Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 11

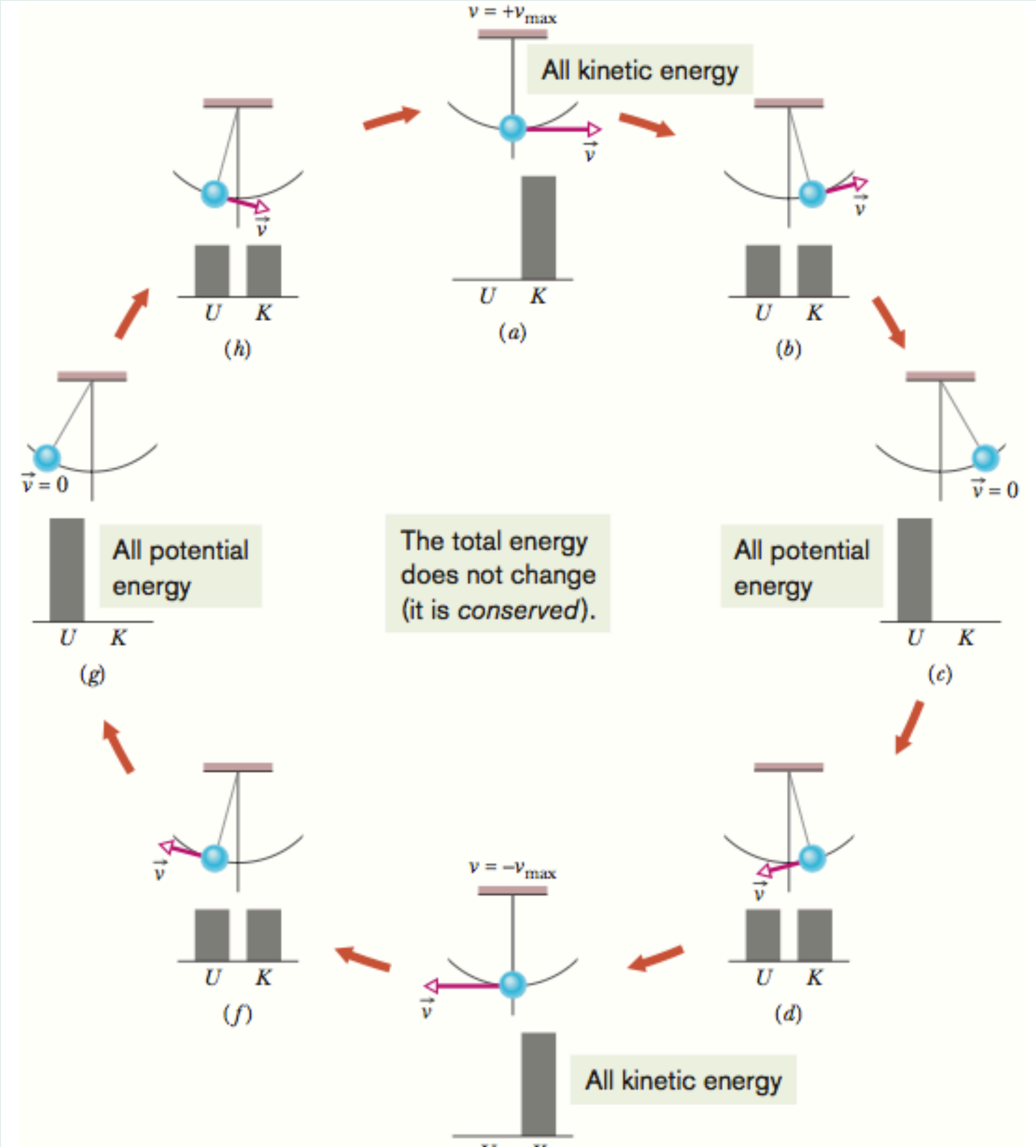
8. Potential Energy. Conservation of Energy

- **Conservation of Mechanical Energy:**

Conservative forces, **isolated** system

$$\Downarrow$$

$$\boxed{U + K = \text{constant}}$$



- ideal cord
- no friction

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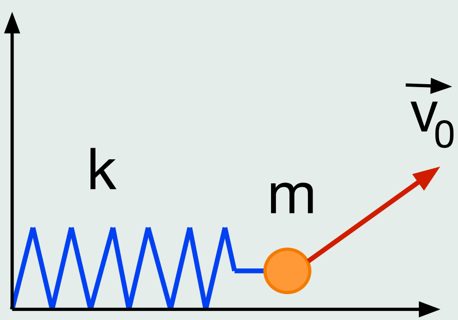
A ball attached to an ideal spring fixed at the origin is lounched on a frictionless horizontal surface with initial velocity $\vec{v}_0$. The spring is initially relaxed. What is the *maximum* possible value of the elongation of the spring during the resulting motion.



$A)\ v_0\sqrt{m/k}$

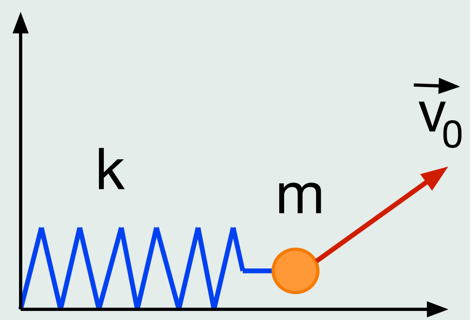
$B)\ v_0\sqrt{k/m}$

$C)\ mg/k$

$D)$ Cannot be determined from the data.

Answer

A ball attached to an ideal spring fixed at the origin is lounched on a frictionless horizontal surface with initial velocity $\vec{v}_0$. The spring is initially relaxed. What is the *maximum* possible value of the elongation of the spring during the resulting motion.



$A)\ v_0\sqrt{m/k}$

$B)\ v_0\sqrt{k/m}$

$C)\ mg/k$

$D)$ Cannot be determined from the data.

Energy conservation:

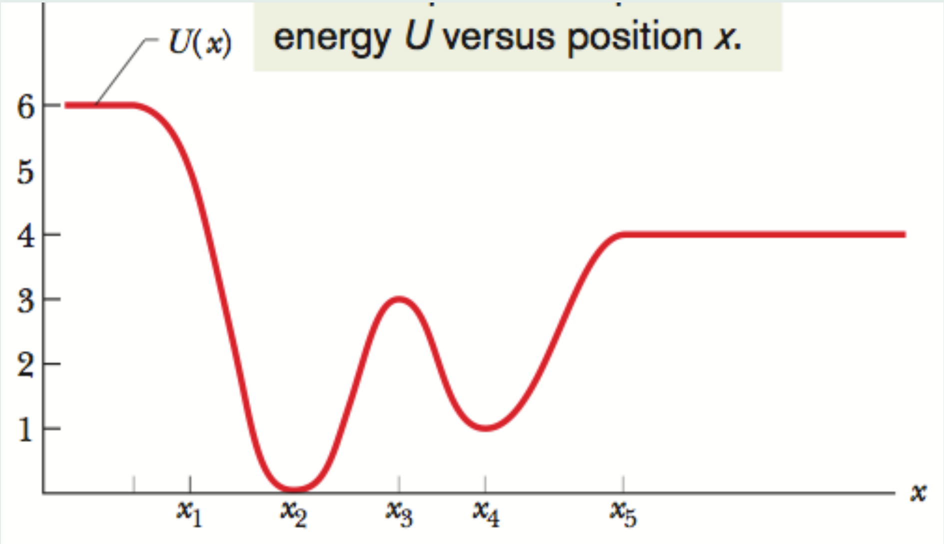
$$mv_0^2/2 = mv^2/2 + k(\Delta l)^2/2 \;\Rightarrow\; (\Delta l)_{\mathsf{max}} = v_0\sqrt{m/k}.$$

- **Reading a potential energy curve**

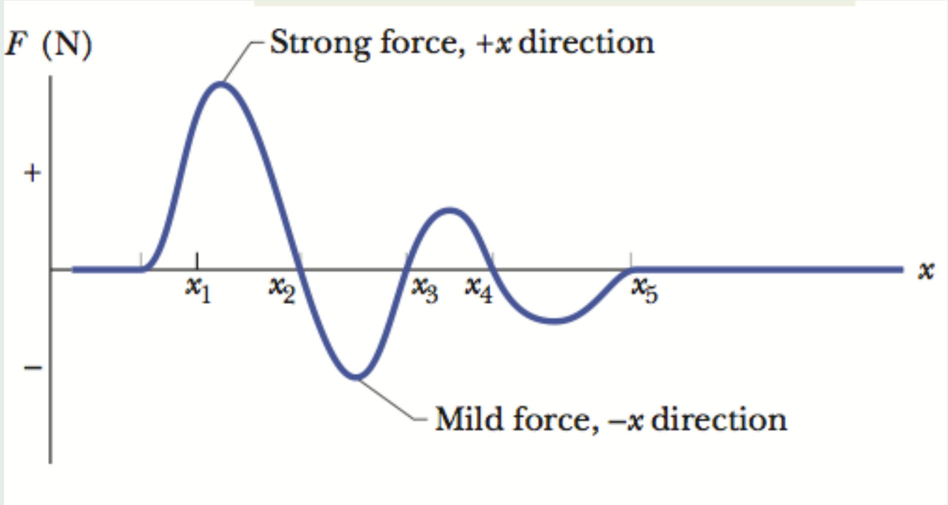
In 1D motion along the x-axis, conservative force:

$$\Delta U = -W = -\int_{x_i}^{x_f} F_x dx \;\Rightarrow\; dU(x) = -F_x(x)dx$$

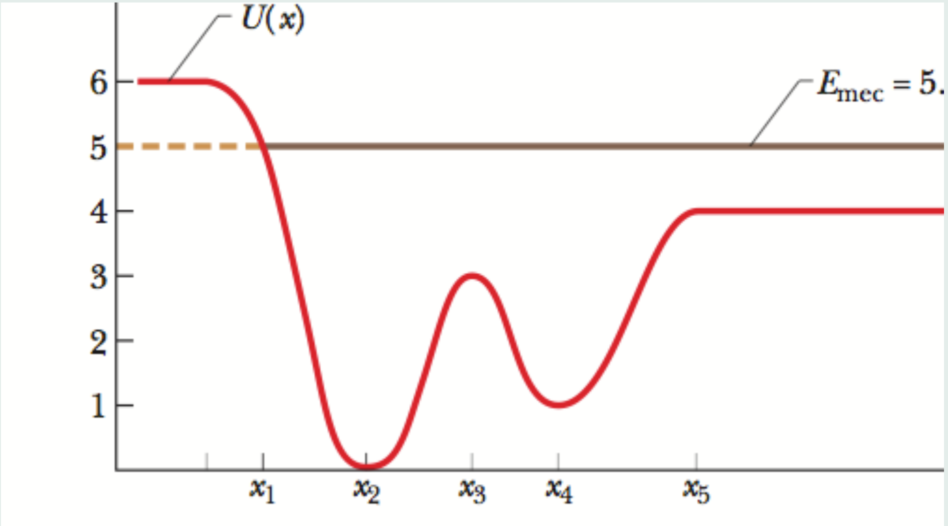
$$\boxed{F_x(x) = -\frac{dU}{dx}(x)}$$



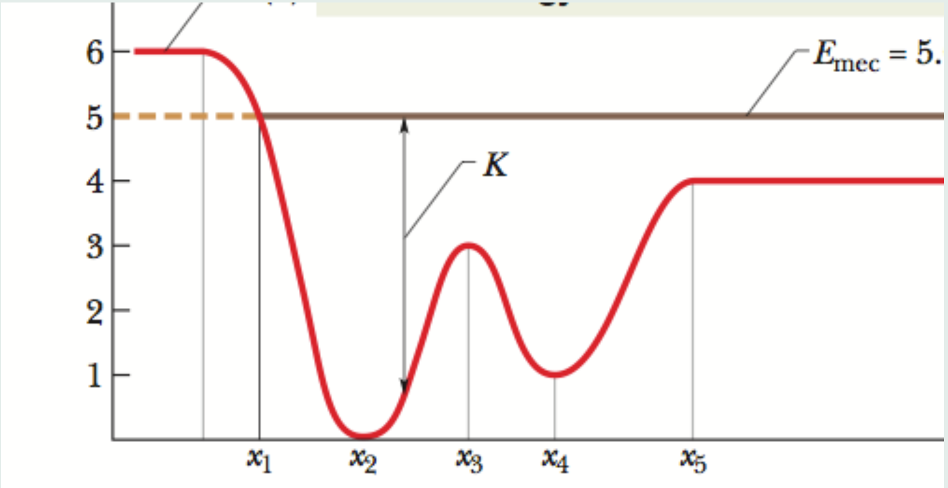
A plot of $U(x)$, the potential energy of a particle confined to move along an x axis. There is no friction, so mechanical energy is conserved.



A plot of the force F(x) acting on the particle, derived from the potential energy plot by taking its slope at various points.



- The total mechanical energy E_{mec} is conserved $\Rightarrow$ flat line.



$$K(x) = E_{\text{mec}} - U(x)$$

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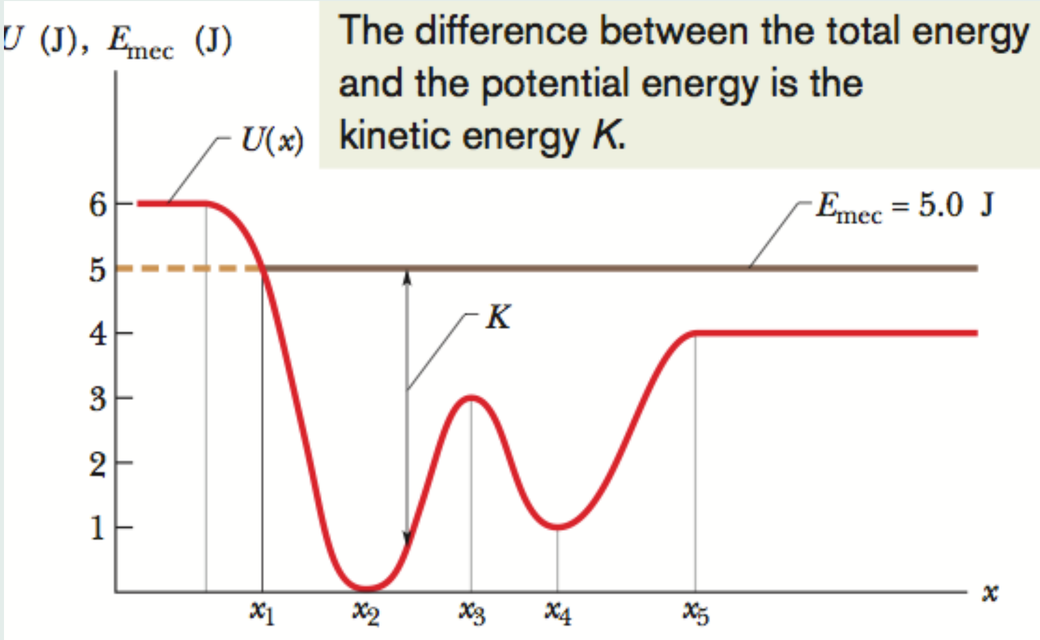
Can the particle ever be at $x < x_1$?



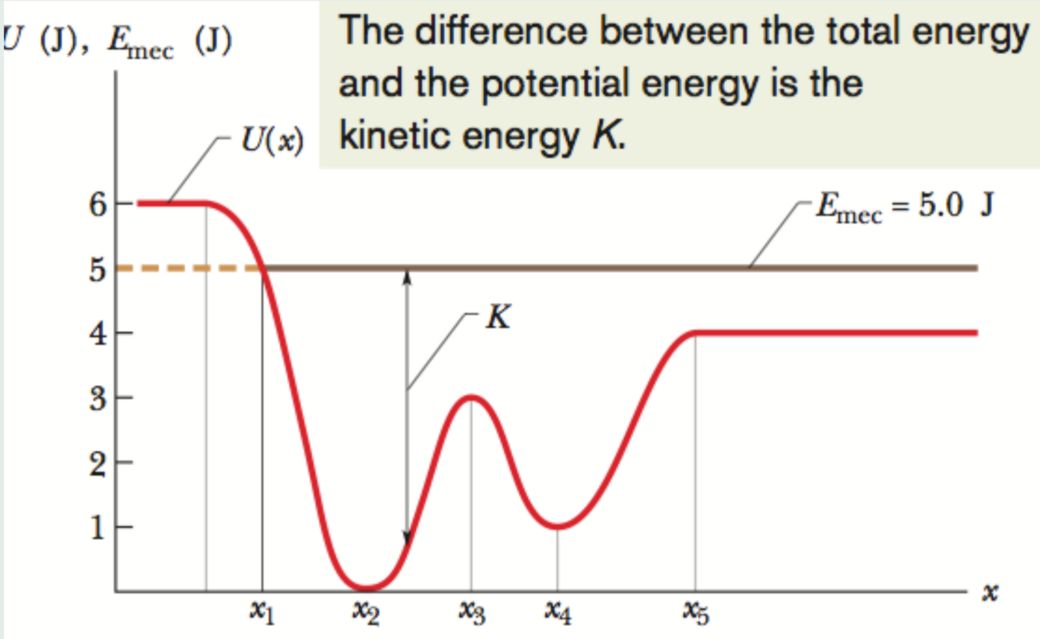
$A)$ Yes.

$B)$ No.

$C)$ Cannot be determined.

Answer

Can the particle ever be at $x < x_1$?



$A)$ Yes.

$B)$ No.

$C)$ Cannot be determined.

$$K = E_{\text{mec}} - U < 0 \quad \text{impossible!} \quad K = mv^2/2 \geq 0$$

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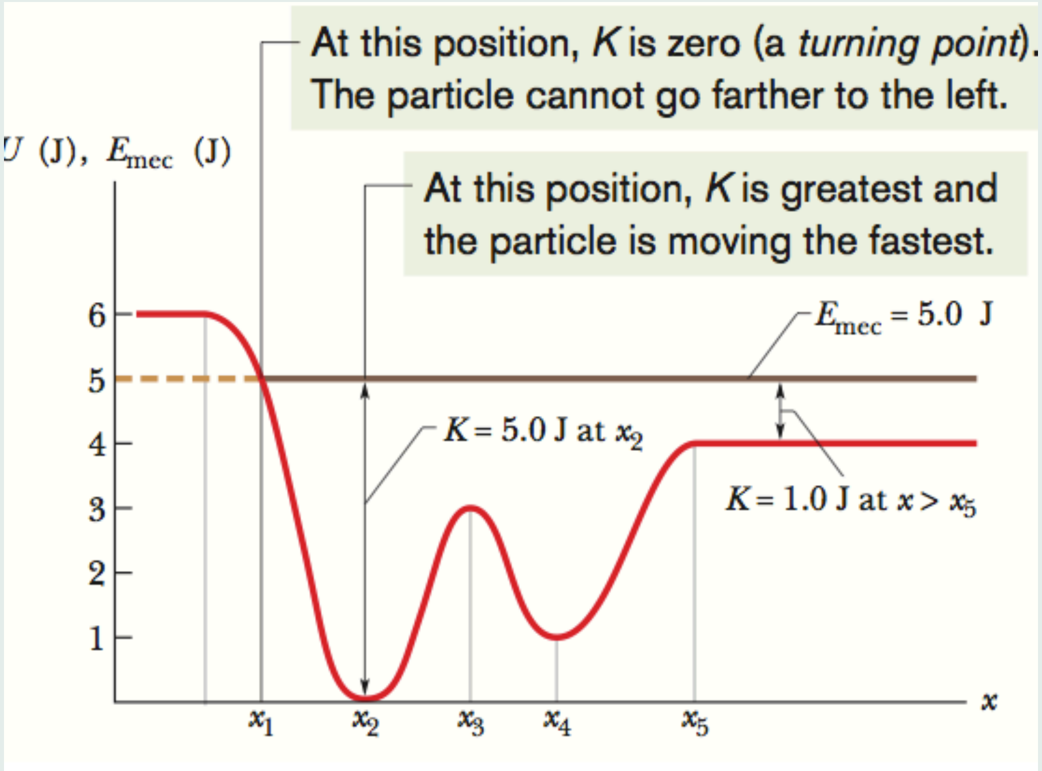
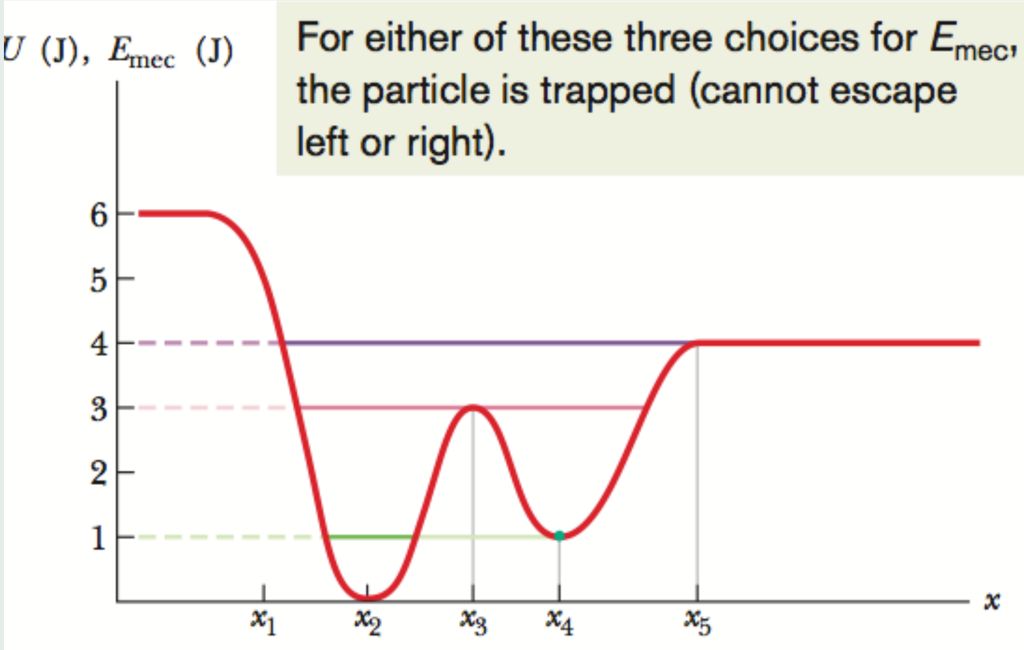


- Turning point:

$$K(x_1) = 0$$

- Classically forbidden region:

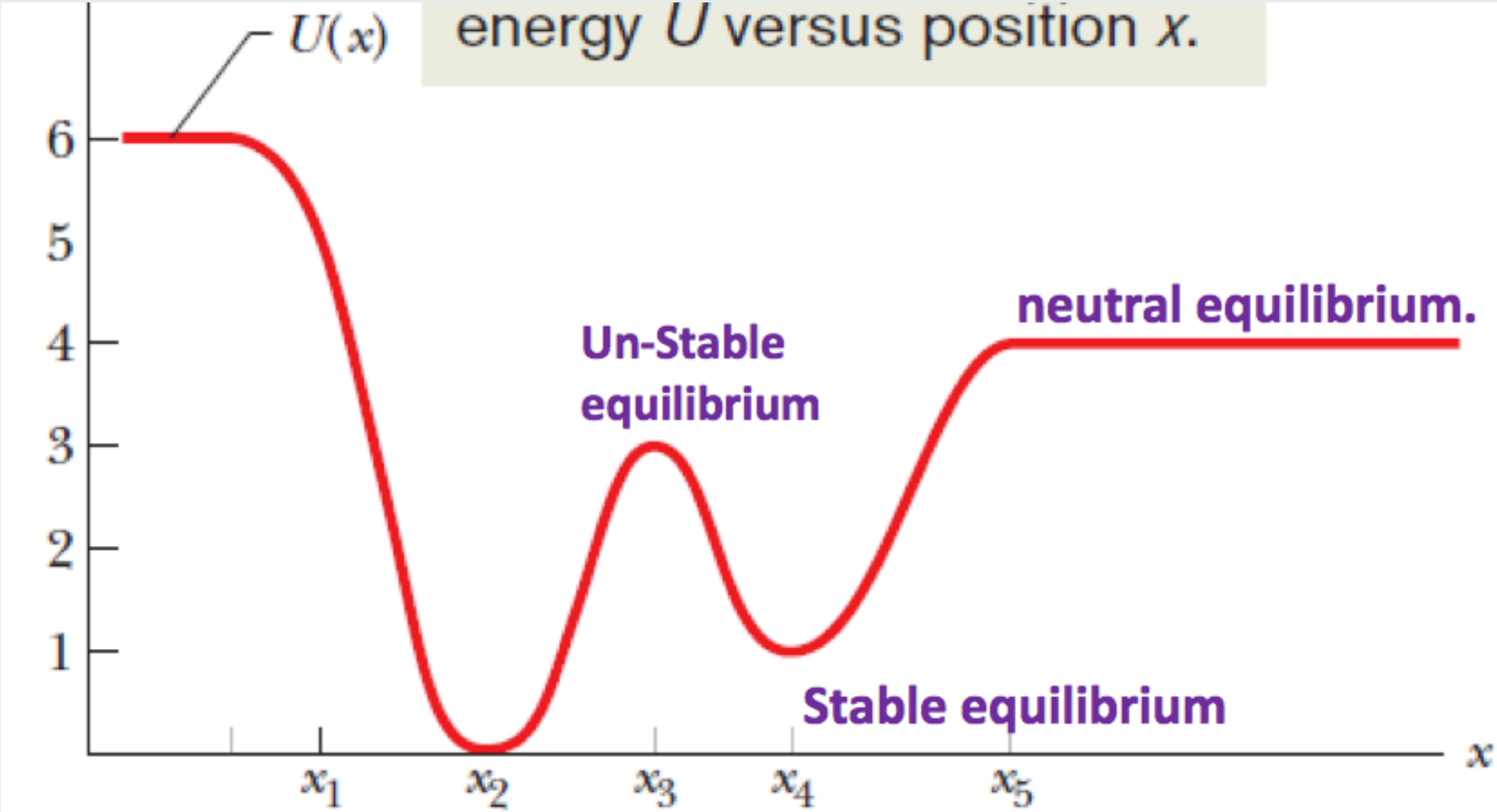
$$x < x_1$$



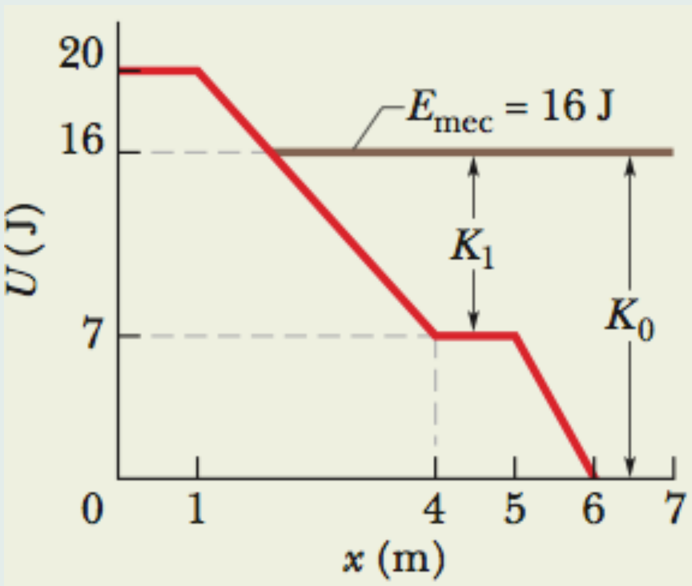
- $E_{\text{mec}} \leq 4\text{J} \Rightarrow$

Trapped particle

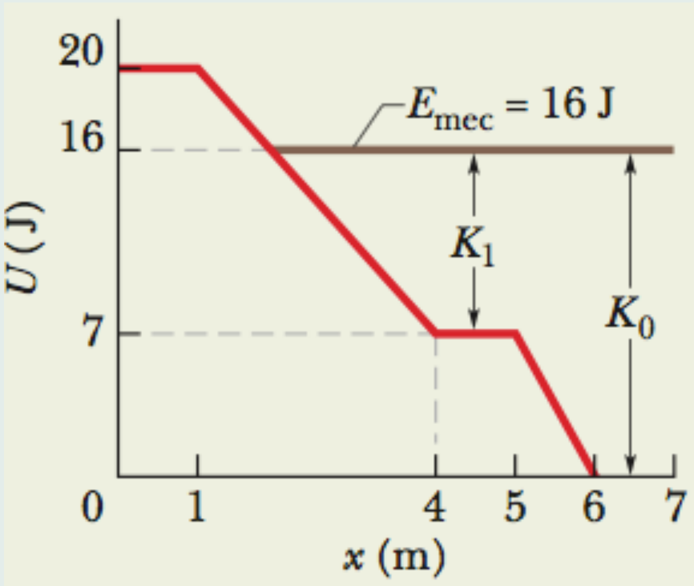
- **Equilibrium Points**: **extrema** of $U(x)$



Example:



- particle of mass $m = 2\text{kg}$ moving along the x-axis
- conservative force derived from attached $U(x)$ graph
- $x = 6.5\text{m}$, $v_{0x} = -4\text{m/s}$
- $x_1 = 4.5\text{m}$, $v_1 = ?$



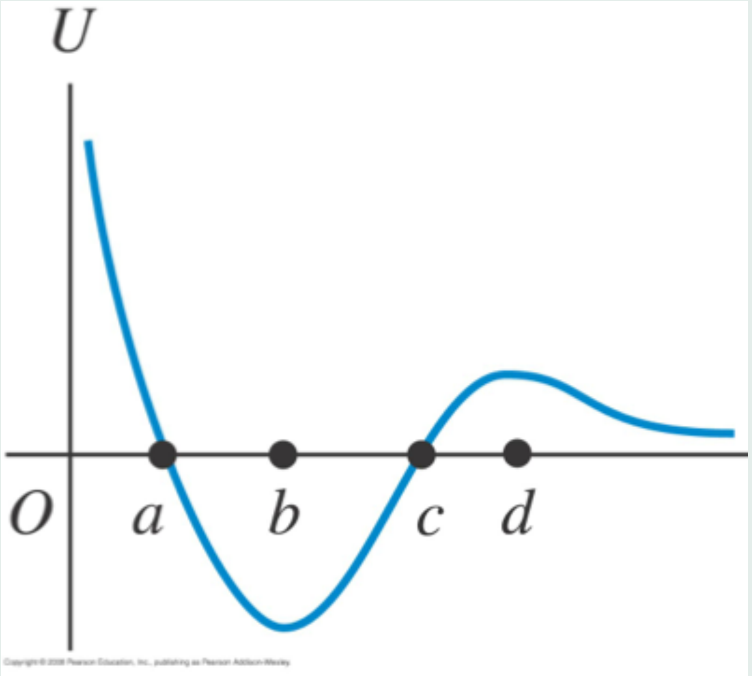
$$x = 6.5\text{m} \;\Rightarrow\; U = 0$$

$$E_{\text{mec}} = K_0 = mv_0^2/2 = 16\text{J}$$

$$K_1 = E_{\text{mec}} - U_1 = 16\text{J} - 7\text{J} = 9\text{J}$$

$$v_1 = \sqrt{\frac{2E_1}{m}} = 3\text{m/s}$$

i-Clicker



A particle is initially at $x = d$ and moves in the negative x-direction. Where does the particle have the greatest *speed*?

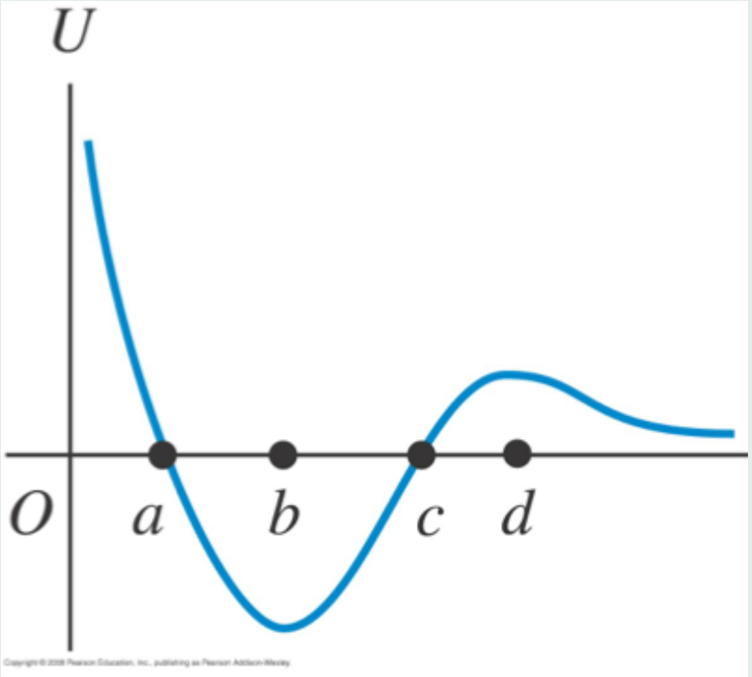
$A)\ x = a$

$B)\ x = b$

$C)\ x = c$

$D)\ x = d$

Answer



$E_{\text{mec}} = U + K = \text{constant}$
$K_{\max} \Leftrightarrow U_{\min}$

A particle is initially at $x = d$ and moves in the negative x-direction. Where does the particle have the greatest *speed*?

$A)\ x = a$

$B)\ x = b$

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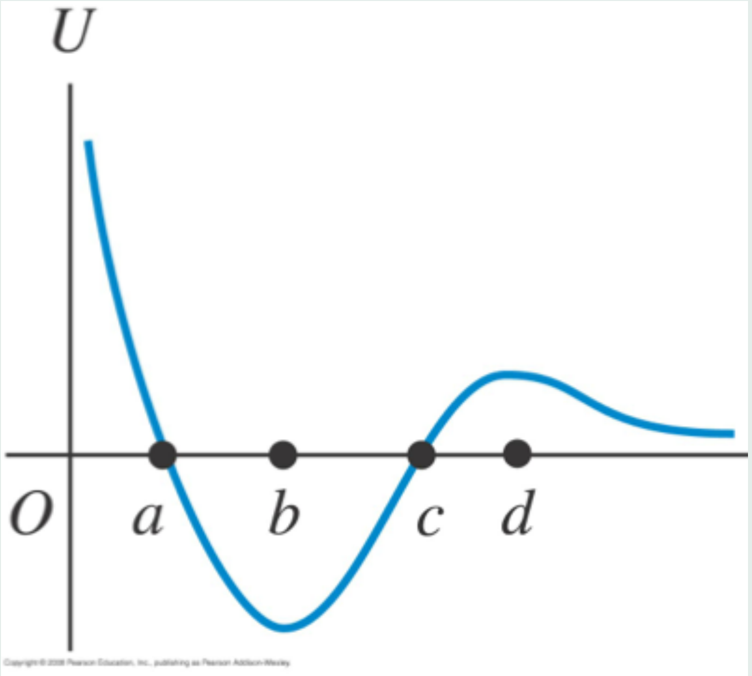
$D)\ x = d$

i-Clicker



A particle is initially at $x = d$ and moves in the negative x-direction. Where is the particle *slowing down*?

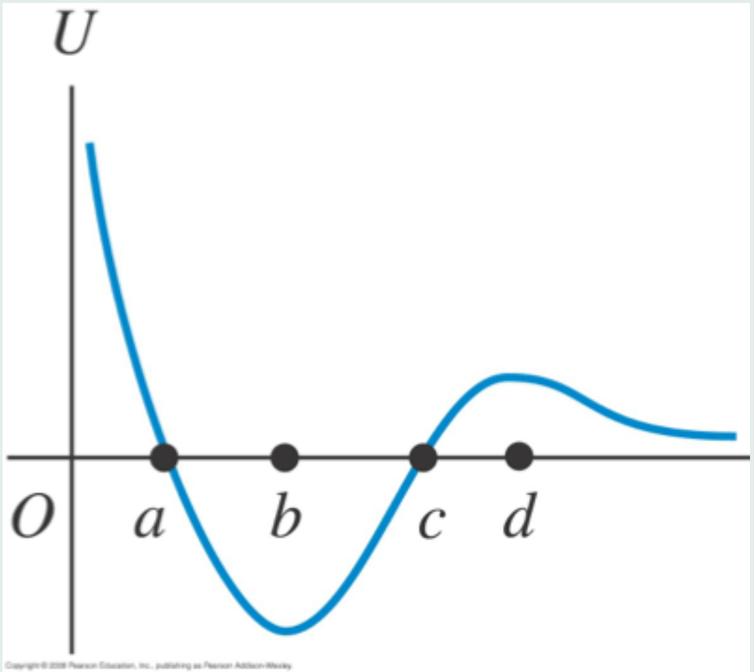
A) $x = a$

B) $x = b$

C) $x = c$

D) *nowhere*

Answer



A particle is initially at $x = d$ and moves in the negative x-direction. Where is the particle *slowing down*?

A) $x = a$

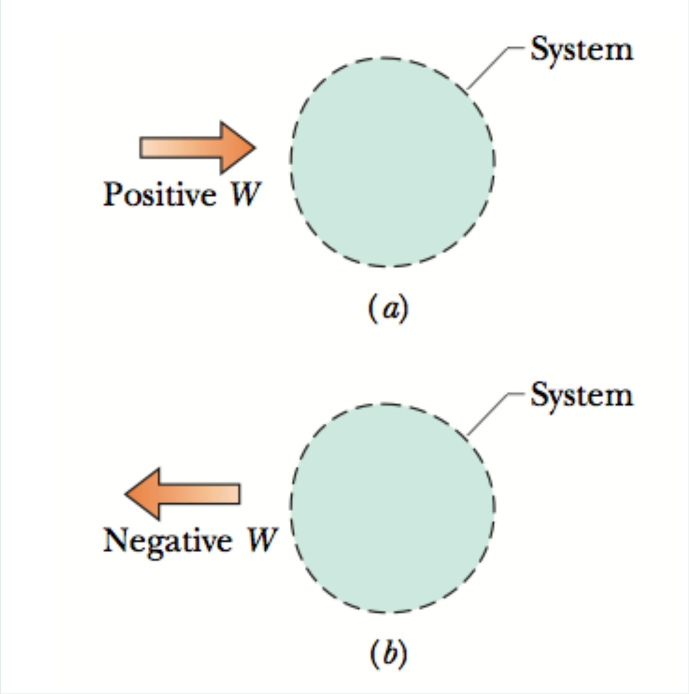
B) $x = b$

C) $x = c$

D) $nowhere$

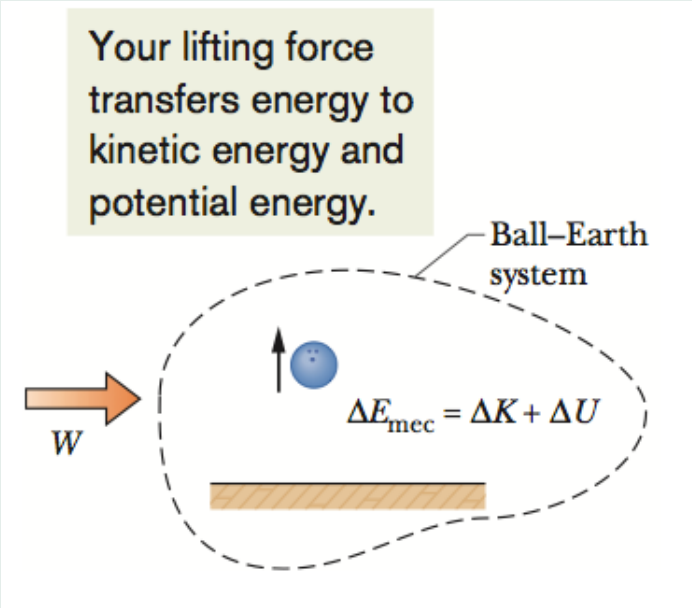
$$a_x > 0 \;\Leftrightarrow\; F_x > 0 \;\Leftrightarrow\; dU/dx < 0$$

- **Work done on a system by an external force**



Work is energy transferred to or from a system by means of an external force acting on that system.

Example:



$$W = \Delta K + \Delta U = \Delta E_{\text{mec}}$$

Applied external force

$$\vec{F} = F\hat{j}$$

Newton's 2nd law:

$$ma_y = F - mg$$

Constant acceleration model:

$$v^2 = v_0^2 + 2a_y\Delta y$$

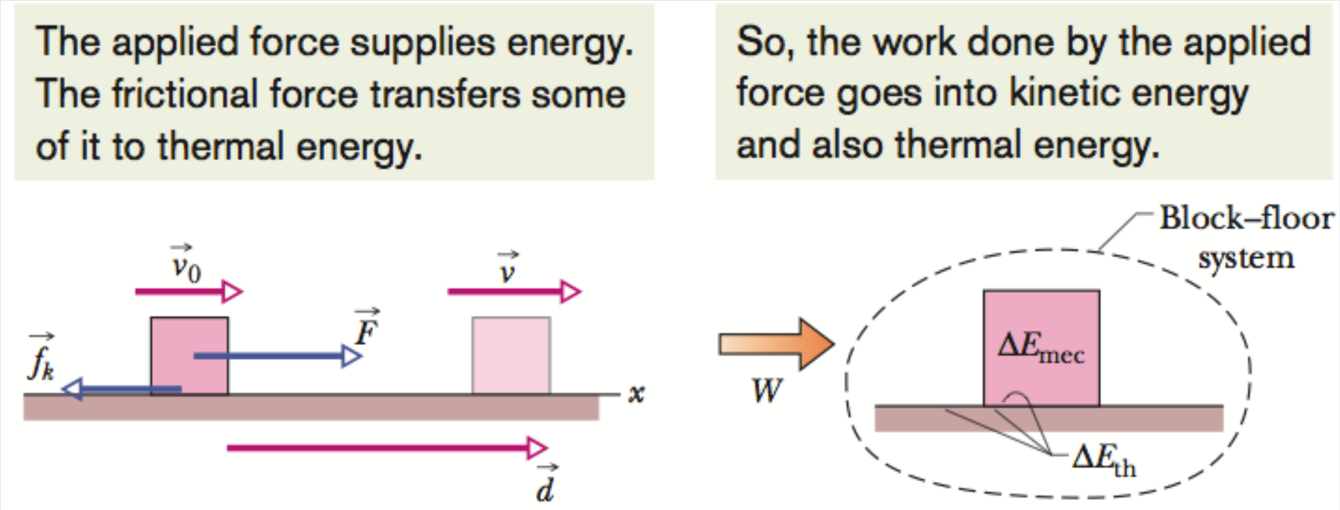
$$F\Delta y = \frac{mv^2}{2} - \frac{mv_0^2}{2} + mg\Delta y$$
$$= \Delta K + \Delta U$$

- **Conservative forces inside the system**

$$W_{\text{ext}} = \Delta K + \Delta U = \Delta E_{\text{mec}}$$

W_{ext} : total work done on the system by external forces

Example:

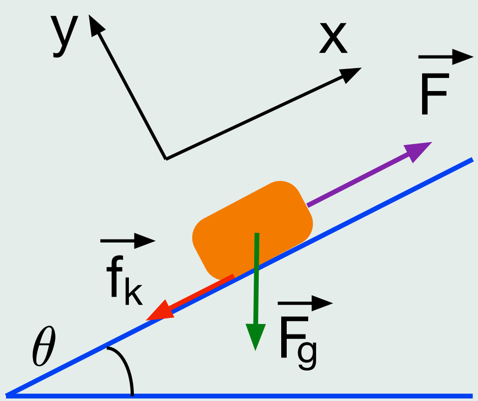


$$F - f_k = ma_x \qquad v^2 = v_0^2 + 2a_x\Delta x$$

$$F\Delta x = \frac{mv^2}{2} - \frac{mv_0^2}{2} + f_k\Delta x$$

$$F\Delta x = \Delta K + f_k\Delta x$$

Example:



$$ma_x = F - f_k - mg\sin\theta$$

$$v^2 = v_0^2 + 2a_x\Delta x$$

$$\begin{aligned} F\Delta x &= \frac{mv^2}{2} - \frac{mv_0^2}{2} + mg\Delta x\sin\theta \\ &\quad + f_k\Delta x \\ &= \Delta K + \Delta U + f_k\Delta x \end{aligned}$$

$$W_F = \Delta K + \Delta U + |W_{\text{friction}}|$$

- **Conservative and dissipative forces inside the system:**

$$W_{\text{ext}} = \Delta U + \Delta K + |W_{\text{dissipative}}|$$

$$W_{\text{ext}} = \Delta E_{\text{mec}} + |W_{\text{dissipative}}|$$

Note: work of friction or drag forces

$$\Downarrow$$

heat

$$\Downarrow$$

Increase in **thermal** energy

$$|W_{\text{dissipative}}| = \Delta E_{\text{th}}$$

Law of Conservation of Energy

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

$$W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{int}$$

- ΔE_{mec} is any change in the mechanical energy of the system,
- ΔE_{th} is any change in the thermal energy of the system,
- ΔE_{int} is any change in any other type of internal energy of the system.

The total energy E of an isolated system cannot change.

$$\Delta E_{\mathsf{mec}} + \Delta E_{\mathsf{th}} + \Delta E_{int} = 0 \quad \text{(isolated system)}$$

- **Power:** the rate at which the energy is transferred by a force from one type to another

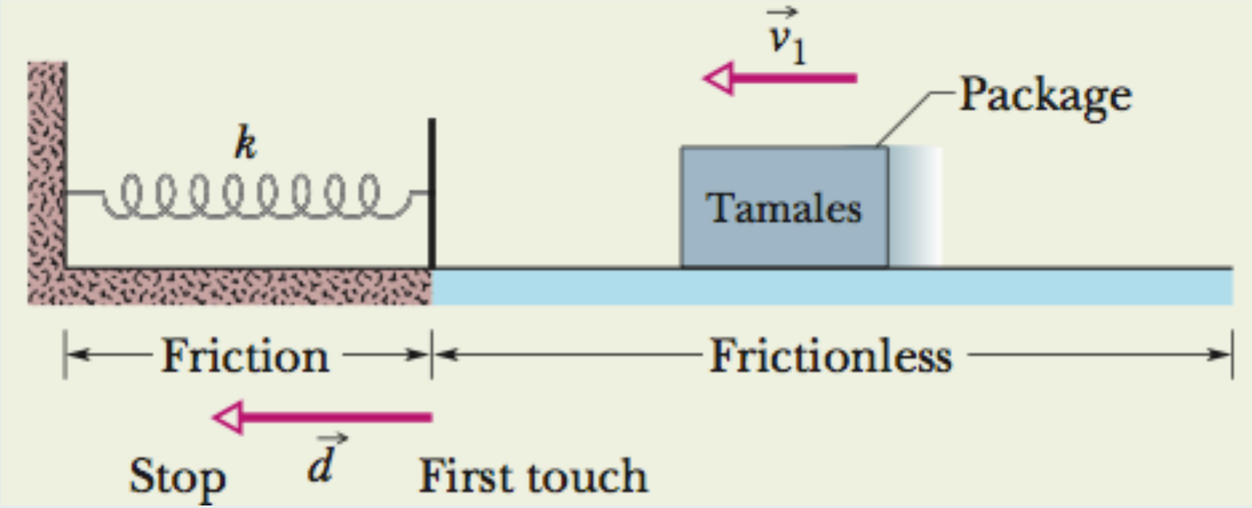
Average power:

$$P_{\text{avg}} = \frac{\Delta E}{\Delta t}$$

Instantaneous power:

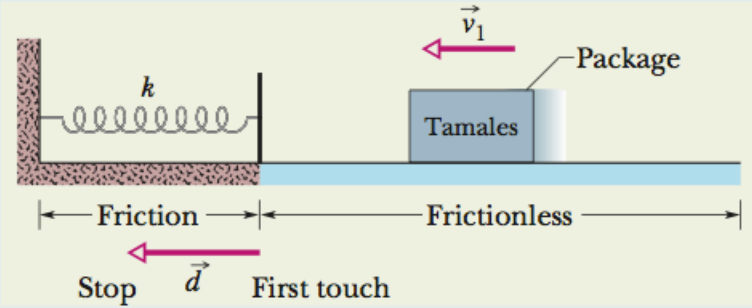
$$P = \frac{dE}{dt}$$

Example: an object slides along a frictionless floor with speed v_1. It hits a relaxed spring placed on rough surface such that the kinetic friction is f_k. What is the distance d when the object stops?



Isolated system:

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} = 0$$



$$(E_{\text{mec}})_i = \frac{mv_1^2}{2} \quad (E_{\text{mec}})_f = \frac{kd^2}{2}$$

$$\Delta E_{\text{mec}} = \frac{kd^2}{2} - \frac{mv_1^2}{2}$$

$$\Delta E_{\text{th}} = f_k d$$

$$\frac{kd^2}{2} + f_k d - \frac{mv_1^2}{2} = 0$$