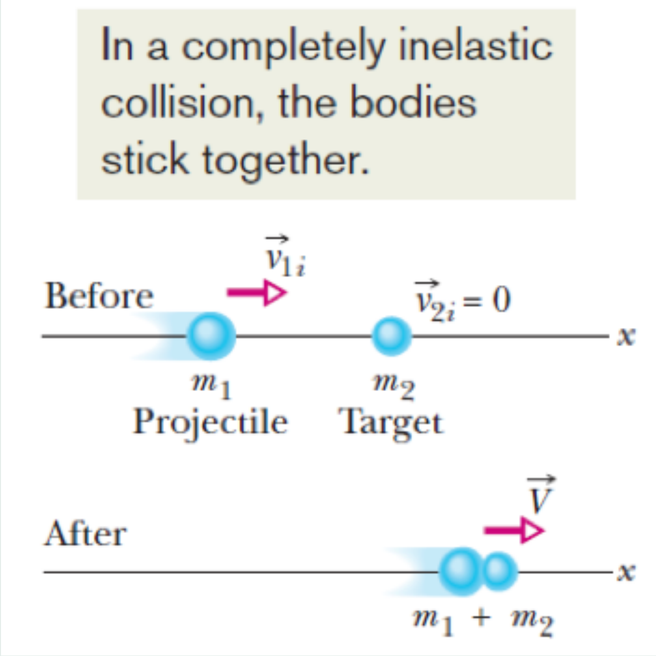
Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 14

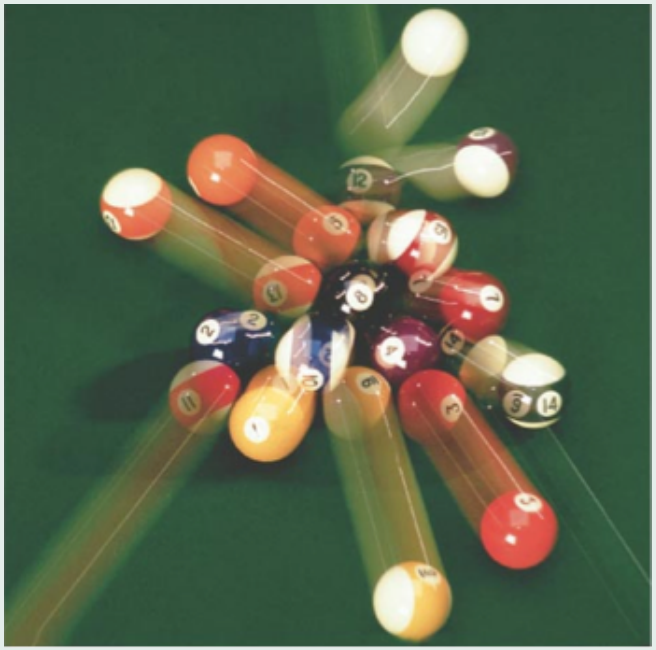
9. Center of Mass. Linear Momentum II

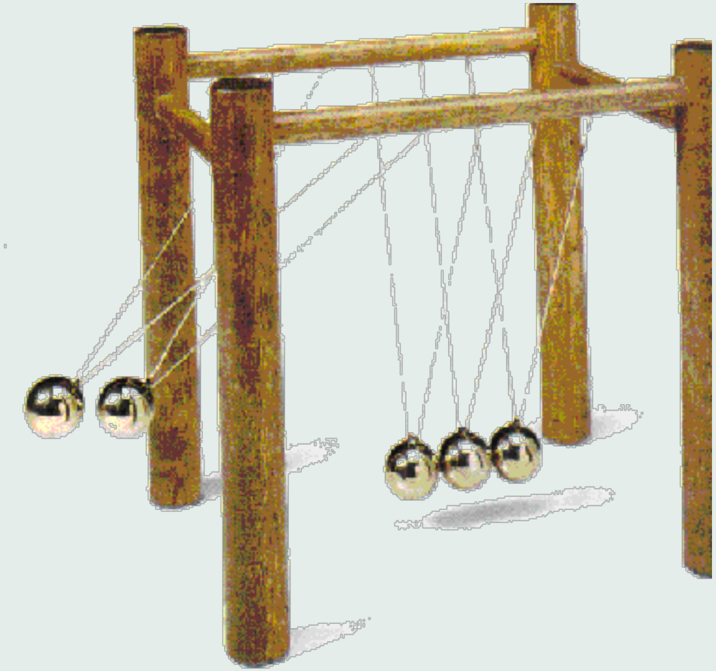
- Previously: inelastic collisions in 1D



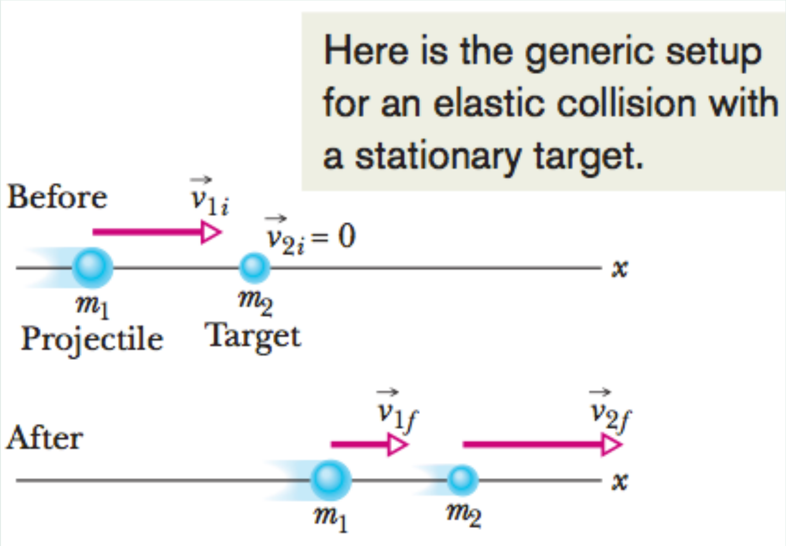
Linear momentum conserved, kinetic energy not conserved (some fraction converted to thermal energy.)

• **Ellastic collisions in 1D: both** linear momentum and kinetic energy are conserved





- **Generic setup – stationary target**



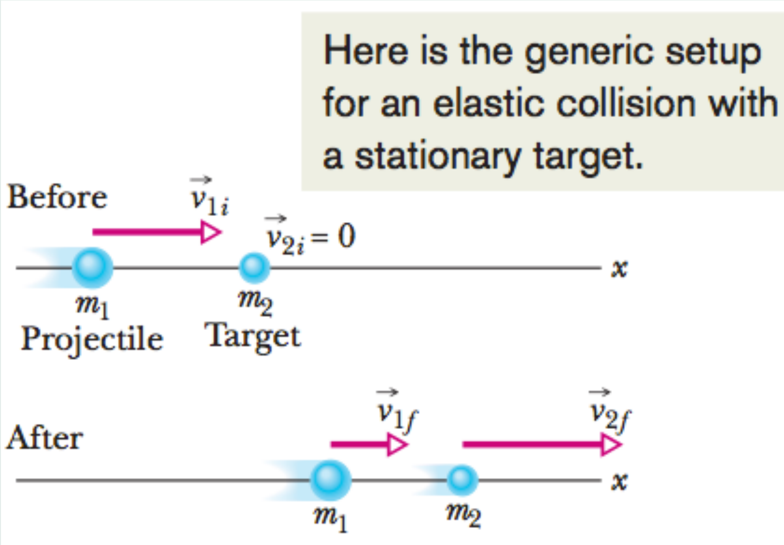
- **The linear momentum of the system is conserved:**

$$\vec{P}_i = \vec{P}_f$$

- **The total kinetic energy of the system is conserved:**

$$K_i = K_f$$

Note: the kinetic energy of each colliding body may change, but the total kinetic energy of the system does not change.



$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1(v_{1i} - v_{1f}) = m_2 v_{2f}$$
$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 v_{2f}^2$$
$$\Downarrow$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

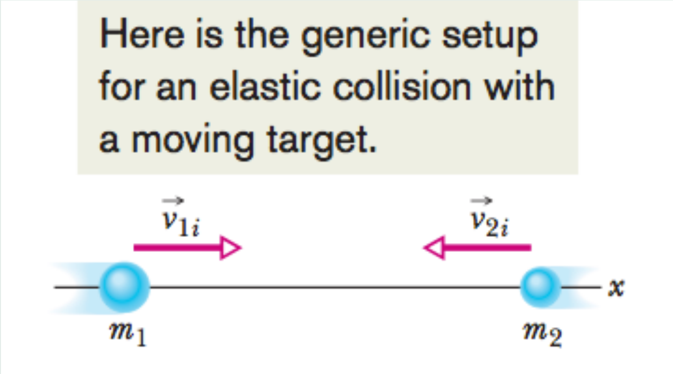
1D elastic collision – stationary target

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} \qquad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Note:

- $v_{2f} > 0$
- $v_{1f} > 0$ if $m_1 > m_2$; $v_{1f} < 0$ if $m_1 < m_2$
- $v_{1f} = 0$, $v_{2f} = v_{1i}$ if $m_1 = m_2$ (identical particles)

- **Generic setup – moving target**



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\Downarrow$$

$$m_1(v_{1i} - v_{1f}) = m_2(v_{2f} - v_{2i})$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$$

$$\Downarrow$$

$$m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})$$

$$\Downarrow$$

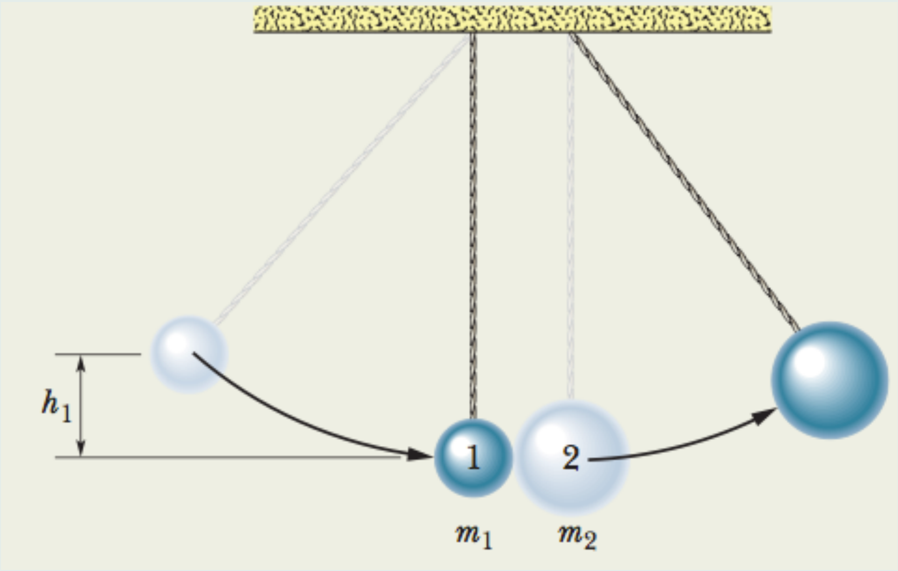
$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

1D elastic collision – moving target

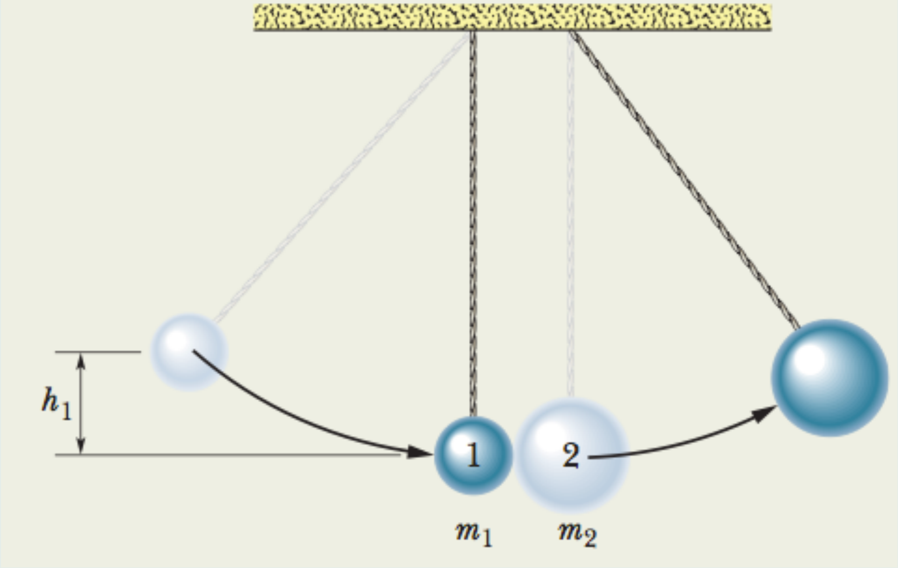
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

- **Example:** two pendulums



- How high will the 1st ball recoil after collision?
- Which way will it swing?
- How high will the 2nd ball swing after collision?



- **Step 1:**

$$mgh_1 = \frac{1}{2}mv_{1i}^2$$

$$v_{1i} = \sqrt{2gh_1}$$

- **Step 2:** collision

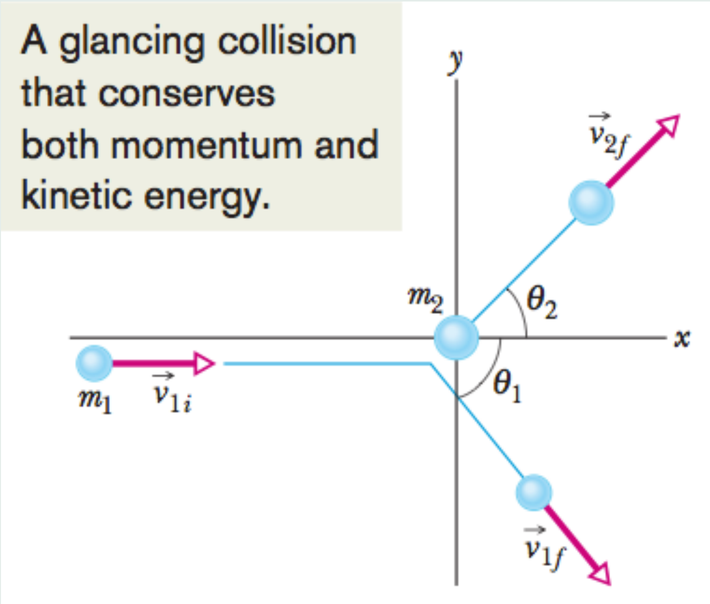
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i}$$

- **Step 3:**

$$m_1gh_{1f} = \frac{1}{2}m_1v_{1f}^2 \qquad m_2gh_{2f} = \frac{1}{2}m_2v_{2f}^2$$

- **Collisions in 2D**



- Linear momentum conserved:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

- Stationary target:

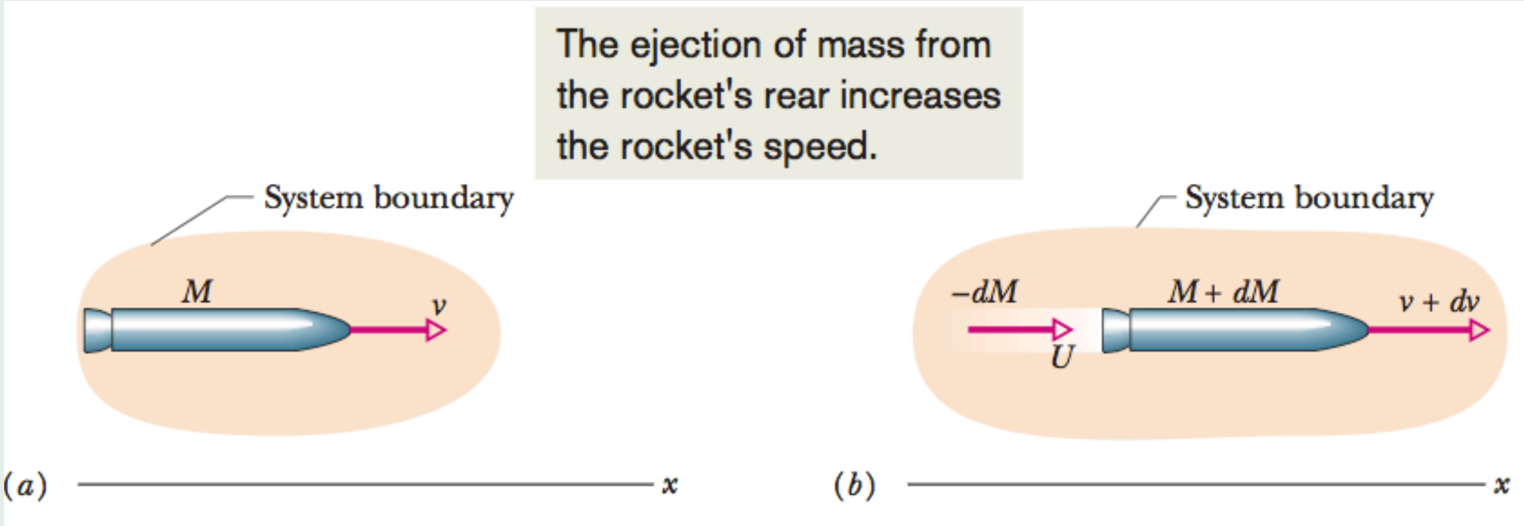
$$\begin{aligned} m_1 v_{1i} =& m_1 v_{1f} \cos\theta_1 \\ &+ m_2 v_{2f} \cos\theta_2 \end{aligned}$$

$$m_1 v_{1f} \sin\theta_1 = m_2 v_{2f} \sin\theta_2$$

- **If elastic**, kinetic energy also conserved:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

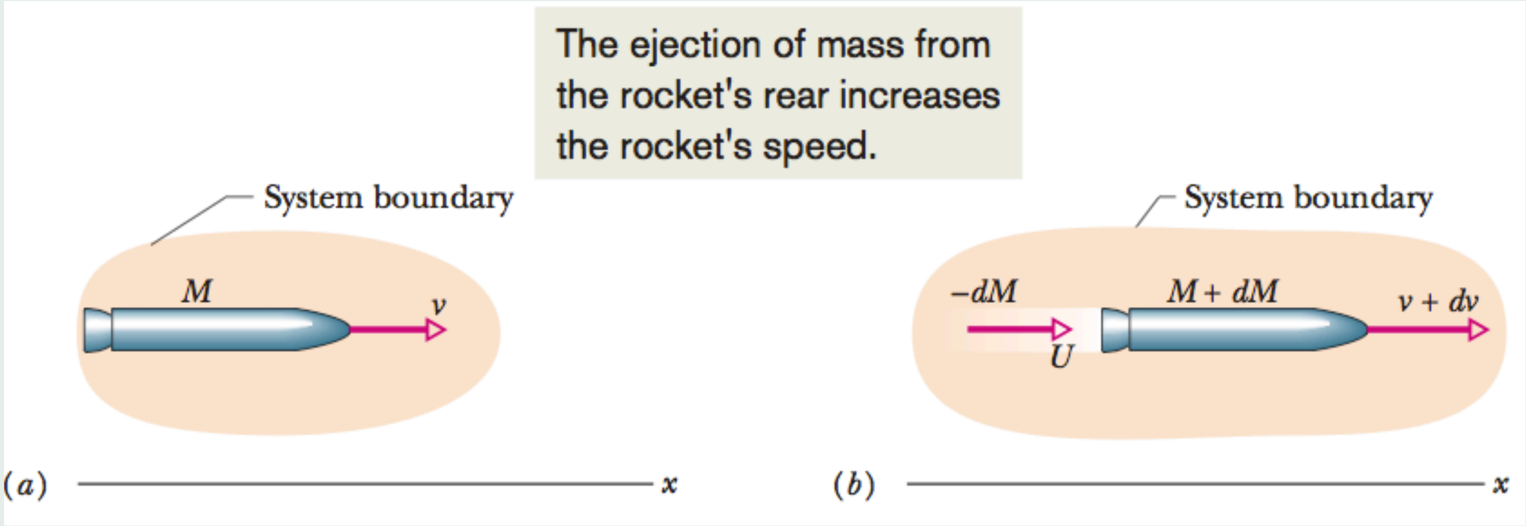
- ## Systems with varying mass



(a) accelerating rocket at time t in inertial frame

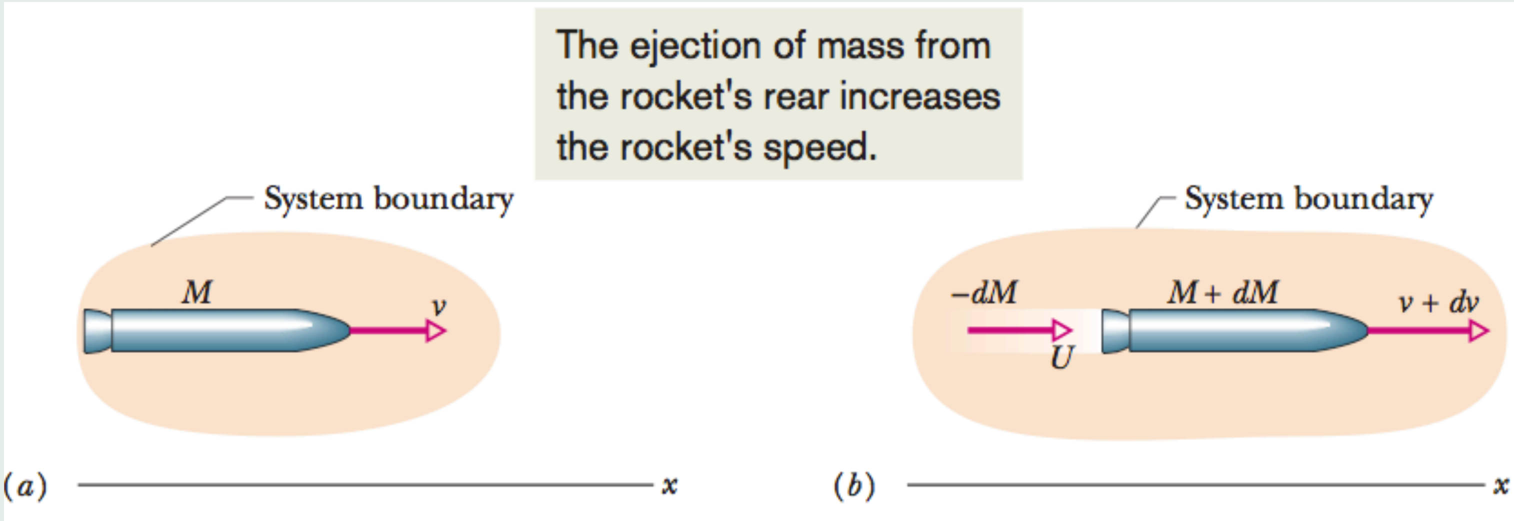
(b) accelerating rocket at time $t + dt$ in the same frame

$$v \to v + dv, \quad dv > 0 \qquad M \to M + dM, \quad dM < 0$$



- Suppose the **relative** speed v_{rel} between the rocket and exhaust products is known.

- How do we find the acceleration?

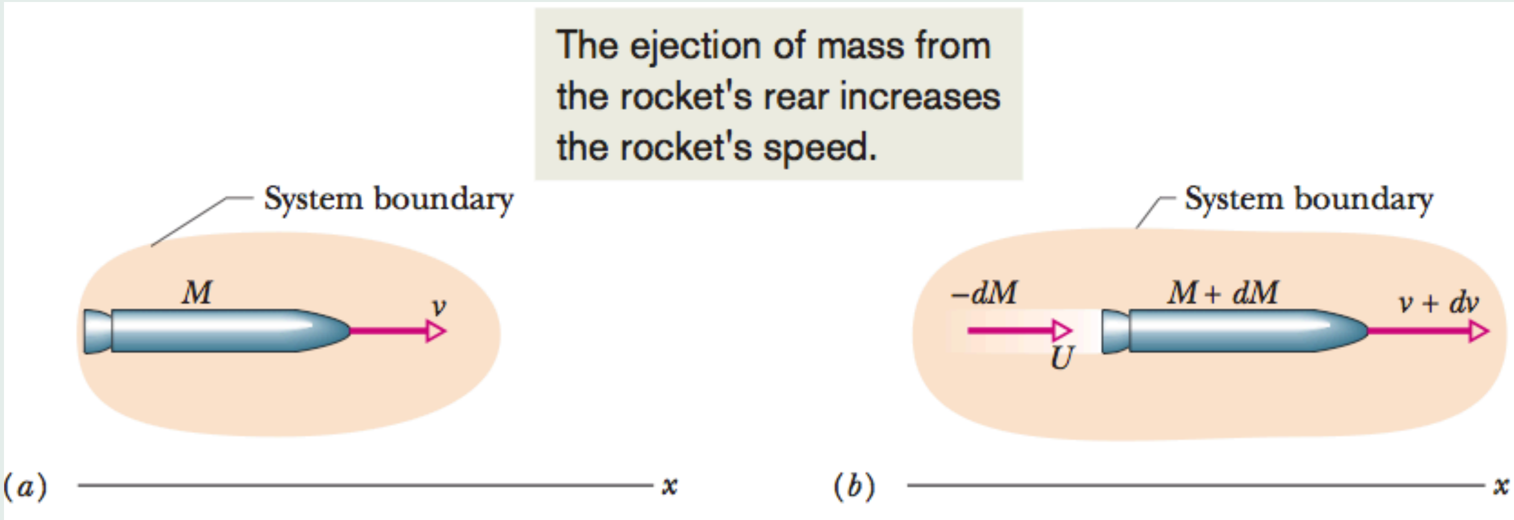


Rocket + exhaust products = isolated closed system

$$\Downarrow$$

Linear momentum conserved

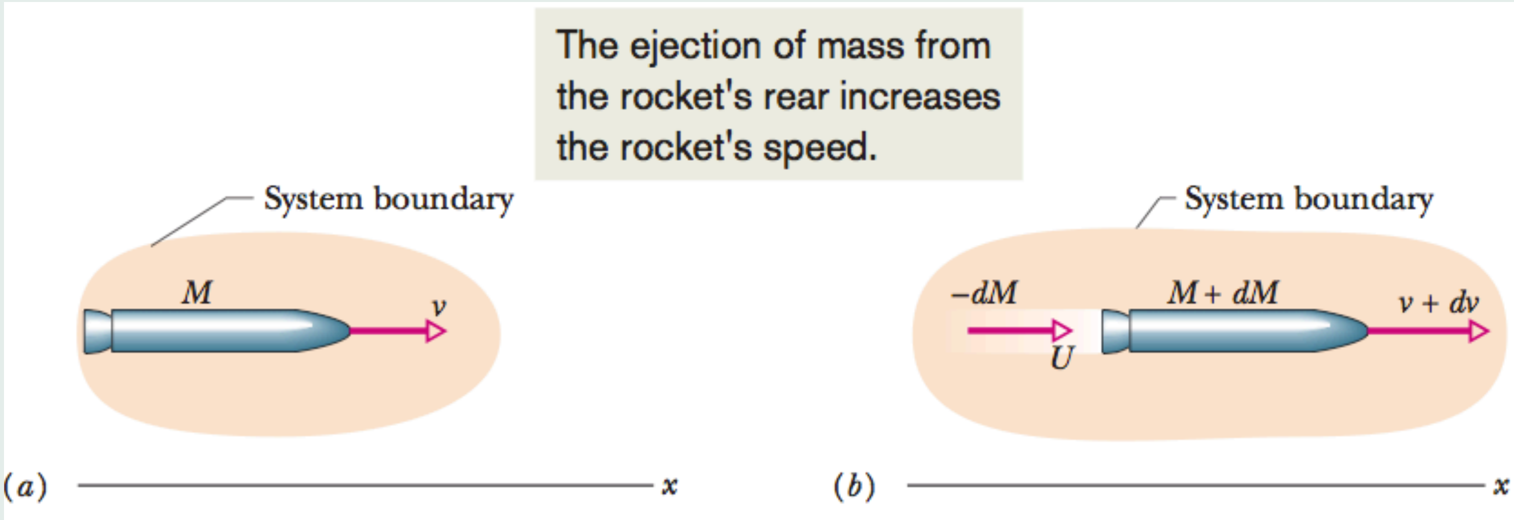
$$\vec{P}_a = \vec{P}_b \qquad P_{a,x} = P_{b,x}$$



$$P_{a,x} = Mv \qquad P_{b,x} = (M + dM)(v_x + dv_x) + (-dM)u_x$$

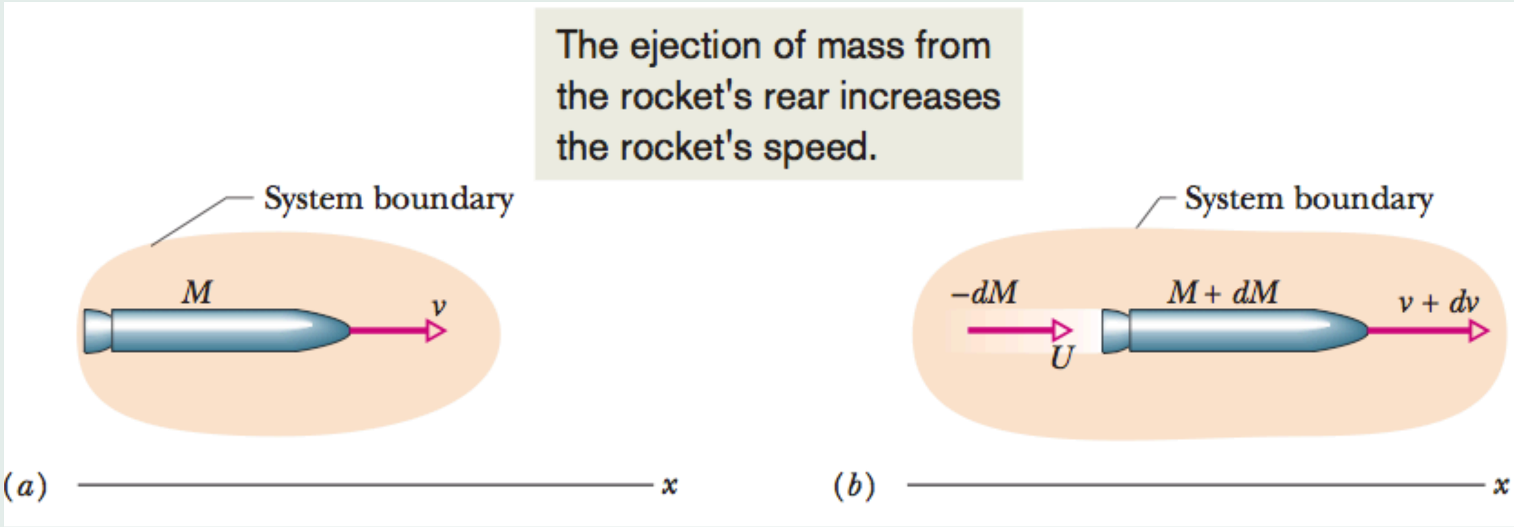
Note: u_x the x-component of the velocity of the exhaust products **relative to the inertial frame**

$$v_x + dv_x = u_x + v_{\text{rel}}$$



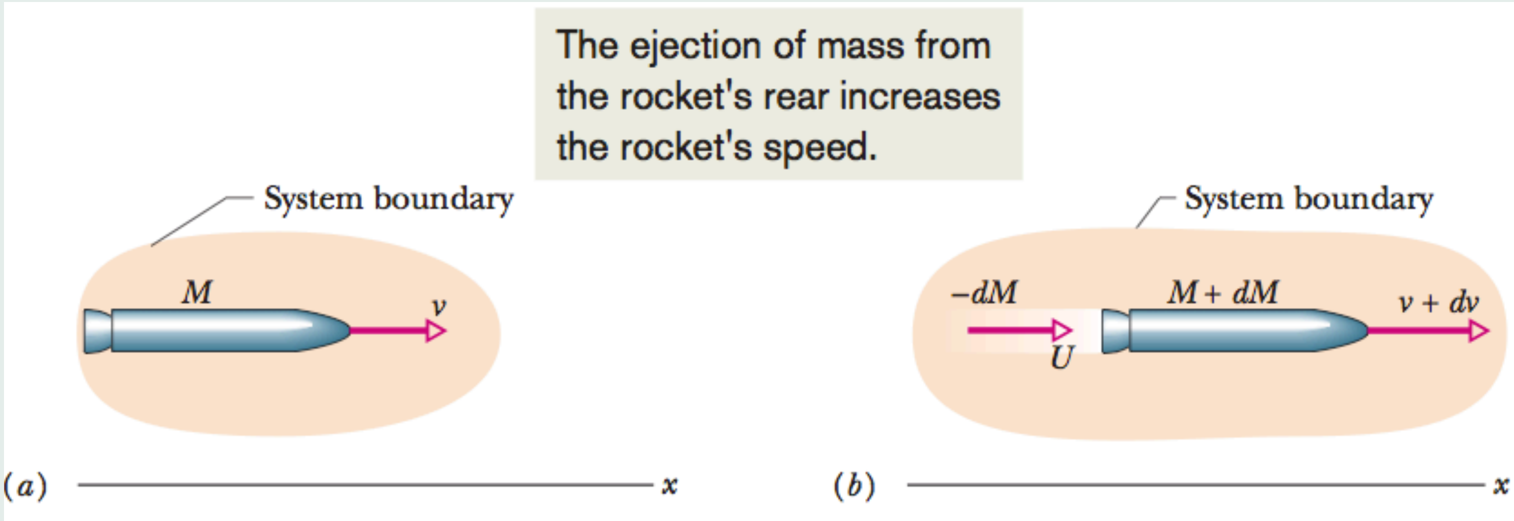
$$Mv = (M + dM)(v_x + dv_x) - (v_x + dv_x - v_{\text{rel}})dM$$

$$M dv_x + v_{\text{rel}} dM = 0 \;\Rightarrow\; M\frac{dv_x}{dt} = -v_{\text{rel}}\frac{dM}{dt}$$



$$Ma_x = -v_{\text{rel}}\frac{dM}{dt} = Rv_{\text{rel}}$$

The 1st rocket equation



$$v_{f,x} - v_{i,x} = v_{\mathrm{rel}} \ln \frac{M_i}{M_f}$$

The 2nd rocket equation

i-Clicker

Rain falls vertically into an open cart rolling horizontally. What happens to the momentum, speed and kinetic energy?

	p	v	K
$A)$	same	same	same
$B)$	increases	same	increases
$C)$	increases	increases	increases
$D)$	same	decreases	same
$E)$	same	decreases	decreases

i-Clicker

Rain falls vertically into an open cart rolling horizontally. What happens to the momentum, speed and kinetic energy?

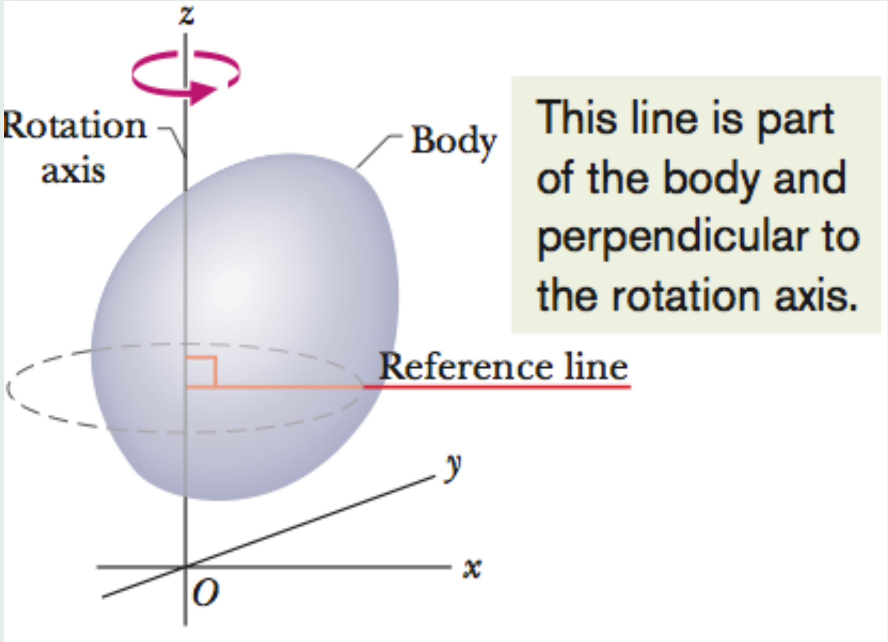
	p	v	K
$A)$	same	same	same
$B)$	increases	same	increases
$C)$	increases	increases	increases
$D)$	same	decreases	same
$E)$	same	decreases	decreases

10. Rotation

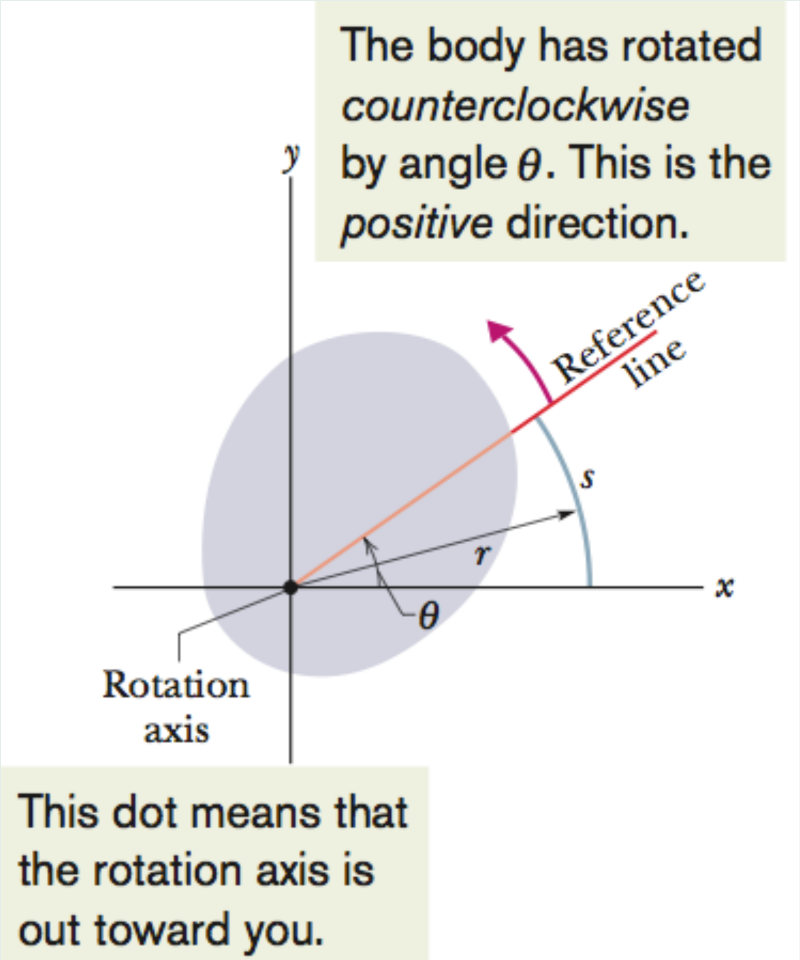
- A **rigid body** is a body that can rotate with all its parts locked together and without any change in its shape.

- A **fixed axis** means that the rotation occurs about an axis that does not move.

- **Rotation variables**



- rotation axis = z-axis
- **reference line:**
  - **perpendicular** to rotation axis.
  - **rotates with the body**



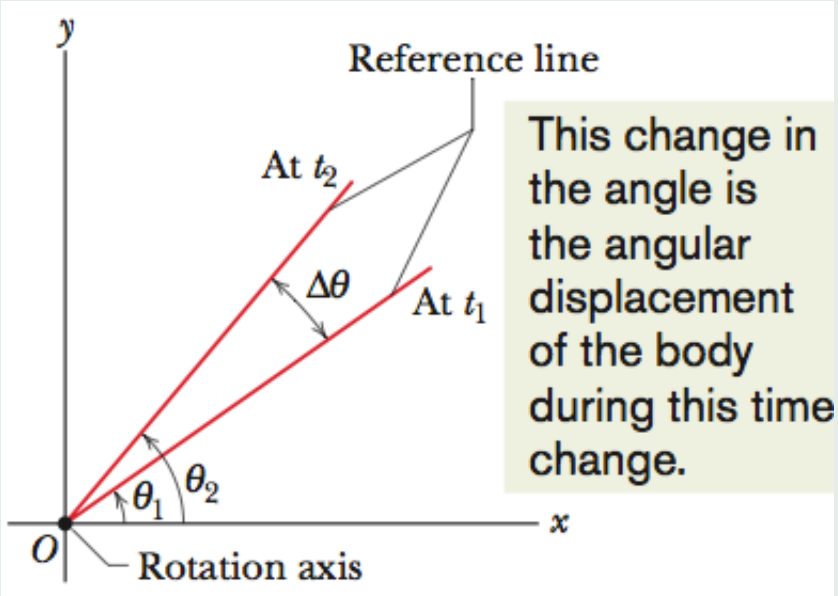
- **Angular position**:

$$\theta = \frac{s}{r}$$

s length of circular arc

r radius of circle

- **Positive direction:** counterclockwise

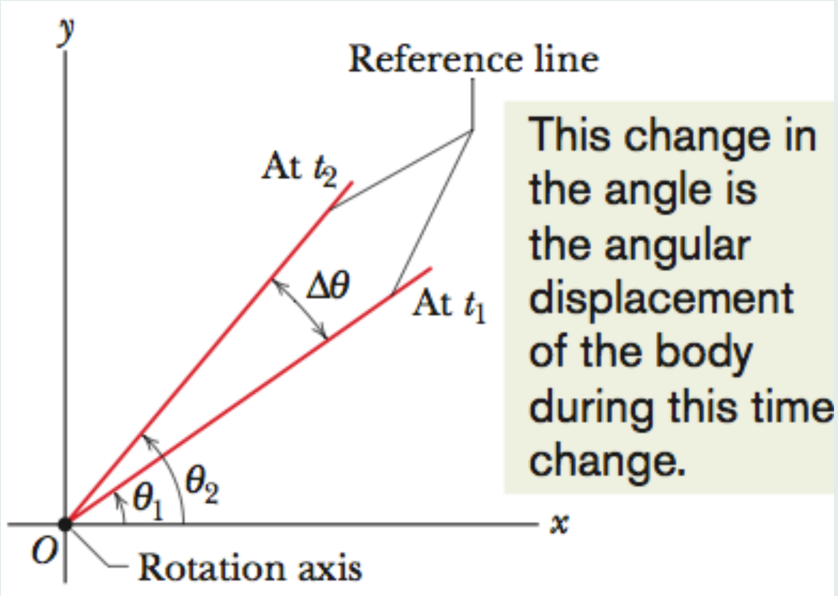


- **Angular displacement**:

$$\Delta\theta = \theta_2 - \theta_1$$

$$\Delta\theta > 0 \quad \textbf{counterclockwise}$$

$$\Delta\theta < 0 \quad \textbf{clockwise}$$



- **Average angular velocity**:

$$\omega_{\mathsf{avg}} = \frac{\Delta\theta}{\Delta t}$$

- **Units:** $\mathrm{rad/s}$.

- **Instantaneous angular velocity**:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- **Average angular acceleration**:

$$\alpha_{\mathsf{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

- **Instantaneous angular acceleration**:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

- **Units:** $\mathrm{rad/s^2}$.

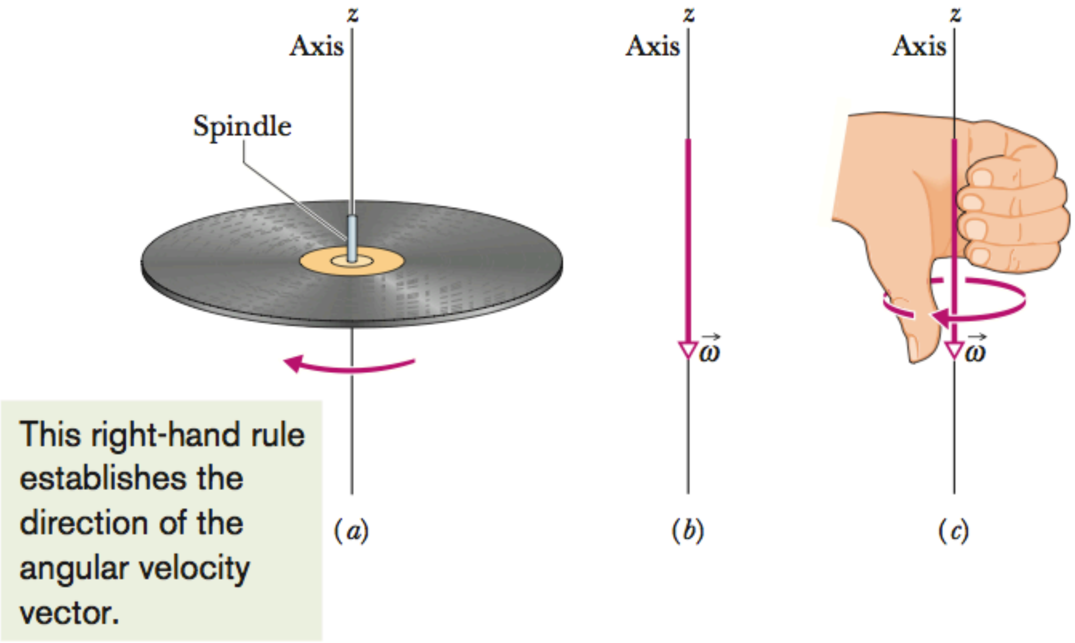


Fig. 10-6 (*a*) A record rotating about a vertical axis that coincides with the axis of the spindle. (*b*) The angular velocity of the rotating record can be represented by the vector $\vec{\omega}$, lying along the axis and pointing down, as shown. (*c*) We establish the direction of the angular velocity vector as downward by using a right-hand rule. When the fingers of the right hand curl around the record and point the way it is moving, the extended thumb points in the direction of $\vec{\omega}$.

Are angular variables vectors?

- **Constant angular acceleration**

Table 10-1

Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable		Angular Equation
(2-11)	$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
(2-15)	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
(2-18)	$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

Constant angular acceleration $\leftrightarrow$ Constant linear acceleration