1.54. **IDENTIFY:** Target variables are \( \vec{A} \cdot \vec{B} \) and the angle \( \phi \) between the two vectors.  
**SET UP:** We are given \( \vec{A} \) and \( \vec{B} \) in unit vector form and can take the scalar product using Eq.(1.19). The angle \( \phi \) can then be found from Eq.(1.18).  
**EXECUTE:**  
(a) \( \vec{A} = 4.00\hat{i} + 3.00\hat{j}, \quad \vec{B} = 5.00\hat{i} - 2.00\hat{j}; \quad A = 5.00, \quad B = 5.39 \)  
\( \vec{A} \cdot \vec{B} = (4.00\hat{i} + 3.00\hat{j}) \cdot (5.00\hat{i} - 2.00\hat{j}) = (4.00)(5.00) + (3.00)(-2.00) = 20.0 - 6.0 = +14.0. \)  
(b) \( \cos \phi = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{14.0}{(5.00)(5.39)} = 0.519; \quad \phi = 58.7^\circ. \)  
**EVALUATE:** The component of \( \vec{B} \) along \( \vec{A} \) is in the same direction as \( \vec{A} \), so the scalar product is positive and the angle \( \phi \) is less than \( 90^\circ \).  

1.55. **IDENTIFY:** For all of these pairs of vectors, the angle is found from combining Equations (1.18) and (1.21), to give the angle \( \phi \) as \( \arccos \left( \frac{\vec{A} \cdot \vec{B}}{AB} \right) = \arccos \left( \frac{A_B + A_B}{AB} \right). \)  
**SET UP:** Eq.(1.14) shows how to obtain the components for a vector written in terms of unit vectors.  
**EXECUTE:** (a) \( \vec{A} \cdot \vec{B} = -22, \quad A = \sqrt{40}, \quad B = \sqrt{13}, \) and so \( \phi = \arccos \left( \frac{-22}{\sqrt{40} \sqrt{13}} \right) = 165^\circ. \)  
(b) \( \vec{A} \cdot \vec{B} = 60, \quad A = \sqrt{34}, \quad B = \sqrt{136}, \quad \phi = \arccos \left( \frac{60}{\sqrt{34} \sqrt{136}} \right) = 28^\circ. \)  
(c) \( \vec{A} \cdot \vec{B} = 0 \) and \( \phi = 90^\circ. \)  
**EVALUATE:** If \( \vec{A} \cdot \vec{B} > 0 \), \( 0 \leq \phi < 90^\circ. \) If \( \vec{A} \cdot \vec{B} < 0 \), \( 90^\circ < \phi \leq 180^\circ. \) If \( \vec{A} \cdot \vec{B} = 0 \), \( \phi = 90^\circ \) and the two vectors are perpendicular.  

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**WORK AND KINETIC ENERGY**

6.1. **IDENTIFY:** Apply Eq.(6.2).  
**SET UP:** The bucket rises slowly, so the tension in the rope may be taken to be the bucket’s weight.  
**EXECUTE:** (a) \( W = F_s = mgs = (6.75 \text{ kg}) (9.80 \text{ m/s}^2)(4.00 \text{ m}) = 265 \text{ J}. \)  
(b) Gravity is directed opposite to the direction of the bucket’s motion, so Eq.(6.2) gives the negative of the result of part (a), or \( -265 \text{ J}. \)  
(c) The total work done on the bucket is zero.  
**EVALUATE:** When the force is in the direction of the displacement, the force does positive work. When the force is directed opposite to the displacement, the force does negative work.  

6.6. **IDENTIFY** and **SET UP:** \( W_{e} = (F \cos \phi)s, \) since the forces are constant. We can calculate the total work by summing the work done by each force. The forces are sketched in Figure 6.6.
EXECUTE: \[ W_i = F_i s \cos \phi \]

\[ W_1 = (1.80 \times 10^3 \text{ N})(0.75 \times 10^3 \text{ m}) \cos 14^\circ \]

\[ W_1 = 1.31 \times 10^3 \text{ J} \]

\[ W_2 = F_2 s \cos \phi = W_1 \]

**Evaluate:** Only the component \( F \cos \phi \) of force in the direction of the displacement does work. These components are in the direction of \( s \) so the forces do positive work.

**6.7.** Identify: All forces are constant and each block moves in a straight line, so \( W = F s \cos \phi \). The only direction the system can move at constant speed is for the 12.0 N block to descend and the 20.0 N block to move to the right.

**Set Up:** Since the 12.0 N block moves at constant speed, \( a = 0 \) for it and the tension \( T \) in the string is \( T = 12.0 \text{ N} \). Since the 20.0 N block moves to the right at constant speed the friction force \( f_k \) on it is to the left and \( f_k = T = 12.0 \text{ N} \).

**Execute:** (a) (i) \( \phi = 0^\circ \) and \( W = (12.0 \text{ N})(0.75 \text{ m}) \cos 0^\circ = 9.00 \text{ J} \).

(ii) \( \phi = 180^\circ \) and \( W = (12.0 \text{ N})(0.75 \text{ m}) \cos 180^\circ = -9.00 \text{ J} \).

(b) (i) \( \phi = 90^\circ \) and \( W = 0 \).

(ii) \( \phi = 0^\circ \) and \( W = (12.0 \text{ N})(0.75 \text{ m}) \cos 0^\circ = 9.00 \text{ J} \).

(iii) \( \phi = 180^\circ \) and \( W = (12.0 \text{ N})(0.75 \text{ m}) \cos 180^\circ = -9.00 \text{ J} \).

(iv) \( \phi = 90^\circ \) and \( W = 0 \).

(c) \( W_{\text{tot}} = 0 \) for each block.

**Evaluate:** For each block there are two forces that do work, and for each block the two forces do work of equal magnitude and opposite sign. When the force and displacement are in opposite directions, the work done is negative.

6.12. **Identify:** \( K = \frac{1}{2} m v^2 \). Use the equations for free-fall to find the speed of the weight when it reaches the ground.

**Set Up:** Estimate that a person has speed 2 m/s when walking and 6 m/s when running. The mass of an electron is \( 9.11 \times 10^{-31} \text{ kg} \). In part (c) take \( y = + \) downward, so \( a_y = +9.80 \text{ m/s}^2 \). Estimate a shoulder height of 1.6 m.

**Execute:** (a) Walking: \( K = \frac{1}{2}(75 \text{ kg})(2 \text{ m/s})^2 = 150 \text{ J} \).

Running: \( K = \frac{1}{2}(75 \text{ kg})(6 \text{ m/s})^2 = 1400 \text{ J} \).

(b) \( K = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(2.19 \times 10^4 \text{ m/s})^2 = 2.2 \times 10^{-18} \text{ J} \).

(c) \( v_y^2 = v_y^2 + 2a_y(y - y_0) \) gives \( v_y = \sqrt{2(9.80 \text{ m/s}^2)(1.6 \text{ m})} = 5.6 \text{ m/s} \).

\( K = \frac{1}{2}(1.0 \text{ kg})(5.6 \text{ m/s})^2 = 16 \text{ J} \).

(d) \( v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(100 \text{ J})}{30 \text{ kg}}} = 2.6 \text{ m/s} \).

**Evaluate:** A walking speed of 2 m/s corresponds to walking a mile in about 13 min. A running speed of 6 m/s corresponds to running a 100 m dash in about 17 s.

6.15. **Identify:** \( W_{\text{tot}} = K_2 - K_1 \). In each case calculate \( W_{\text{tot}} \) from what we know about the force and the displacement.

**Set Up:** The gravity force is \( mg \), downward. The friction force is \( f_k = \mu_n n = \mu_k mg \) and is directed opposite to the displacement. The mass of the object isn't given, so we expect that it will divide out in the calculation.

**Execute:** (a) \( K_1 = 0 \). \( W_{\text{tot}} = W_{\text{grav}} = mgs \).

\( mgs = \frac{1}{2} m v_1^2 \) and \( v_1 = \sqrt{2gs} = \sqrt{2(9.80 \text{ m/s}^2)(95.0 \text{ m})} = 43.2 \text{ m/s} \).

(b) \( K_2 = 0 \) (at the maximum height). \( W_{\text{tot}} = W_{\text{grav}} = -mgs \).

\( -mgs = -\frac{1}{2} m v_2^2 \) and \( v_2 = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.00 \text{ m/s}^2)}{2(0.220)(9.80 \text{ m/s}^2)}} = 5.80 \text{ m/s} \).

(c) \( K_1 = \frac{1}{2} m v_1^2 \), \( K_2 = 0 \). \( W_{\text{tot}} = W_f = -\mu_k ngs \).

\( -\mu_k ngs = -\frac{1}{2} m v_3^2 \) and \( s = \frac{v_3^2}{2\mu_k g} = \frac{(5.00 \text{ m/s}^2)^2}{2(0.220)(9.80 \text{ m/s}^2)} = 5.80 \text{ m} \).

(d) \( K_1 = \frac{1}{2} m v_1^2 \), \( K_2 = \frac{1}{2} m v_2^2 \). \( W_{\text{tot}} = W_f = -\mu_k ngs \).

\( \frac{1}{2} m v_3^2 = -\mu_k ngs + \frac{1}{2} m v_4^2 \)

\( v_4 = \sqrt{v_3^2 - 2\mu_k g s} = \sqrt{(5.00 \text{ m/s}^2)^2 - 2(0.220)(9.80 \text{ m/s}^2)(2.90 \text{ m})} = 3.53 \text{ m/s} \).
(e) $K_1 = \frac{1}{2} mv_1^2$. $K_2 = 0$. $W_{grav} = -mg y_2$, where $y_2$ is the vertical height. $-mg y_2 = -\frac{1}{2} mv_2^2$ and

$$y_2 = \frac{v_2^2}{2g} = \frac{(12.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 7.35 \text{ m}.$$ 

**Evaluate:** In parts (c) and (d), friction does negative work and the kinetic energy is reduced. In part (a), gravity does positive work and the speed increases. In parts (b) and (e), gravity does negative work and the speed decreases. The vertical height in part (e) is independent of the slope angle of the hill.