Rutgers University
Department of Physics & Astronomy

01:750:271 Honors Physics I
Fall 2015

Lecture 10

Midterm I summary

100	90	80	70	60	50	40	30	20
39	43	56	28	11	5	3	0	1

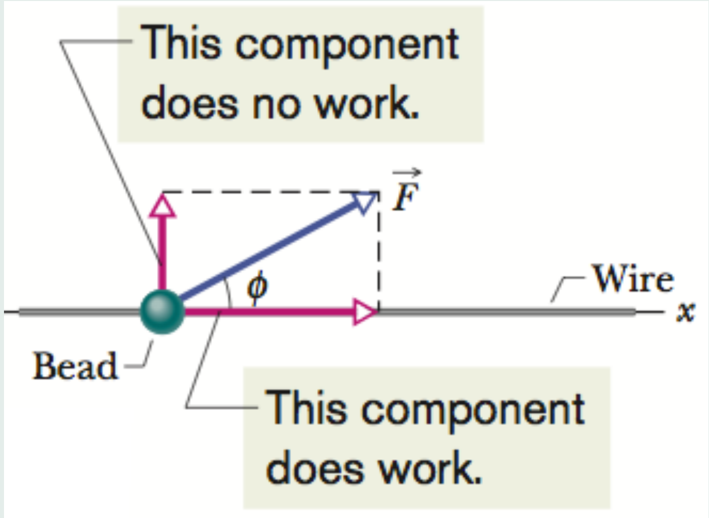
Average: 82.00

7. Kinetic energy and work

- **Kinetic Energy:** energy associated to the motion of an object

$$\boxed{K = \frac{1}{2} m v^2}$$

- **Work done by an applied force**



- Work done by the force $\vec{F}$

$$W = Fd\cos\phi = \vec{F}\cdot\vec{d}$$

$$\vec{d} = (\Delta x)\widehat{i}$$

displacement vector

- **Work-Kinetic Energy Theorem**

$$\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}.$$

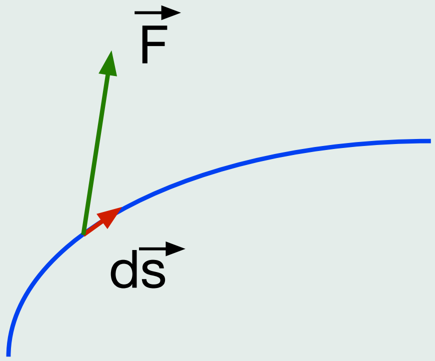
$$\Delta K = W_{\text{net}} \qquad K_f = K_i + W_{\text{net}}$$

$$W_{\text{net}} = \sum W = \sum \vec{F}\cdot\vec{d} = \left(\sum \vec{F}\right)\cdot\vec{d} = \vec{F}_{\text{net}}\cdot\vec{d}$$

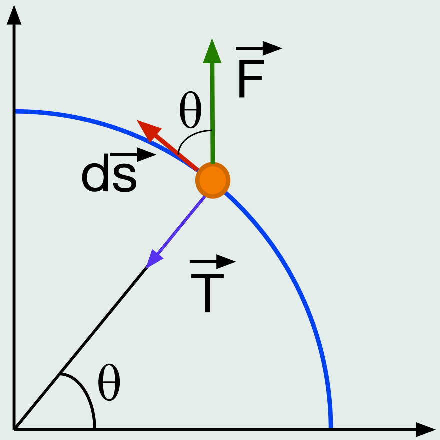
- **Variable force, curved trajectory**

$$W = \int_{\text{trajectory}} dW = \int_{\text{trajectory}} \vec{F} \cdot d\vec{s}$$

$d\vec{s} = \vec{v}dt$ infinitesimal displacement



• **Example:** A ball tied at the end of a string of length r moves on a circular trajectory under an applied force $\vec{F} = F\hat{j}$.



$$\vec{F}_{\text{net}} = \vec{F} + \vec{T}$$

$$\vec{F}_{\text{net}} \cdot d\vec{s} = \vec{F} \cdot d\vec{s} = F ds \cos\theta$$

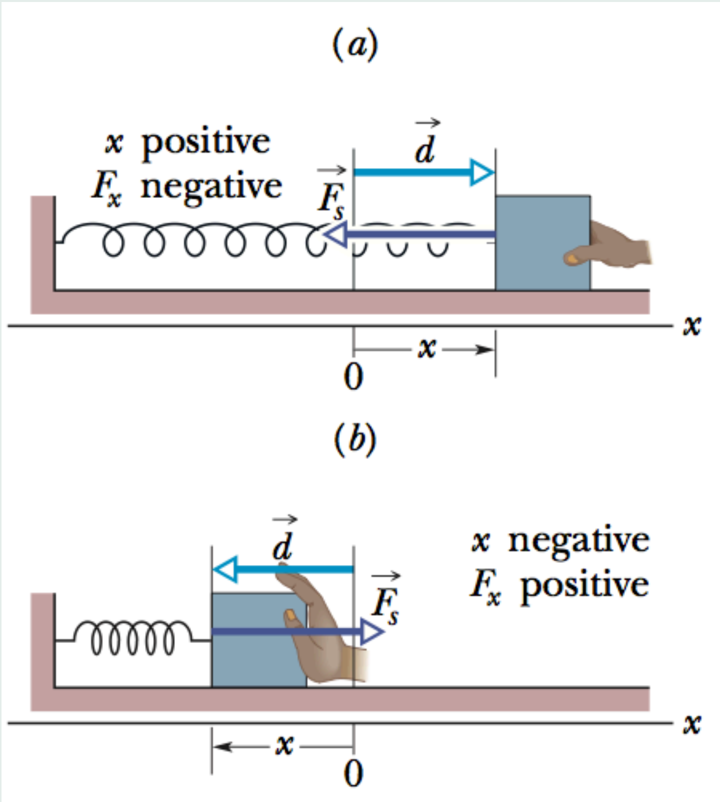
$$W = \int_0^{\pi/2} F \cos\theta \, r d\theta$$

$$= Fr \int_0^{\pi/2} \cos\theta \, d\theta$$

$$= Fr \big(\sin(\pi/2) - \sin(0)\big) = Fr$$

$$\frac{mv^2}{2} = \frac{mv_0^2}{2} + Fr \;\Rightarrow\; v = \sqrt{v_0^2 + \frac{2Fr}{m}}$$

- **Work done by a spring force**

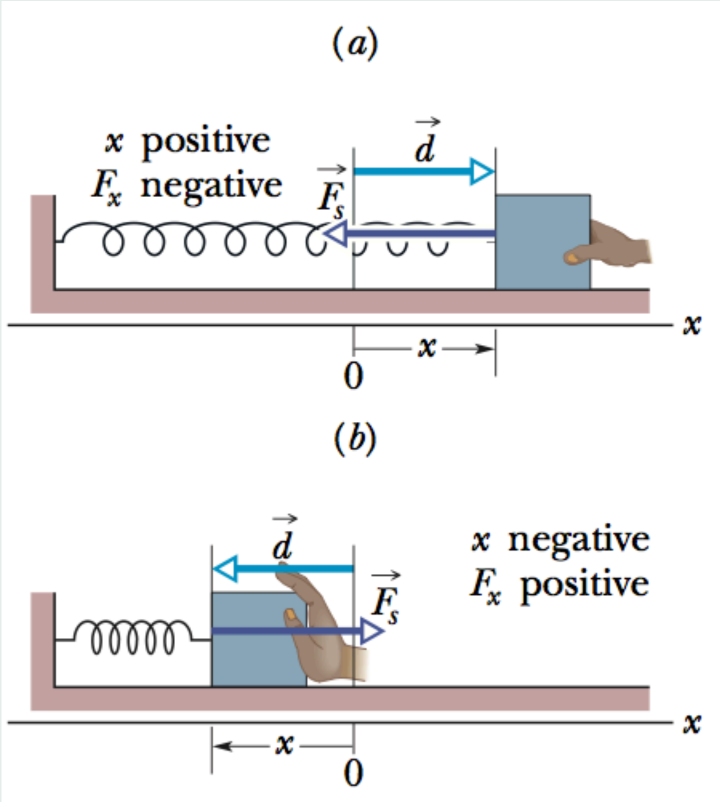


- **Hooke's Law**

$$\vec{F}_s = -k\vec{d}$$

- always **opposed** to displacement (restoring force)

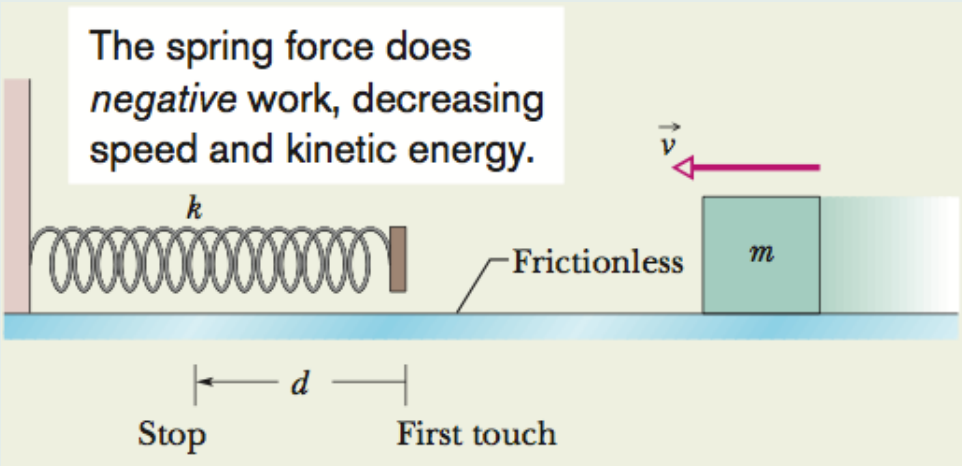
- $k > 0$ **spring constant**

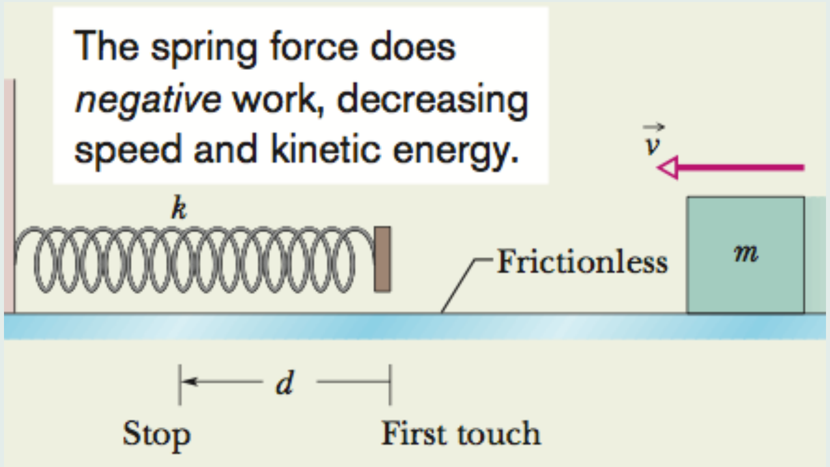


$$
\begin{aligned}
W_s &= \int_{x_i}^{x_f} F_x dx \\
&= \int_{x_i}^{x_f} -kx dx \\
&= (-k) \int_{x_i}^{x_f} x dx \\
&= (-k/2)\left(x_f^2 - x_i^2\right)
\end{aligned}
$$

$$
\boxed{W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2}
$$

• **Example:** an object of mass m slides across a horizontal frictionless surface with speed v. It then runs into and compresses a spring of spring constant k. When the object is momentarily stopped by the spring, by what distance d is the spring compressed?





Total work done by the spring force:

$$W_s = \frac{kx_i^2}{2} - \frac{kx_f^2}{2} = -\frac{kd^2}{2}$$

Work-kinetic energy theorem

$$W_s = K_f - K_i = -\frac{mv^2}{2}$$

$$\frac{mv^2}{2} = \frac{kd^2}{2} \quad \Rightarrow \quad d = v\sqrt{\frac{m}{k}}$$

- **Power**: time rate at which work is done by a force.

If a force does an amount of work W in an amount of time Δt, the **average power** during that time interval is:

$$P_{\text{average}} = \frac{W}{\Delta t}$$

The **instantaneous power** P is the instantaneous time rate of doing work

$$P = \frac{dW}{dt} \qquad dW = \vec{F} \cdot d\vec{s} = \vec{F} \cdot \vec{v} dt \qquad \boxed{P = \vec{F} \cdot \vec{v}}$$

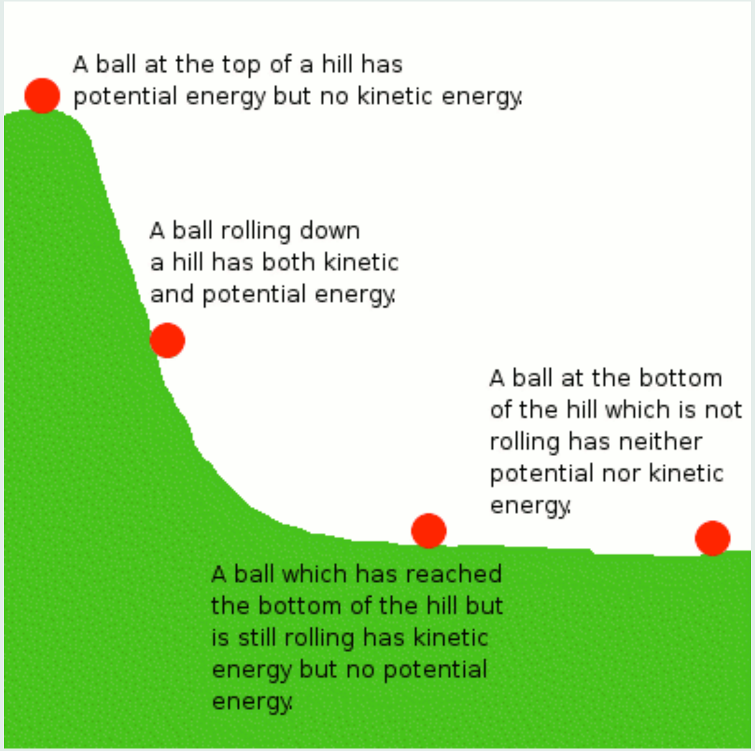
- Units: Watt

$$1 \ \text{Watt} = 1\,\text{W} = 1\,\text{J/s}$$

8. Potential energy. Conservation of energy

- **Potential energy:** energy associated with the configuration of a system of objects that exert forces on one another.

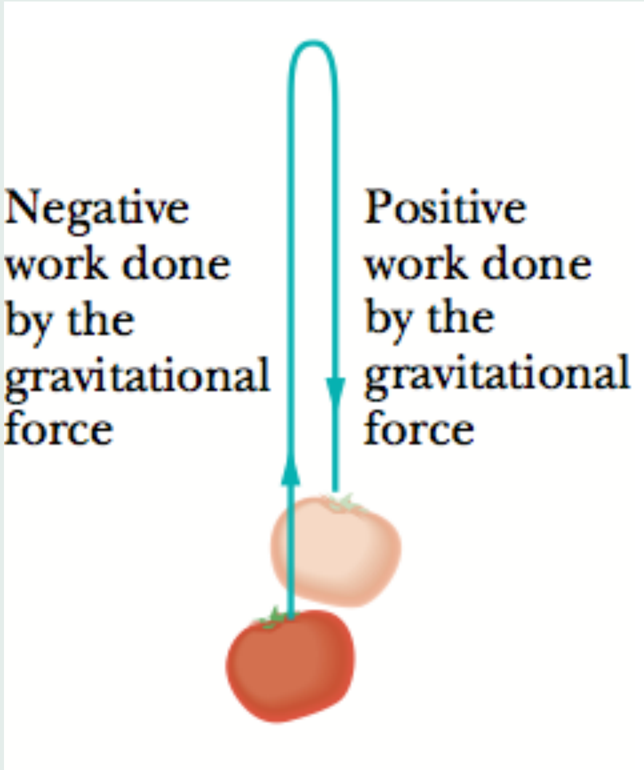
- can be converted into **kinetic energy** by allowing the system to evolve freely



Elastic potential energy

Gravitational potential energy

- **Work and potential energy**



First part of motion:

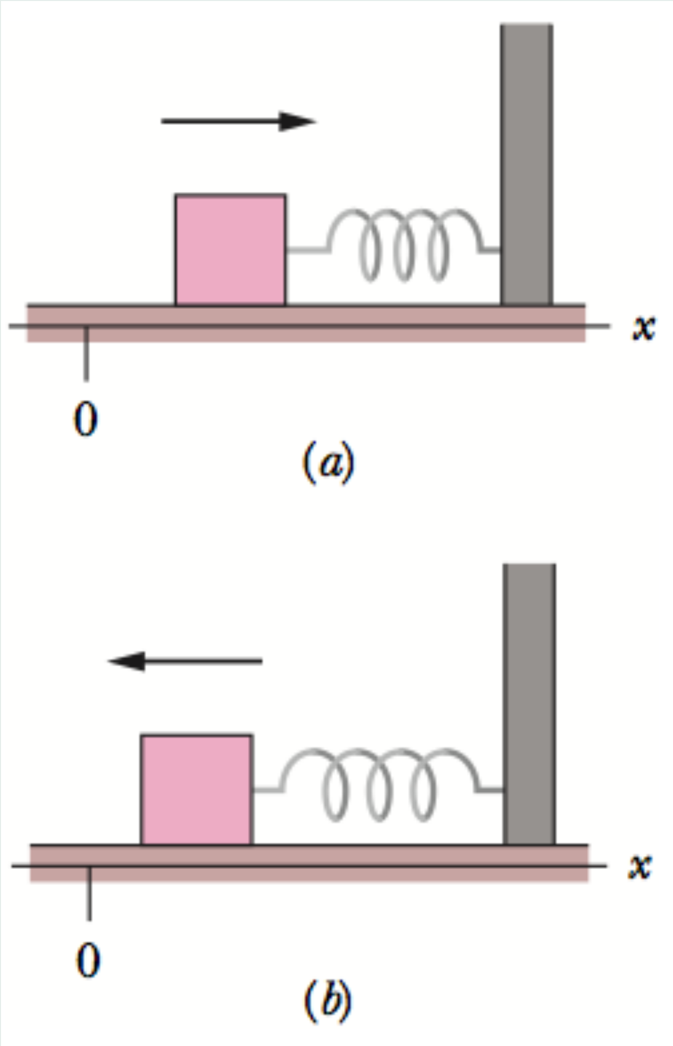
$$W_{F_g} = \Delta K < 0, \qquad K \searrow$$

energy transferred from kinetic energy to gravitational potential energy.

Second part of motion:

$$W_{F_g} = \Delta K > 0, \qquad K \nearrow$$

energy transferred from gravitational potential energy to kinetic energy.



First part of motion:

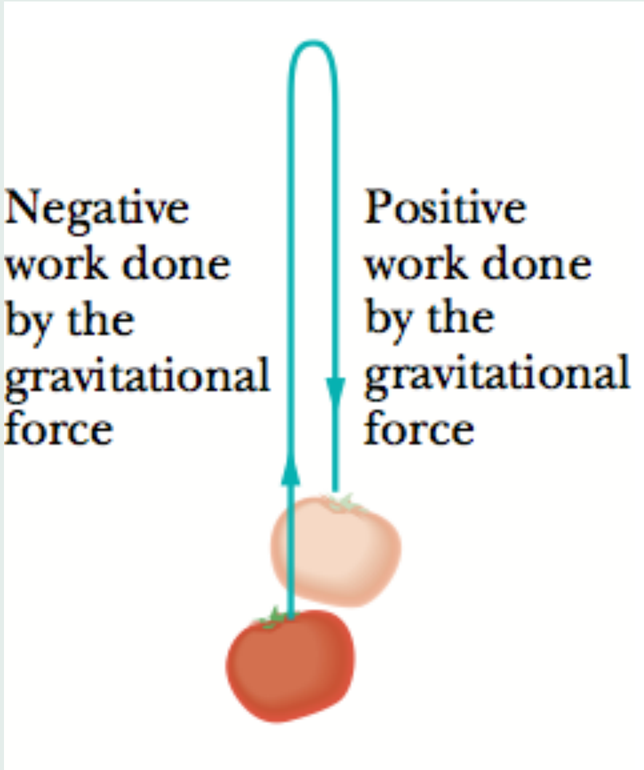
$$W_{F_s} = \Delta K < 0, \qquad K \searrow$$

energy transferred from kinetic energy to elastic potential energy.

Second part of motion:

$$W_{F_s} = \Delta K > 0, \qquad K \nearrow$$

energy transferred from elastic potential energy to kinetic energy.



First part of motion:

$$W_{F_g} = mg(y_0 - y_{\mathsf{max}})$$

$$\Delta K = -\frac{mv_0^2}{2}$$

Constant acceleration model:

$$\frac{mv_0^2}{2} = mg(y_{\mathsf{max}} - y_0)$$

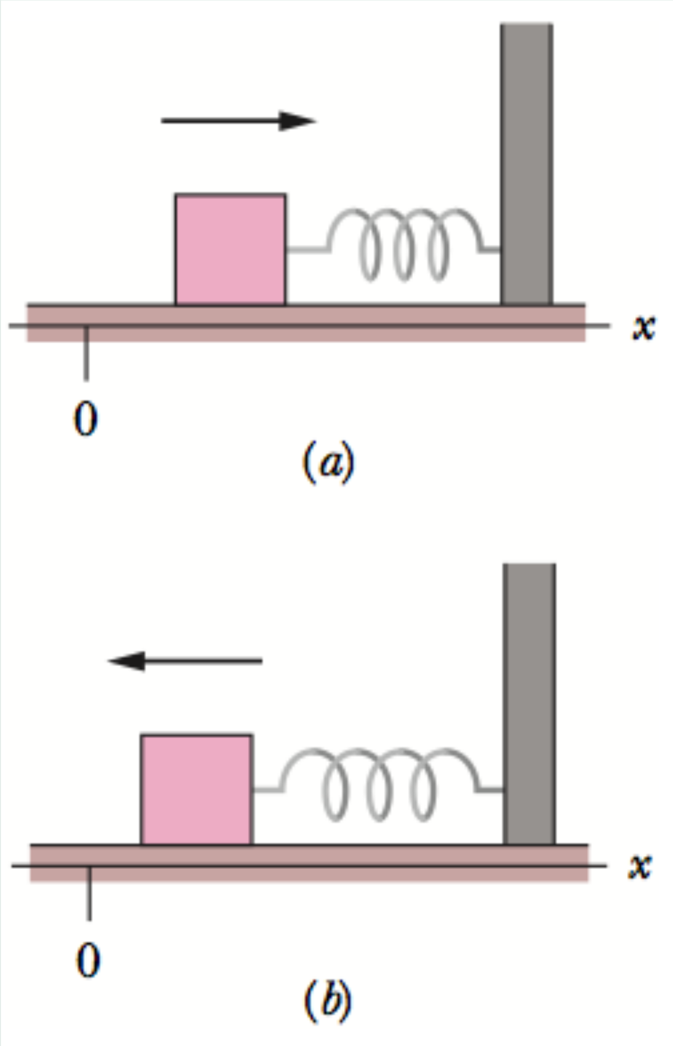
Second part of motion:

$$W_{F_g} = mg(y_{\mathsf{max}} - y_0)$$

$$\Delta K = \frac{mv^2}{2}$$

Constant acceleration model:

$$\frac{mv^2}{2} = mg(y_{\mathsf{max}} - y_0)$$



First part of motion:

$$W_{F_s} = -\frac{kx_{\max}^2}{2}$$

$$W_{F_s} = \Delta K = -\frac{mv_0^2}{2}$$

$$\frac{mv_0^2}{2} = \frac{kx_{\max}^2}{2}$$

Second part of motion:

$$W_{F_s} = \frac{kx_{\max}^2}{2}$$

$$W_{F_s} = \Delta K = \frac{mv^2}{2}$$

$$\frac{mv^2}{2} = \frac{kx_{\max}^2}{2}$$

- Note that in both examples examples

$$W_{\text{1−st part}} = -W_{\text{2−nd part}}$$

Gravitational force:

$$\Delta(mgy) = -W_{F_g} \qquad K + mgy = \text{constant}$$

Elastic force:

$$\Delta(kx^2/2) = -W_s \qquad K + \frac{kx^2}{2} = \text{constant}$$

Naturally led to:

- **Gravitational potential energy:**

$$U_g = mgy$$

- **Elastic potential energy:**

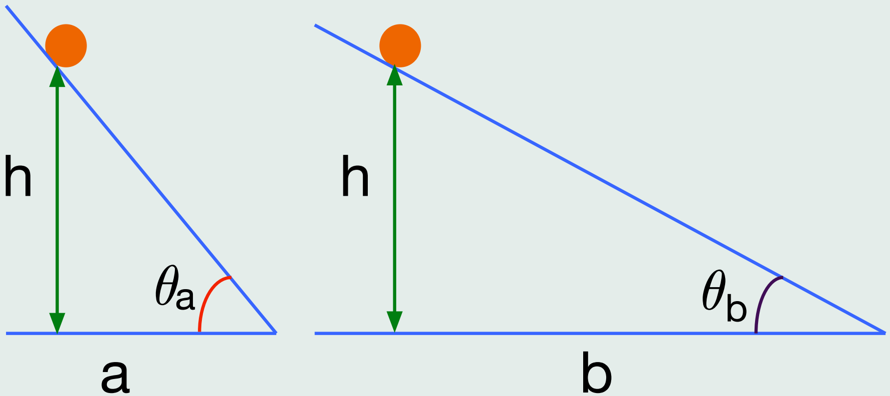
$$U_s = \frac{kx^2}{2}$$

- **Energy conservation:**

$$K + U_g = \text{constant} \qquad K + U_s = \text{constant}$$

i-Clicker

Which of the following statements is true?



$A)\ W_{F_g}^{(a)} > W_{F_g}^{(b)}$

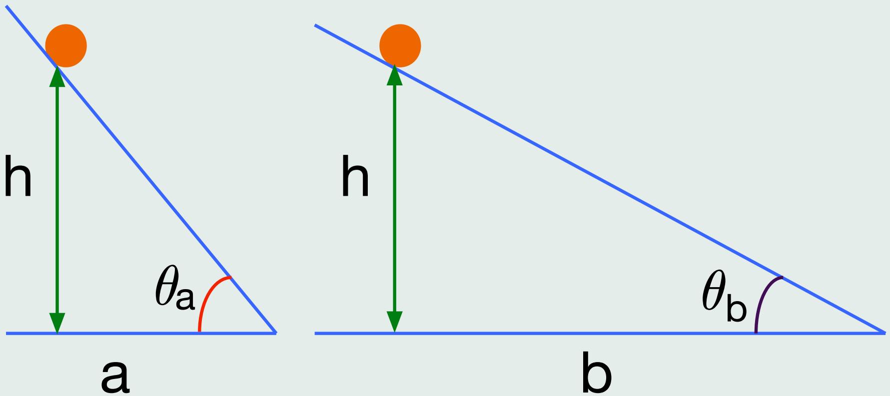
$B)\ W_{F_g}^{(a)} < W_{F_g}^{(b)}$

$C)\ W_{F_g}^{(a)} = W_{F_g}^{(b)}$

$D)$ none of the above.

Answer

Which of the following statements is true?



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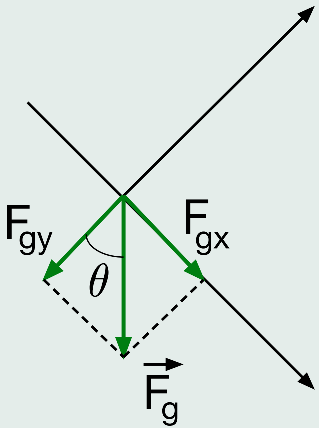


$$W = \vec{F}_g \cdot \vec{d} = F_{gx} \Delta x$$

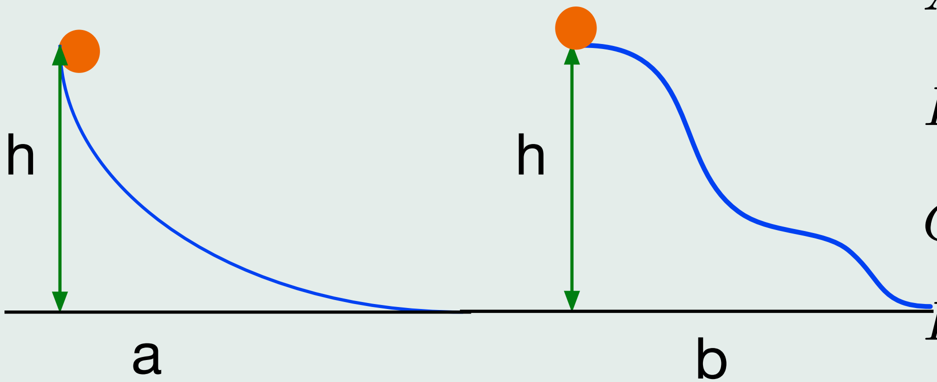
$$F_{gx} = mg\sin\theta$$

$$\Delta x = \frac{h}{\sin\theta}$$

$$W = mgh$$

i-Clicker

Which of the following statements is true?



A) $W_{F_g}^{(a)} > W_{F_g}^{(b)}$

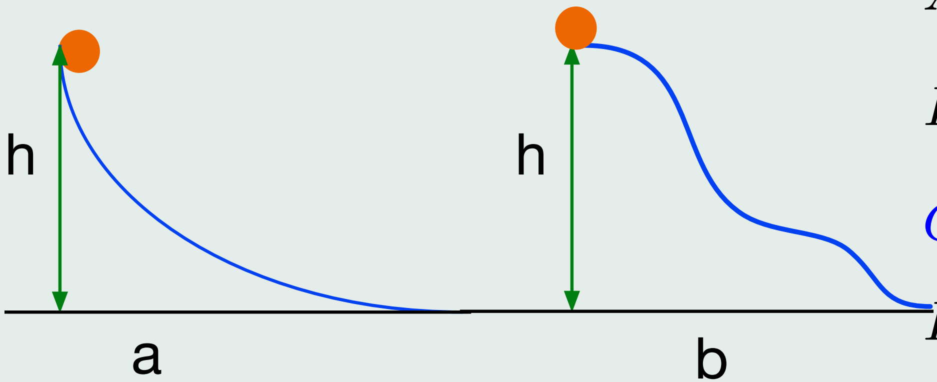
B) $W_{F_g}^{(a)} < W_{F_g}^{(b)}$

C) $W_{F_g}^{(a)} = W_{F_g}^{(b)}$

D) none of the above.

Answer

Which of the following statements is true?



$A)\ W_{F_g}^{(a)} > W_{F_g}^{(b)}$

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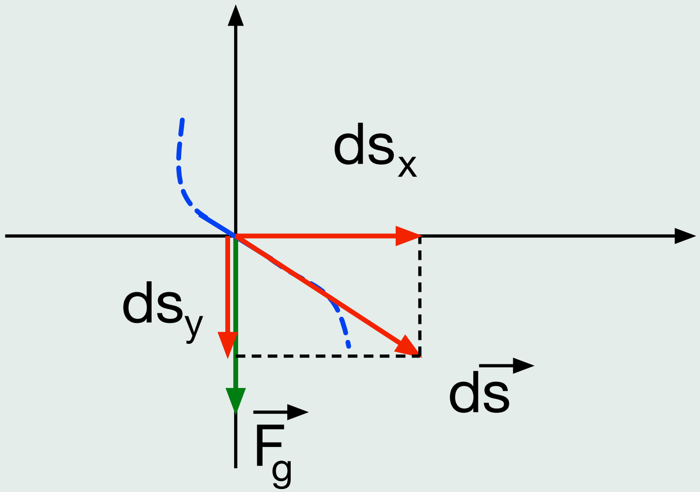


$$W_g = \int \vec{F}_g \cdot d\vec{s}$$

$$\vec{F}_g \cdot d\vec{s} = mgds_y$$

$$\begin{aligned} W_g &= \int_0^h mgds_y \\ &= mg \int_0^h ds_y \\ &= mgh. \end{aligned}$$

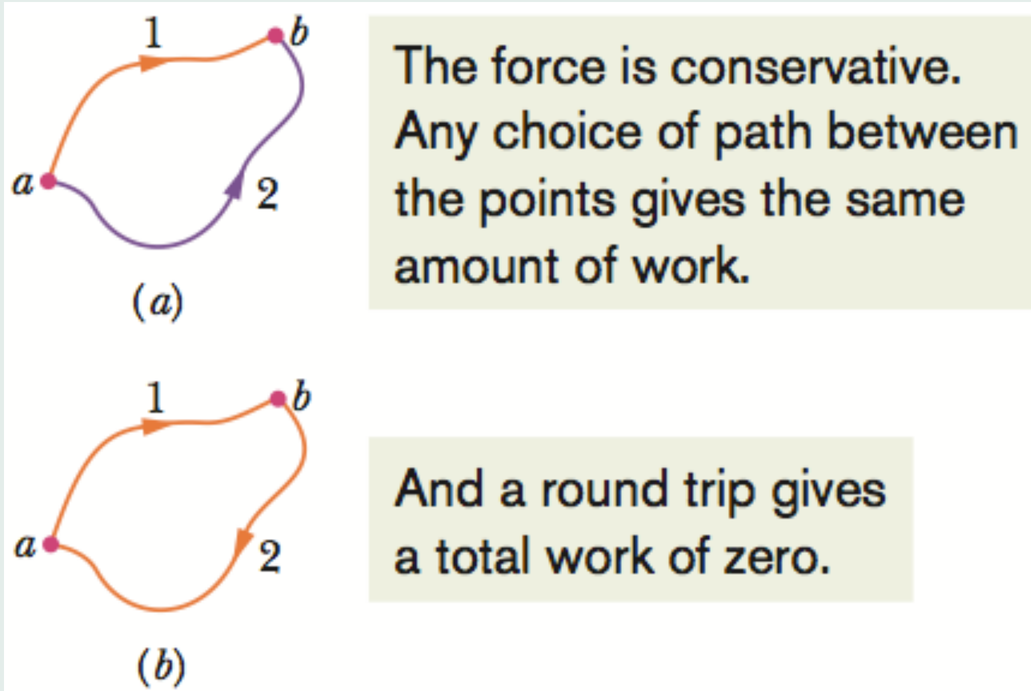
$$W = mgh$$

- **Conservative Forces**

The work done by the force depends only on the initial and final position of the object, not on the path in between.

$$\Updownarrow$$

The net work done by a conservative force on a particle moving around any closed path is zero.



Consequence: when the configuration change is reversed the work changes sign:

$$W_{a\to b} = -W_{b\to a}$$

- **Examples:** gravitational force, elastic force

- **Potential energy for conservative forces:** define U such that:

$$\Delta U = U_f - U_i = -W_{i\to f}$$

Note:

- $W_{i\to f}$ is path independent, hence this is a consistent relation

- Choosing $U_0 = 0$ for some reference configuration:

$$U_a = -W_{0\to a}$$

- Gravitational potential energy

$$\Delta U_g = mg(y_f - y_i)$$

Reference configuration: ground level

$$U_g(0) = 0 \;\Rightarrow\; U = mgy$$

- Elastic potential energy

$$\Delta U_s = \frac{k}{2}(x_f^2 - x_i^2)$$

Reference configuration: relaxed spring

$$U_s(0) = 0 \;\Rightarrow\; U_s = \frac{kx^2}{2}$$

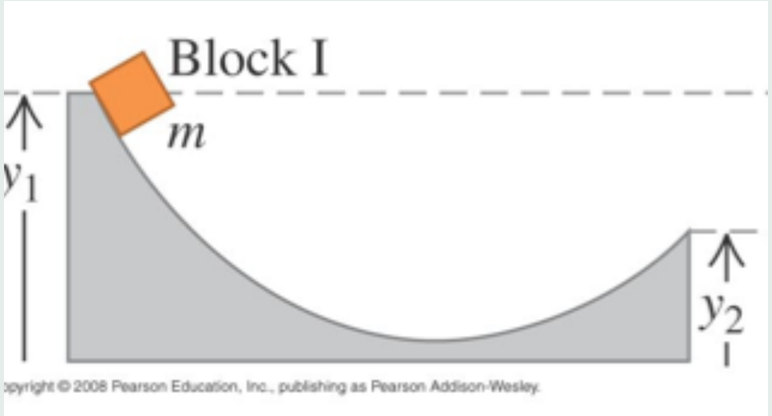
Conservation of Mechanical Energy

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

$$\text{Conservative forces, isolated system} \Rightarrow U + K = \text{constant}$$

i-Clicker

A block of mass m slides down a curved slope as shwon below. What is the final speed of the block?



$A)$ $v = \sqrt{2gy_1}$

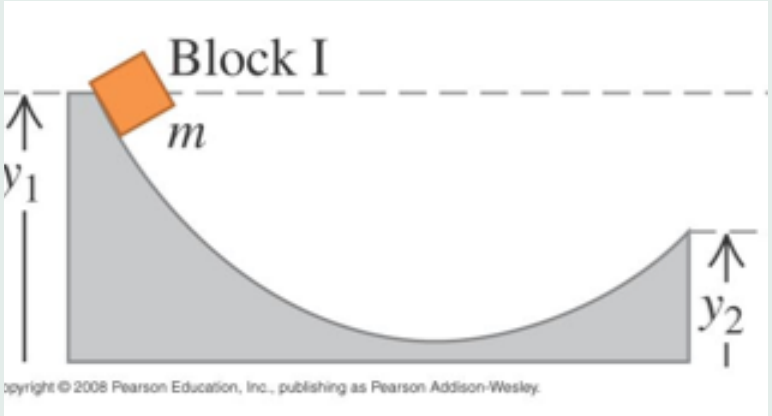
$B)$ $v = \sqrt{2gy_2}$

$C)$ $v = \sqrt{2g(y_1 - y_2)}$

$D)$ none of the above

Answer

A block of mass m slides down a frictionless curved slope as shown below. What is the final speed of the block?



$A)$ $v = \sqrt{2gy_1}$

$B)$ $v = \sqrt{2gy_2}$

$C)$ $v = \sqrt{2g(y_1 - y_2)}$

$D)$ none of the above

Energy conservation:

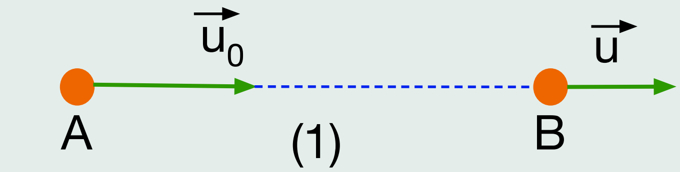
$$mgy_1 = mgy_2 + mv^2/2 \;\Rightarrow\; v = \sqrt{2g(y_1 - y_2)}$$

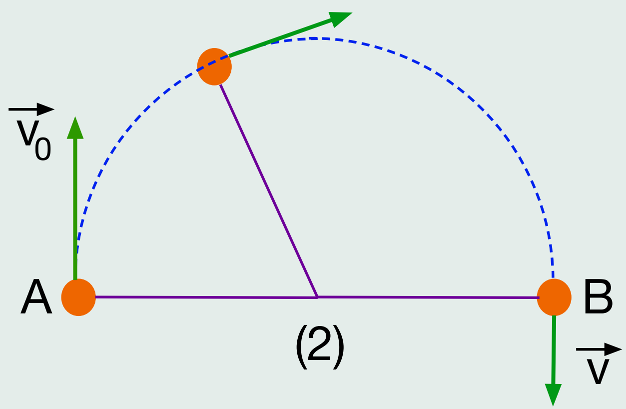
- **Non-conservative (dissipative) forces**:
  - W depends on the path
  - There is **no** potential energy U associated to a configuration such that

$$\Delta U = -W$$

  - Examples: kinetic friction, drag

Example:





- Suppose an object is launched from A to B on a rough horizontal surface with kinetic friction coefficient μ_k

(1) along a straight line

(2) on a circular trajectory (tied to a string)

$$W^{(1)}_{A\to B} = W^{(2)}_{B\to A} \ ?$$

$$W^{(1)}_{A\to B} = -\mu_k m g d_{AB}$$

$$W^{(2)}_{A\to B} = \int_A^B \vec{f}_k \cdot d\vec{s} = -\frac{\pi}{2}\mu_k m g d_{AB}$$

In conclusion:

$$W^{(1)}_{A\to B} \neq W^{(2)}_{B\to A}$$