downward motion

- Kinematic constraint: $|a_y| = R|\alpha|$.



$$a_y = -|a_y|, \qquad \alpha = -|\alpha|$$

$$-mR|\alpha| = T - mg$$

$$-\frac{1}{2}MR^2|\alpha| = -TR$$

$$\Downarrow$$

$$\left(m + \frac{M}{2}\right) R|\alpha| = mg$$

$$|\alpha| = \frac{2m}{2m+M}\frac{g}{R}$$

$$|a_y| = \frac{2m}{2m+M}g$$

$$T = \frac{1}{2}M|a_y|$$

$$= \frac{mM}{2m+M}g$$

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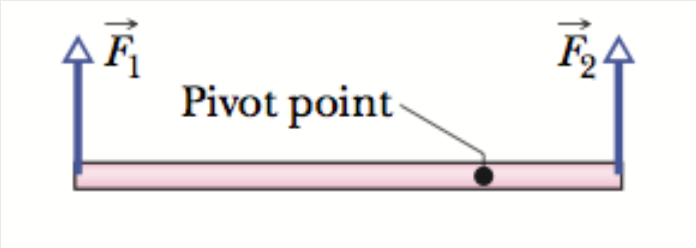
i-Clicker

Two horizontal forces act on a horizontal bar that can pivot about a vertical axis as shown below. Suppose the bar is stationary.



If the angle between $\vec{F}_2$ and the bar is decreased from 90°, should the magnitude of $\vec{F}_2$ be

$A)$ made larger

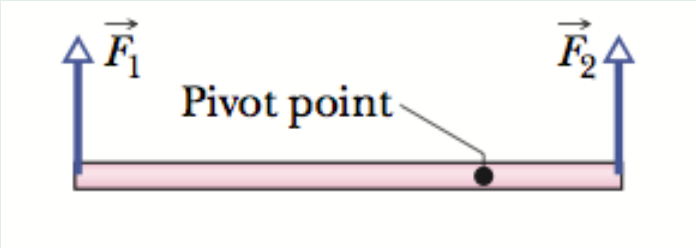
$B)$ made smaller

$C)$ left the same

in order for the bar to remain stationary ?

Answer

Two horizontal forces act on a horizontal bar that can pivot about a vertical axis as shown below. Suppose the bar is stationary.



If the angle between $\vec{F}_2$ and the bar is decreased from 90°, should the magnitude of $\vec{F}_2$ be

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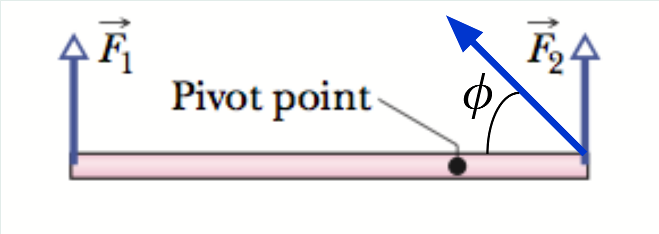


- Initially $\phi = 90^\circ$, stationary bar

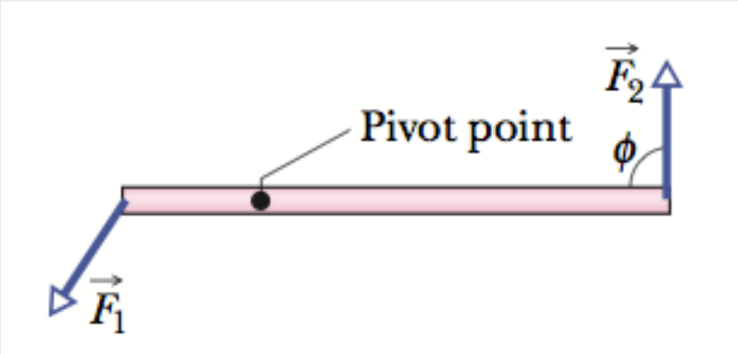
$$\tau_1 + \tau_2 = 0$$
$$F_1 l_1 = F_2 l_2$$

- $\phi \searrow$, stationary bar

$$\tau_1 + \tau_2 = 0$$
$$F_1 l_1 = F_2' l_2 \sin\phi$$
$$\Downarrow$$
$$F_2' = \frac{F_1 l_1}{l_2 \sin\phi} > F_2$$

i-Clicker

Two horizontal forces act on a horizontal bar that can pivot about a vertical axis as shown below. Rank the following values of ϕ according to the magnitude of the angular acceleration of the bar, greatest first: $90°$, $70°$, $120°$.



$A)$ $70°,\ 90°,\ 120°.$

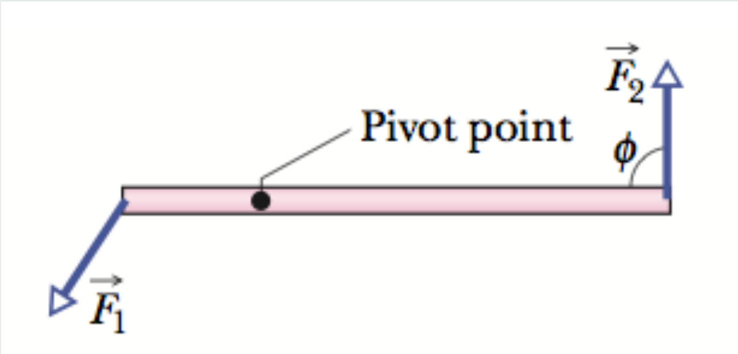
$B)$ $120°,\ 90°,\ 70°$

$C)$ $90°,\ 120°,\ 70°$

$D)$ $90°,\ 70°,\ 120°$

Answer

Two horizontal forces act on a horizontal bar that can pivot about a vertical axis as shown below. Rank the following values of ϕ according to the magnitude of the angular acceleration of the bar, greatest first: 90°, 70°, 120°.



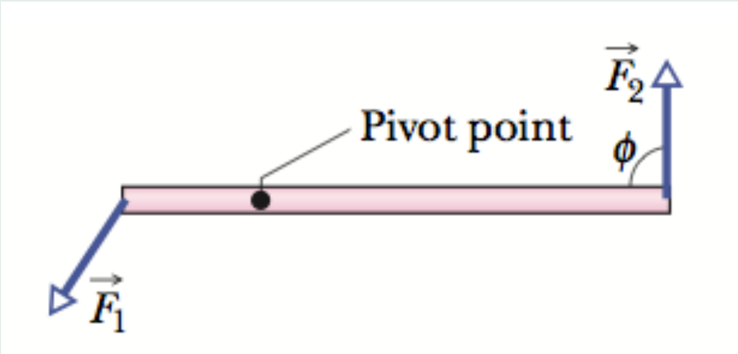
$A)$ 70°, 90°, 120°.

$B)$ 120°, 90°, 70°

$C)$ 90°, 120°, 70°

$D)$ 90°, 70°, 120°

Newton's rotation law:



$$I\alpha = \tau_{\text{net}} = \tau_1 + \tau_2$$

$$I\alpha = \tau_1 + F_2 l_2 \sin\phi$$

$$\sin\phi_1 > \sin\phi_2$$

$$\Downarrow$$

$$\alpha_1 > \alpha_2$$