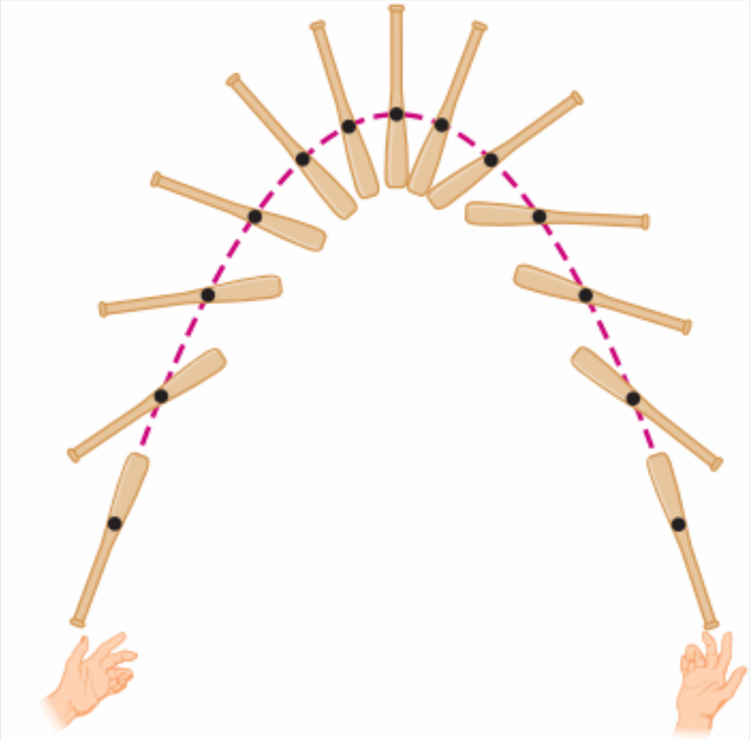
# Rutgers University
# Department of Physics & Astronomy

## 01:750:271 Honors Physics I
## Fall 2015

Lecture 12

# 9. Center of Mass. Linear Momentum I



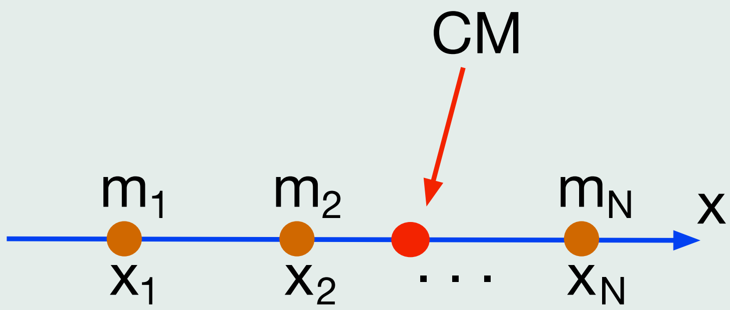
The **center of mass** of a system of particles is the point that moves as though:

(1) all of the systems mass were concentrated there and

(2) all external forces were applied there.

The center of mass of the baseball bat follows a parabolic path, but all other points follow more complicated paths.

- **The center of mass: a system of particles**

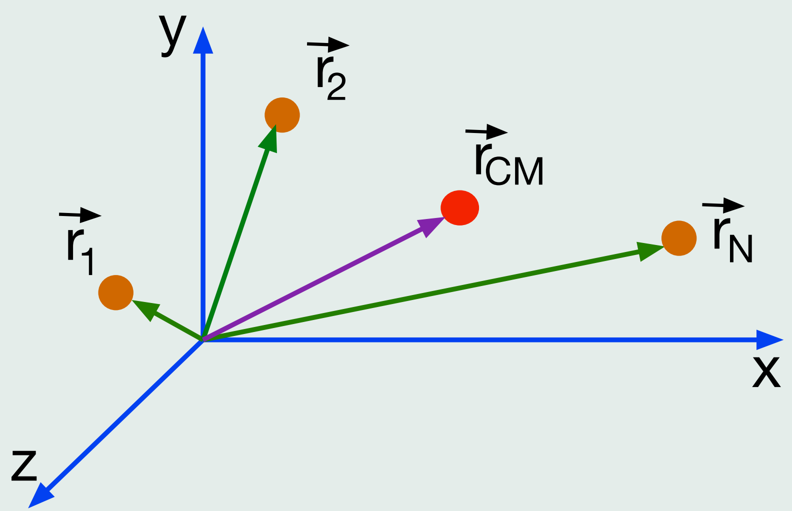



Discrete linear distribution of particles

**Note:** in general ● does not have to be one of the particles in the system.

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_N x_N}{m_1 + m_2 + \cdots + m_N} = \frac{1}{M} \sum_{i=1}^{N} m_i x_i$$

- **Three dimensions:** discrete distribution of particles



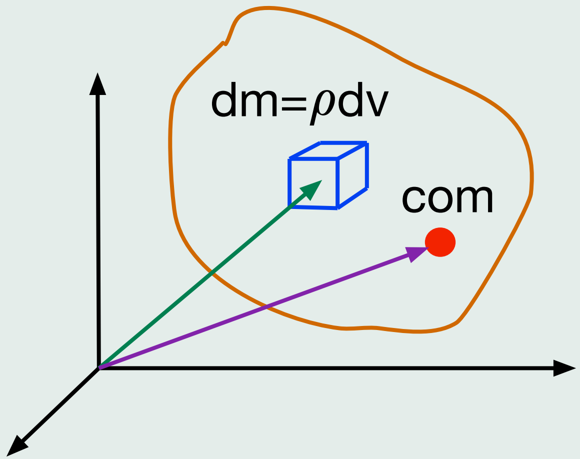

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$$

$$y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$$

$$z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i$$

$$\vec{r}_{\text{com}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N}{m_1 + m_2 + \cdots + m_N} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i$$

- **Solid body:** continuous mass distribution




$$\vec{r}_{\text{com}} = x_{\text{com}}\widehat{i} + y_{\text{com}}\widehat{j} + z_{\text{com}}\widehat{k}$$

$$x_{\text{com}} = \frac{1}{M}\int x dm \quad y_{\text{com}} = \frac{1}{M}\int y dm \quad z_{\text{com}} = \frac{1}{M}\int z dm$$

$$M = \int dm \qquad \text{total mass of the object}$$

## Mass density:

$$\rho = \frac{dm}{dv} \qquad dv = dxdydz \quad \text{volume element}$$

**Uniform** mass distribution:

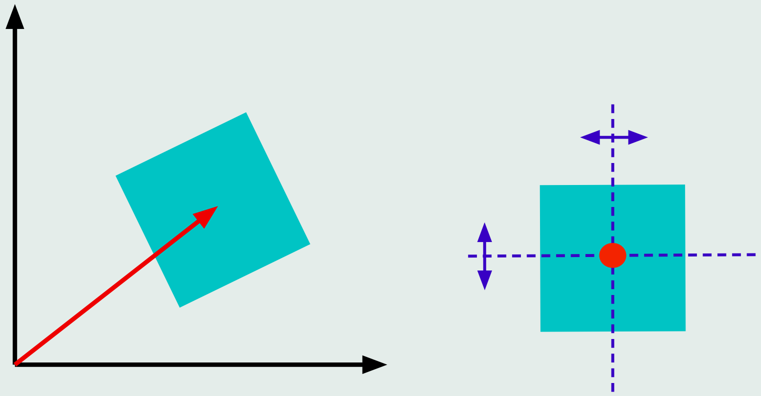
$\rho =$ **constant** as function of $\vec{r}$

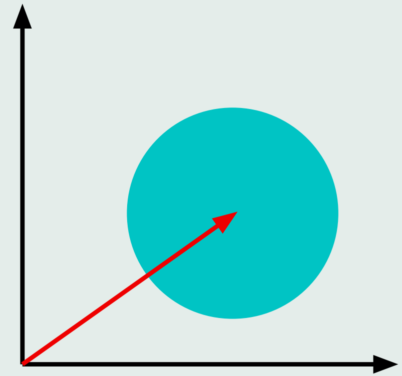
$$\Downarrow$$

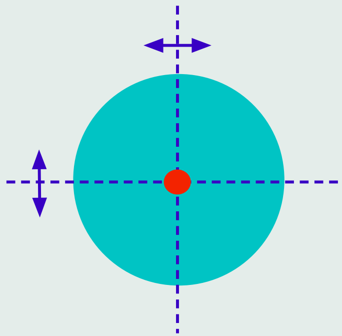
$$x_{\text{com}} = \frac{1}{V}\int x dv \quad y_{\text{com}} = \frac{1}{V}\int y dv \quad z_{\text{com}} = \frac{1}{V}\int z dv$$

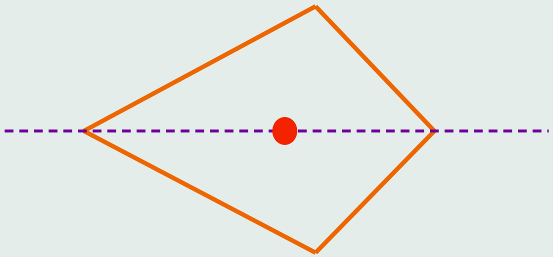
$$V = \int dv = \int dxdydz \qquad \text{total volume}$$

- **Symmetry:** if the object has a symmetry axis and $\rho =$ **constant** then its com must lie on that axis

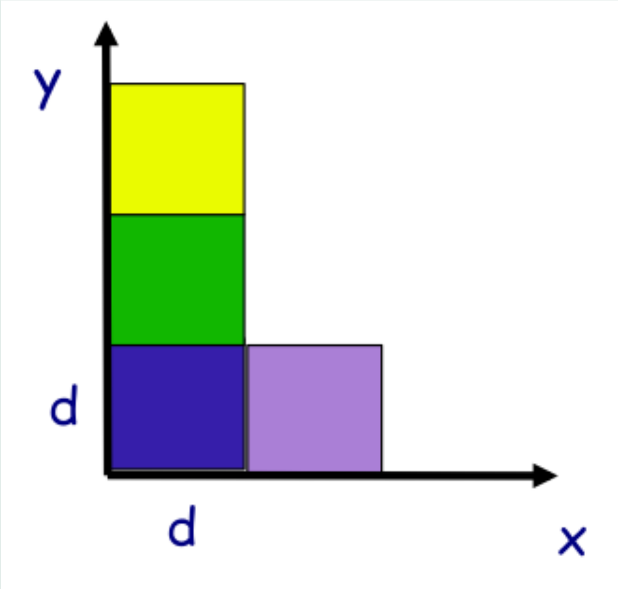








- **Example**



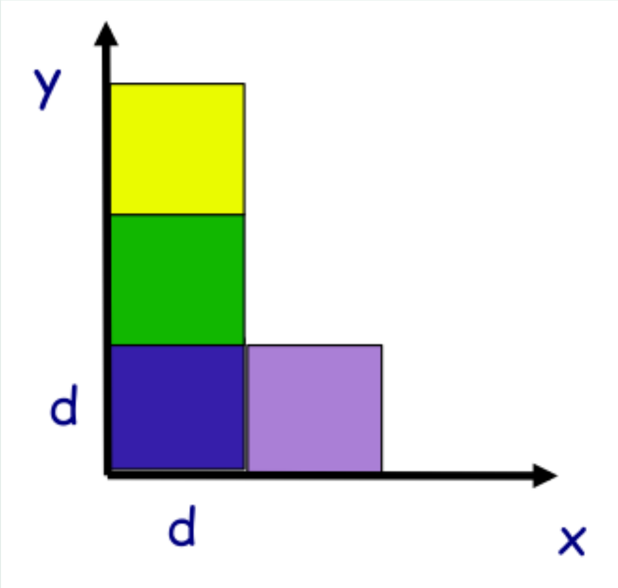

Consider an L shape made up of 4 blocks each a square with sides of length d.

- Assume all blocks are uniform and have the same mass $M$

- Treat this as a 2-dimensional problem – ignore width in the $z$-direction.

$$x_{\text{com}} = \frac{3M(d/2) + M(3d/2)}{4M} = \frac{3}{4}d$$

# i-Clicker



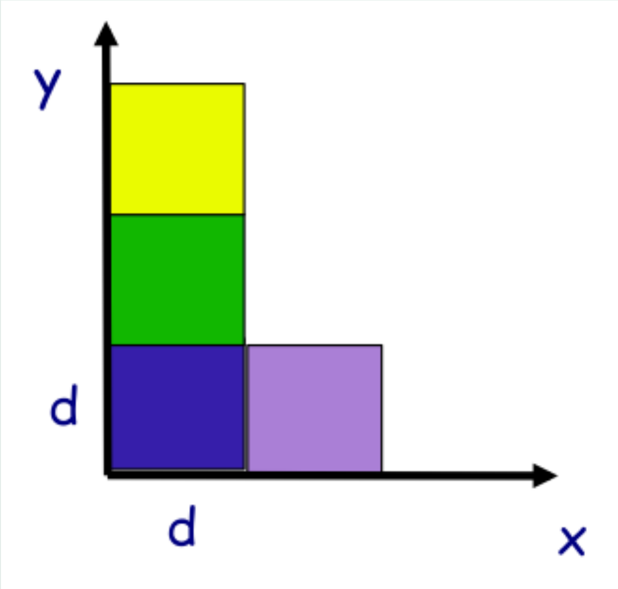

Consider an L shape made up of 4 blocks each a square with sides of length d.

- Assume all blocks are uniform and have the same mass $M$
- Treat this as a 2-dimensional problem - ignore width in the $z$-direction.

What is $y_{\text{com}}$ ?

$A)\ d \qquad B)\ 7d/8 \qquad C)\ 5d/4 \qquad D)\ 3d/4 \qquad E)\ d/2$

## Answer




Consider an L shape made up of 4 blocks each a square with sides of length d.

- Assume all blocks have the same mass M
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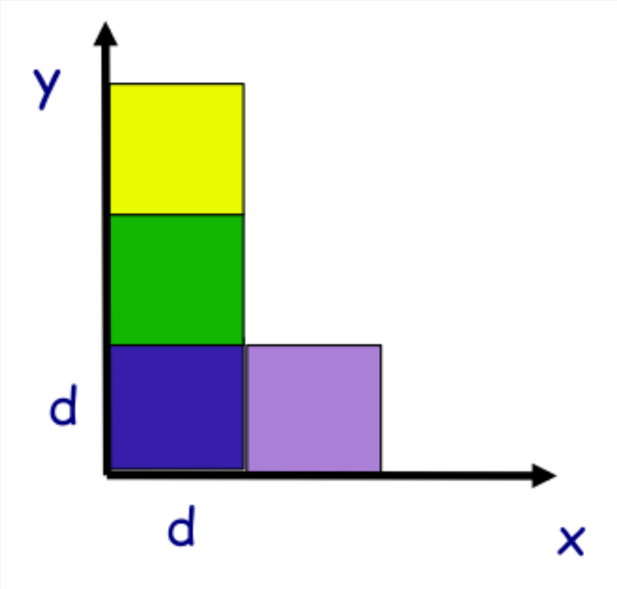
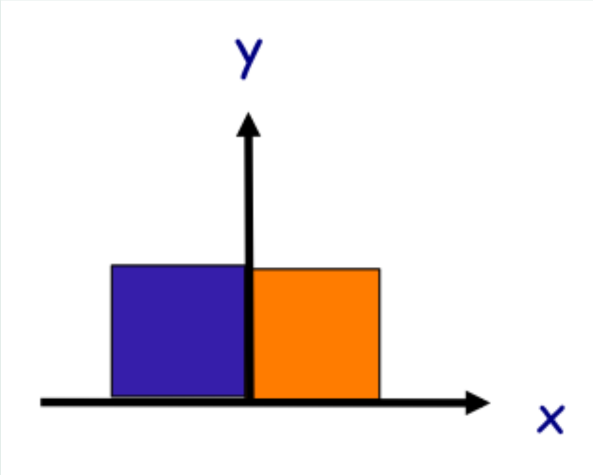



$$y_{\text{com}} = \frac{2M(d/2) + M(3d/2) + M(5d/2)}{4M} = \frac{5}{4}d$$

- **Example:**

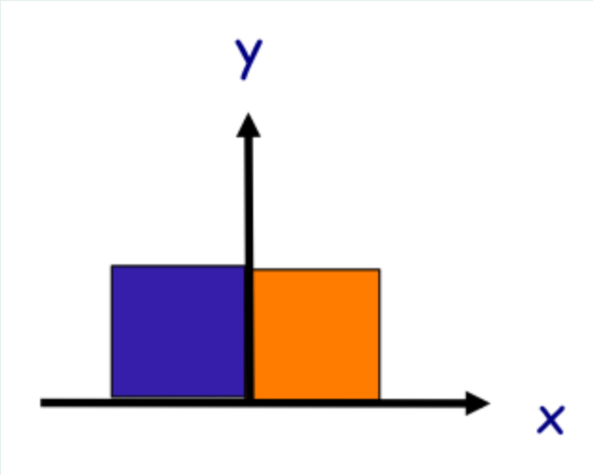



Consider the two uniform squares shown each with sides of length d.

- Assume $m_{\text{blue}} = 2\,\text{kg}$, $m_{\text{orange}} = 1\,\text{kg}$
- Treat this as a 2-dimensional problem.

$$x_{\text{com}} = \frac{m_{\text{blue}} x_{\text{blue}} + m_{\text{orange}} x_{\text{orange}}}{m_{\text{blue}} + m_{\text{orange}}} = -\frac{d}{6}$$

## i-Clicker

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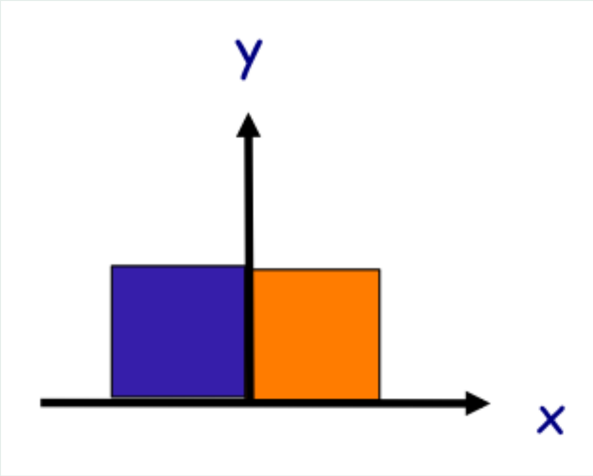
$A)\ d \quad B)\ d/4 \quad C)\ 0 \quad D)\ 3d/2 \quad E)\ d/2$

## Answer

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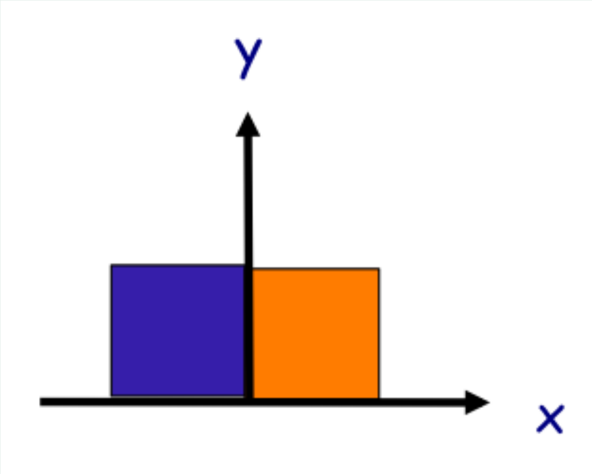
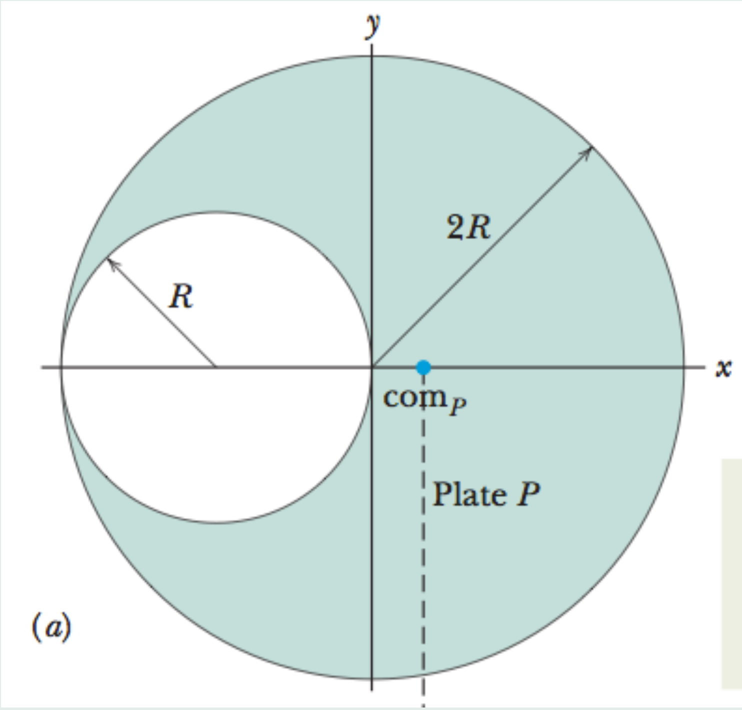


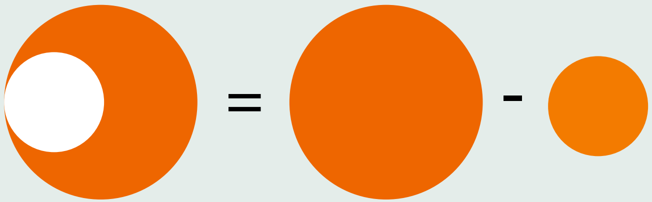

$$y_{\text{com}} = \frac{m_{\text{blue}} y_{\text{blue}} + m_{\text{orange}} y_{\text{orange}}}{m_{\text{blue}} + m_{\text{orange}}} = \frac{d}{2}$$

- **Subtraction example**




A metal plate $P$ is has the shape of a disk of radius $R$ with a hole of radius $R$. Assuming uniform density, $\rho = \text{constant}$, find $x_{\text{com}}$.

Treat the problem as two dimensional ignoring thickness in the $z$-direction.



- $P$: holed disk
- $D$: whole big disk
- $H$: the small disk removed from $D$

$$P = D - H$$

$$x^P_{\text{com}} = \frac{\int_P x dx dy}{\int_P dx dy} = \frac{X_P}{A_P}$$

$$X_P = \int_D x dx dy - \int_H x dx dy = X_D - X_H$$

$$A_P = \int_D dx dy - \int_H dx dy = A_D - A_H$$

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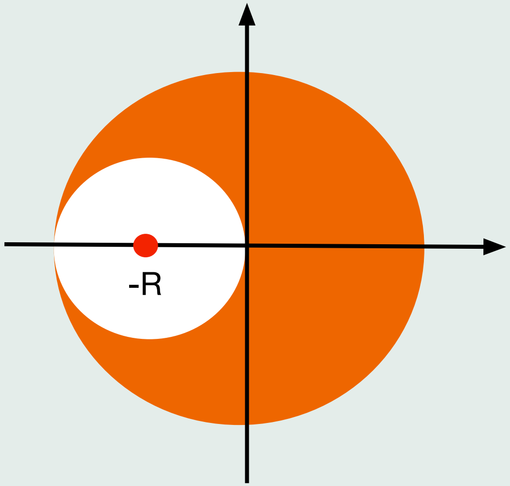



$$x^D_{\text{com}} = 0 \qquad x^H_{\text{com}} = -R$$

$$x^D_{\text{com}} = \frac{X_D}{A_D} \qquad x^H_{\text{com}} = \frac{X_H}{A_H}$$

$$X_D = 0, \qquad X_H = -RA_H$$

$$x^P_{\text{com}} = \frac{X_D - X_H}{A_D - A_H} = R\frac{A_H}{A_D - A_H} = \frac{R}{3}$$

- **Newton's 2nd law for a system of particles**

$$\boxed{\vec{F}_{\text{net}} = M\vec{a}_{\text{com}}}$$

$$F_{\text{net},x} = Ma_{\text{com},x}$$

$$F_{\text{net},y} = Ma_{\text{com},y}$$

$$F_{\text{net},z} = Ma_{\text{com},z}$$

- $\vec{F}_{\text{net}}$ is the net force of all **external** forces acting on the system. **Internal** forces are not included.

- $M$ is the total mass of the system. If $M$ is constant, the system is said to be **closed**.

- $\vec{a}_{\text{com}}$ is the acceleration of the center of mass of the system.

The internal forces of the explosion cannot change the path of the com.

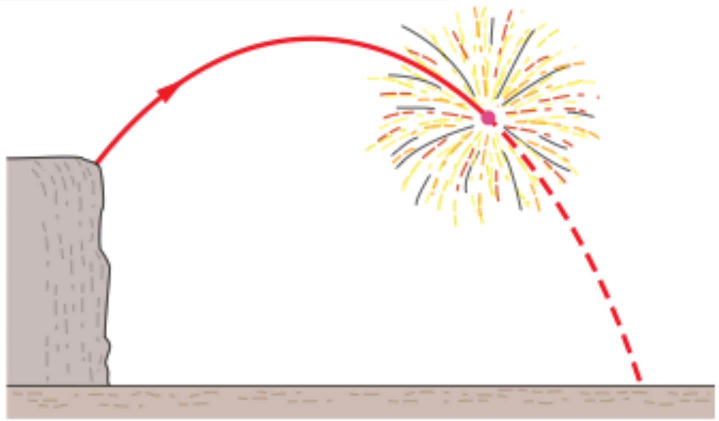


**Fig. 9-5** A fireworks rocket explodes in flight. In the absence of air drag, the center of mass of the fragments would continue to follow the original parabolic path, until fragments began to hit the ground.

# Derivation

$$M\vec{r}_{\text{com}} = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N$$

$$\Downarrow \;\; d/dt$$

$$M\vec{v}_{\text{com}} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_N\vec{v}_N$$

$$\Downarrow \;\; d/dt$$

$$M\vec{a}_{\text{com}} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots + m_N\vec{a}_N$$

$$M\vec{a}_{\text{com}} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N$$

$$M\vec{a}_{\text{com}} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N$$

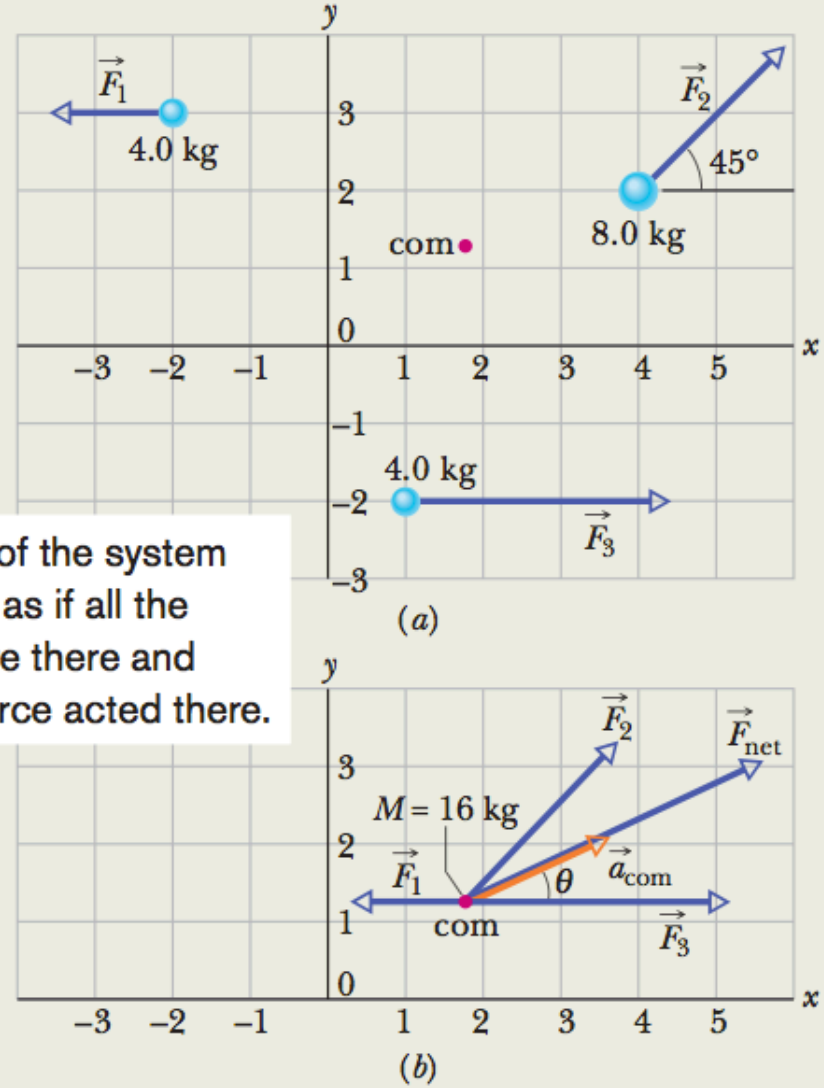
$\vec{F}_i =$ the sum of **all** forces acting on particle $i$, including **internal** ones.

$$\vec{F}_i = \vec{F}_{i,\text{ext}} + \vec{F}_{i,\text{int}}$$

**Newton's 3rd law:**

$$\sum_{i=1}^{N} \vec{F}_{i,\text{int}} = 0$$

$$\boxed{M\vec{a}_{\text{com}} = \vec{F}_{1,\text{ext}} + \vec{F}_{2,\text{ext}} + \cdots + \vec{F}_{N,\text{ext}}}$$

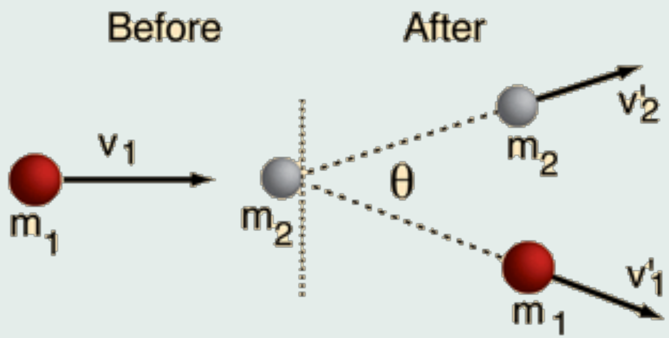



The com of the system will move as if all the mass were there and the net force acted there.

$$M\vec{a}_{\text{com}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$M = m_1 + m_2 + m_3$$

- **Linear momentum**




**How can we predict the outcome of a collision?**

Assuming energy is conserved

$$K_1 + K_2 = K_1' + K_2'$$

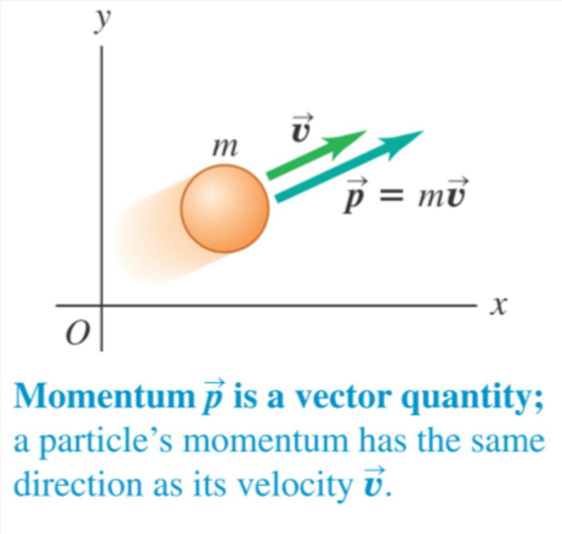
**Not sufficient!**

Energy is a **scalar** quantity.

**No direction!**

# Linear momentum of a particle

$$\boxed{\vec{p} = m\vec{v}}$$



The rate of change of the momentum is equal to the net force acting on the particle:

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}_{\text{net}}}$$

$$\frac{d(m\vec{v})}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}_{\text{net}}$$

- **Linear momentum of a system of particles**

The linear momentum of a system of particles is equal to the product of the total mass $M$ of the system and the velocity of the center of mass.

$$\boxed{\vec{P} = M\vec{v}_{\text{com}}}$$

$$\vec{P} = \sum_{i=1}^{N} \vec{p}_i = \sum_{i=1}^{N} m_i \vec{v}_i = M\vec{v}_{\text{com}}$$

**The rate of change of the momentum is equal to the net external force acting on the system:**

$$\boxed{\frac{d\vec{P}}{dt} = \vec{F}_{\text{ext}}}$$

# • Collision and Impulse

The collision of a ball with a bat collapses part of the ball. *(Photo by Harold E. Edgerton. ©The Harold and Esther Edgerton Family Trust, courtesy of Palm Press, Inc.)*

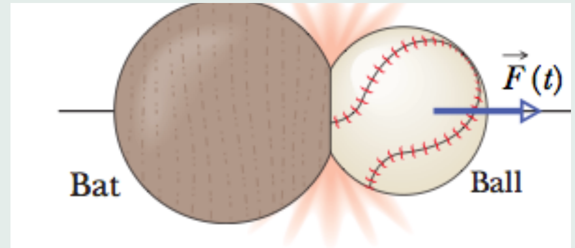



**Fig. 9-8** Force $\vec{F}(t)$ acts on a ba as the ball and a bat collide.

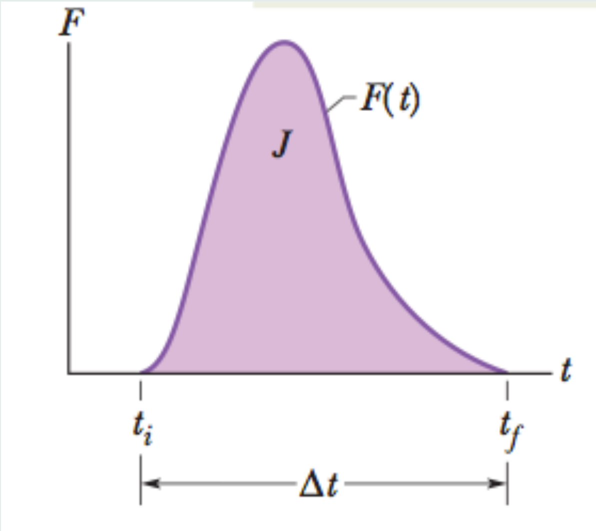
The figure depicts the collision at one instant. The ball experiences a force $F(t)$ that varies during the collision and changes the linear momentum of the ball.

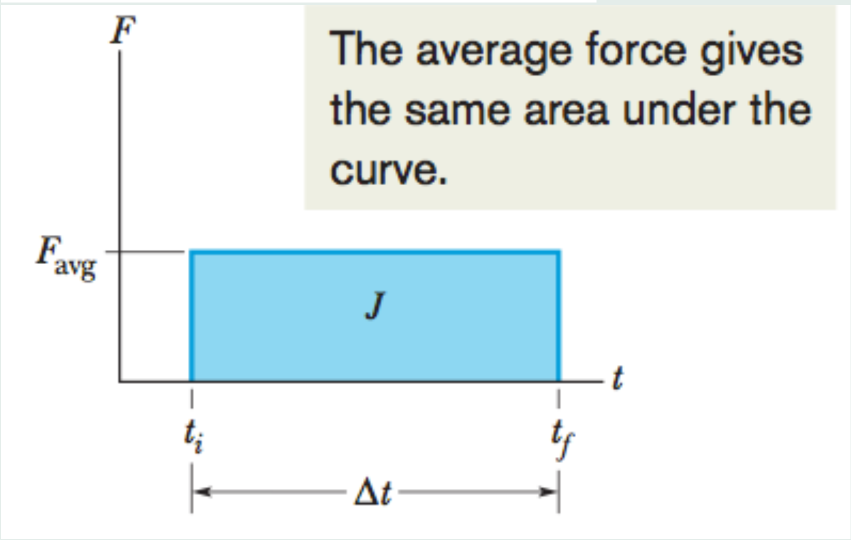
The change in linear momentum is related to the force by Newton's second law:

$$\vec{F} = \frac{d\vec{p}}{dt} \;\Rightarrow\; \Delta\vec{p} = \int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F} dt$$

- **Impulse:**

$$\boxed{\vec{J} = \int_{t_i}^{t_f} \vec{F} dt}$$




- The magnitude of $\vec{J}$ equals the area under the curve $F(t)$.

- Average force

$$\vec{F}_{\text{average}} = \frac{\vec{J}}{\Delta t}$$

- Newton's 3rd law:

$$\vec{F}_{\text{ball}}(t) + \vec{F}_{\text{bat}}(t) = 0$$

at all times. Hence:

$$\vec{J}_{\text{bat}} = -\vec{J}_{\text{ball}}$$

$$|\vec{J}_{\text{bat}}| = |\vec{J}_{\text{ball}}|$$

# i-Clicker

Consider two ways to slow a toy car from $15\mathrm{km/h}$ to a complete stop.

$(i)$ The car slams into a wall and stops.

$(ii)$ The car gradually plows into a tube of gelatine and comes to a gradual halt.

In which case is the magnitude of the impulse bigger?

$A)$ Case (i)

$B)$ Case (ii)

$C)$ They are equal.

$D)$ Cannot be decided.

# Answer

Consider two ways to slow a toy car from $15\mathrm{km/h}$ to a complete stop.

$(i)$ The car slams into a wall and stops.

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In which case is the magnitude of the impulse bigger?

$A)$ Case (i)

$B)$ Case (ii)

$C)$ They are equal.

$D)$ Cannot be decided.

$\vec{J} = \Delta\vec{p}$, the same in both cases.