Dissipation Due to a "Valley Wave" Channel in the Quantum Hall Effect of a Multivalley Semiconductor

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When the quantized Hall effect occurs at a semiconductor surface such as Si(110), where the carriers have a time-reversal valley degeneracy, there should be a spontaneous valley polarization at appropriate values of the filling factor ν. There can be dissipation at $T=0$ due to radiation of Goldstone bosons ("valley waves") at impurity sites, provided that the current density exceeds a critical value j_c determined by the intervalley electron-electron scattering or other terms which modify the valley-wave dispersion at long wavelengths. The dissipation above j_c is described by a constant ρ_{xx}, which should be small but measurable, and sensitive to the density of neutral impurities.

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In this note we consider the possible modification of the quantized Hall effect when the band structure of the two-dimensional electron system has a valley degeneracy, due to time-reversal invariance, which is not lifted by the surface potential. [An example is a metal-oxide-semiconductor field-effect transistor on the (110) surface of silicon.] We discuss in particular the case of filling factor $\nu=1$, such that there is precisely one electron per quantum of magnetic flux, but similar considerations would apply to the fractional quantized Hall effect at filling factor $\nu=\frac{1}{3}$ and to various other values of ν. As has been noted previously, if one treats the Hamiltonian for this system in an effective-mass approximation, and if one neglects intervalley scattering terms and any other terms which distinguish the two valleys, then the ground state has a broken SU(2) symmetry due to a spontaneous "valley polarization," and low-lying Goldstone modes ("valley waves") exist as a result.[1] One may ask, in particular, to what extent these valley waves will spoil the ability of the system to carry an electric current at $T=0$ without dissipation, which is generally associated with the quantized Hall effect.

Using an analysis linear in the valley-wave amplitude, we argue below that there exists a critical current density j_c, whose value is small and determined by terms in the Hamiltonian which violate the SU(2) symmetry, such that for currents j above j_c there is a resistivity ρ_{xx} which is independent of j, while ρ_{xx} vanishes rapidly for $j<j_c$. The value of ρ_{xx} is determined by intervalley scattering due to impurities in the layer, and it may be varied by several orders of magnitude by variation of the concentration of neutral impurities, which have little effect on the resistance in normal circumstances.

In the absence of the applied magnetic field, an electron in an inversion layer is characterized by a two-dimensional band structure $\epsilon(\mathbf{k})$. We consider a case where $\epsilon(\mathbf{k})$ has minima at points $\mathbf{k}=\pm\mathbf{Q}/2$, which are not at the center or edge of the Brillouin zone, and therefore are degenerate by time-reversal symmetry. [We choose our coordinate system such that the surface lies in the x-y plane, with $\mathbf{Q}$ in the x direction. We also use the notation $\mathbf{R}=(\mathbf{r},z)$, where $\mathbf{r}$ is the projection of the position $\mathbf{R}$ on the x-y plane.] When the magnetic field is present, and there is one electron per flux quantum, the most important terms in the Hamiltonian can be written as

$$H_{\mathrm{eff}}=\frac{1}{2}\hbar\omega_c\sum_{l\tau}a_{l\tau}^{\dagger}a_{l\tau}+\frac{1}{2L^2}\sum_{\mathbf{q}}v(q)[\hat{\rho}(\mathbf{q})\hat{\rho}(-\mathbf{q})-f(\mathbf{q})\hat{\rho}(0)]+\frac{1}{2L^2}\sum_{\mathbf{q}}U(\mathbf{q})\hat{\rho}(\mathbf{q}). \tag{1}$$

Here the index $\tau=\pm1$ distinguishes the electron valleys at wave vector $\pm\mathbf{Q}/2$, the operator $a_{l\tau}^{\dagger}$ creates an electron in the lowest Landau level of valley τ with an envelope wave function $\phi_l(\mathbf{r})$ in the x-y plane, and $\hat{\rho}(\mathbf{q})$ is the Fourier transform of the density operator at wave vectors small compared to the reciprocal lattice,

$$\hat{\rho}(\mathbf{q})=\sum_{\tau l,l'}a_{l\tau}^{\dagger}a_{l'\tau}\rho_{ll'}(\mathbf{q}), \tag{2}$$

where

$$\rho_{ll'}(\mathbf{q})=\int d^2r\,\phi_{l'}^{*}(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}\phi_l(\mathbf{r}).$$

The term $U(\mathbf{q})$ in Eq. (1) is the long-wavelength part of the scattering potential due to impurities, while $v(q) \approx 2\pi e^2/\epsilon q$ is the two-dimensional Fourier transform of the electron-electron interaction. The quantity L^2 is the area of the system while the remaining quantities in (1) are given by $\omega_c = eB/(m_x m_y c^2)^{1/2}$, and

$$f(\mathbf{q}) = \exp[-l^2(q_x^2 m_y + q_y^2 m_x)/(4 m_x m_y)^{1/2}], \quad (3)$$

where m_x and m_y are the effective masses in the x and y directions, $l = (\hbar c/eB)^{1/2}$, and B is the magnetic field. We have assumed that the Zeeman energy is large, so that all spins are aligned parallel to the field, and we have chosen, for simplicity, to omit from (1) interaction terms which can mix in higher Landau levels. Of course, we assume that l is very large compared to the lattice spacing. The envelope functions $\phi_l(\mathbf{r})$ may be any convenient basis functions for the lowest Landau level, in the effective-mass approximation, and we may use any convenient gauge.

Let us now introduce an "isospin operator"

$$\mathbf{S} \equiv \frac{1}{2} \sum_{\tau\tau' l} \boldsymbol{\sigma}_{\tau\tau'} a_{l\tau}^\dagger a_{l\tau'},$$

where $\boldsymbol{\sigma}$ is the set of Pauli spin matrices. Then $\mathbf{S}$ commutes with H_{eff}, and the components of $\mathbf{S}$ satisfy the usual angular momentum commutators. When there is precisely one electron per flux quantum, the ground state of H_{eff} is given exactly by a single Slater determinant of the form

$$|\psi\rangle = \prod_l [a_{l,1}^\dagger \cos\tfrac{1}{2}\theta + a_{l,-1}^\dagger \sin\tfrac{1}{2}\theta\, e^{i\phi}]|0\rangle, \quad (4)$$

where $|0\rangle$ is the vacuum state. The energy is independent of the parameters θ and ϕ, while the expectation value of $\mathbf{S}$ has the form

$$\langle S_z \rangle = \tfrac{1}{2} N \cos\theta, \quad \langle S_x + iS_y \rangle = \tfrac{1}{2} N \sin\theta\, e^{i\phi}, \quad (5)$$

where N is the number of electrons in the system. The lowest-lying excitations are Goldstone modes associated with long-wavelength variations in the orientation of the isospin vector, which we term "valley waves," and which are analogous to spin waves in a Heisenberg ferromagnet. The excitation energy is proportional to the square of the wave vector q, in the limit of long wavelengths, and the spectrum can be calculated exactly for the model described by H_{eff} without impurities (see Ref. 1, Kallin and Halperin,[2] and Bychkov, Iordanskii, and Éliashberg[3]). For the case of isotropic electron mass ($m_x = m_y = m^*$) the spectrum for $ql << 1$ has the form

$$\omega_0(q) = 2\pi J l^2 q^2, \quad (6)$$

where the "exchange constant" is calculated to be $\hbar J = (3/2^{11}\pi)^{1/2} e^2/\epsilon l$.

The full Hamiltonian of the system may be written as $H = H_{\text{eff}} + H'$, where H' contains terms that violate SU(2) symmetry. For example, there will be terms in the electron-electron interaction of the form $a_{l_1,1}^\dagger a_{l_2,-1} a_{l_3,-1}^\dagger a_{l_4,1}$ in which one electron is scattered from valley 1 to valley -1, while the second is scattered from -1 to 1. There also are small symmetry-breaking terms that arise from the fact that the form factors of the Bloch functions have an opposite dependence on wave vector, in the two valleys, and from the fact that terms in the kinetic energy proportional to the cube of the wave vector have opposite signs in the two valleys. An impurity at point $(\mathbf{r}_j, z_j)$ gives rise to a symmetry-breaking term of form

$$H_j' = 2[u(z_j) e^{-i\mathbf{Q}\cdot\mathbf{r}_j} s_+(\mathbf{r}_j) + \text{H.c.}], \quad (7)$$

where

$$\mathbf{s}(\mathbf{r}) \equiv \frac{1}{2} \sum_{ll'} \sum_{\tau\tau'} \phi_l^*(\mathbf{r}) \phi_{l'}(\mathbf{r}) \boldsymbol{\sigma}_{\tau\tau'} a_{l\tau}^\dagger a_{l'\tau'}$$

is the isospin density operator at point $\mathbf{r}$, projected onto the lowest Landau level. Because the phase factor $e^{-i\mathbf{Q}\cdot\mathbf{r}_j}$ is a rapdily varying function of the position of the impurity, the perturbation H_j' behaves like a field which is randomly oriented in the x-y plane of isospin space. The coefficient $u(z_j)$ is the matrix element of the impurity potential between Bloch wave states in the electron layer at wave vectors $\mathbf{Q}/2$ and $-\mathbf{Q}/2$ in the plane. The relevant Fourier components of the impurity potential are those at large wave vectors $\mathbf{Q} + \mathbf{G}$, where $\mathbf{G}$ is a reciprocal-lattice vector, and are qualitatively the same for neutral or charged impurities. Moreover, the matrix element $u(z_j)$ vanishes rapidly if z_j is outside of the region occupied by the electron layer.

In the absence of impurity scattering, the Hamiltonian H conserves separately the number of electrons in each of the two valleys, and therefore commutes with operator S_z. For $B = 0$, this follows directly from wave-vector conservation parallel to the interface. For $B \neq 0$, the envelope wave function $\phi_l(\mathbf{r})$ contains a Gaussian spread in the momentum. The amplitude for a transition from one valley to the other is of order $\exp(-\text{const} \times Q^2 l^2)$, which is the overlap integral between states in the lowest Landau level of two different valleys, and is negligibly small for the field strengths of interest.

To lowest order in the symmetry-breaking H', if impurity scattering is neglected, the ground-state energy depends on θ through a term of the form

$$E(\theta) = -\hbar\Delta_0 \langle S_z \rangle^2 / N = -\tfrac{1}{4} N \hbar \Delta_0 \cos^2\theta,$$

where Δ_0 is a constant. If $\Delta_0 > 0$, then the ground state has all electrons condensed in one valley, so that $\langle \mathbf{S} \rangle$ is parallel to the $\pm z$ direction (Ising case). If $\Delta_0 < 0$ (x-y case), then the minimum-energy state has $\theta = \pi/2$. The energy remains independent of the orientation angle ϕ in the x-y plane.

The effect of Δ_0 on the valley-wave spectrum may be deduced immediately from the behavior of the analogous ferromagnetic systems with small anisotropy.[4] In the Ising case there is a gap in the spectrum, and one finds

$$\omega(k) = \omega_0(k) + \Delta_0, \tag{8a}$$

where ω_0 is the unperturbed spectrum, given by (6). For the x-y case, the frequency spectrum is

$$\omega(k) = [\omega_0(k) + |\Delta_0|]^{1/2}[\omega_0(k)]^{1/2}, \tag{8b}$$

which is linear in the limit[5] $k \rightarrow 0$ (see Fig. 1).

Now suppose that there is an electric field in the x-y plane, and hence a Hall current resulting from the drift velocity $\mathbf{v}_d = (\mathbf{E}\times\mathbf{B})c/B^2$. In the frame of reference moving with the electrons, the electric field vanishes, but the impurities move backwards with velocity $-\mathbf{v}_d$. If v_d exceeds the critical velocity v_c, given by $v_c = (8\pi J l^2 \Delta_0)^{1/2}$ or $(2\pi J l^2 |\Delta_0|)^{1/2}$ for the Ising or x-y case, respectively, dissipation can occur via Cherenkov radiation of valley waves by the moving impurities. If the density of impurities is small, the emission rate can be calculated by Fermi's "golden rule." In the Ising case, we obtain a dissipation rate

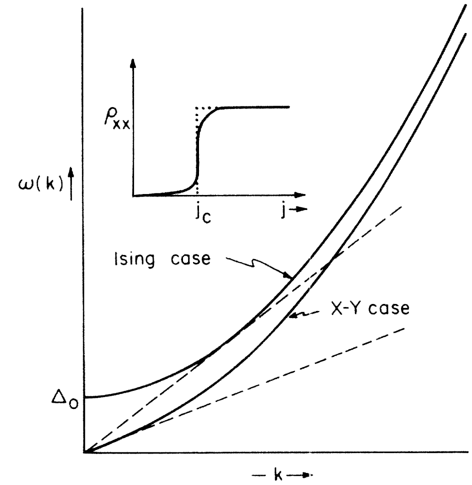

FIG. 1. A sketch of the valley-wave dispersion for the Ising and x-y ground states at small ql. The effect of the symmetry-breaking terms is to introduce a gap Δ_0 in the Ising case and a linear dispersion in the x-y case. The slope of the dashed tangent line is the critical velocity, in each case. Inset: Sketch of the longitudinal resistivity vs the current density j in the Ising case.

$$\frac{dE}{dt} = \frac{8\pi N}{\hbar L^2}\sum_j |u(z_j)|^2 \int \delta[\omega(q) - \mathbf{v}_d\cdot\mathbf{q}]\omega(q)\frac{d^2q}{(2\pi)^2} = \frac{v_d^2}{8\pi^3\hbar J^2 l^6}\sum_j |u(z_j)|^2, \tag{9}$$

for $v_c < v_d << Jl$. (The maximum velocity of the valley waves is of order Jl.) The dissipation for $v_d > v_c$ is equivalent to a resistivity

$$\rho_{xx} = (1/2\pi l^2 \hbar J^2 e^2)\textstyle\sum_j |u(z_j)|^2/L^2. \tag{10}$$

In the x-y case, the matrix element for valley-wave creation depends on q through a Bogoliubov transformation, which relates $\mathbf{s}(\mathbf{r})$ to the valley-wave creation operators.[4] The final result for the dissipation differs somewhat from the Ising case, but the resistivity ρ_{xx} reaches a saturation value, for $v_d >> v_c$, which is just $\frac{1}{2}$ of the value given in Eq. (10).

In addition to providing the mechanism for dissipation, the symmetry-breaking impurity field H_j' may have an effect on the ground state and the excitation spectrum. Even for small values of the impurity concentration, the random field will cause destruction of long-range order in the x-y case.[6] On average, the random field will lower the energy of the Ising orientation relative to the x-y orientation.[7] Thus, a sufficient concentration of impurities can lead to Ising orientation, even if the pure system would have isospin in the x-y plane.[7] The impurities will tend to increase the gap Δ_0 in the Ising case.

Finally, impurities may broaden the valley-wave spectrum, and may lead to some density of localized modes, at low energies. It seems unlikely that localized modes can contribute directly to the dissipation ρ_{xx}. However, when terms beyond lowest order in the impurity concentration are taken into account the dissipation may not be precisely zero for drift velocity v_d below the critical velocity v_c. (We neglect in our discussion the extremely small dissipation which is always present, in principle, from large values of $\mathbf{q}$, where the excitation spectrum is flat.)

We conclude by applying these general results to the Si(110) inversion layer, where our coordinate system is $\hat{z}\parallel(110)$, $\hat{x}\parallel(1\bar{1}0)$, and $\hat{y}\parallel(001)$. Then the lowest-lying energy state for an electron in the inversion layer with wave vector $\mathbf{k}$ near $\mathbf{Q}/2$ is constructed from the bulk Bloch states by

$$\psi_{\mathbf{k}}(\mathbf{R}) \approx \tfrac{1}{2}\sqrt{2}g(z)[e^{i\delta}\phi^B_{\mathbf{k},1}(\mathbf{R}) + e^{-i\delta}\phi^B_{\mathbf{k},2}(\mathbf{R})], \tag{11}$$

where $\phi^B_{\mathbf{k},1}$ is a bulk Bloch state in the (100) valley whose wave vector projected onto the surface plane is equal to $\mathbf{k}$, $\phi^B_{\mathbf{k},2}$ is a similar state in the $(0\bar{1}0)$ valley, the phase shift δ depends on the boundary condition at the surface, and $g(z)$ is an envelope function[8] with the approximate form $g(z) = 2^{-1/2}b^{3/2}ze^{-bz/2}$. For $\mathbf{k}$ near $-\mathbf{Q}/2$ we use the time-reversed states constructed from $(\bar{1}00)$ and (010). From Ref. 8, for surface carrier density $n \approx 1.5\times10^{11}/\mathrm{cm}^2$, or $l \approx 100$ Å, we take $b^{-1} \approx 20$ Å. We choose an impurity potential

$V_{\rm imp}$ such that $\langle \psi_{\mathbf{Q}/2} | V_{\rm imp} | \psi_{-\mathbf{Q}/2} \rangle$ is 1 eV (here $\psi_{\mathbf{k}}$ is normalized to unity in one atomic volume). This scattering strength is intermediate between our estimated values for substitutional Ge and C impurities. The value of ρ_{xx} is proportional to the impurity concentration n_I, and for $n_I = 10^{18}\ \mathrm{cm}^{-3}$, we find $\rho_{xx} = 2.8\ \Omega$ for the Ising case, when $v_d > v_c$.

We have also made an estimate of the valley-wave gap parameter at low impurity concentration, and we find $\hbar\Delta_0 \approx 10^{-6}$ meV.[9] This estimate is based on a Hartree-Fock calculation of the ground-state energy in the presence of the electron-electron interaction, as a function of the mixing angle θ of Eq. (4).[10] More precisely this leads to a value of the form

$$\Delta_0 = \frac{e^2}{\epsilon l}\left(\frac{1}{Ql}\right)^3 \frac{b\Gamma}{Q},$$

where Γ is a number of order unity. An empirical pseudopotential approach[11] was used to determine the Bloch functions in Eq. (11). While the sign of Δ_0 indicates an Ising ground state, the uncertainties in the calculation and in the choice of screened interaction preclude a definitive prediction. The above value of Δ_0 gives a critical velocity $v_c \approx 8\times10^3$ cm/sec, or a critical current density $j_c \simeq 7\times10^{-5}$ A/cm.

In summary, we make several predictions of novel behavior which should be observable in the quantum Hall effect of a multivalley semiconductor. These include the existence of a critical current density j_c above which dissipationless current flow ceases, a saturation of the longitudinal resistivity above j_c, and a strong dependence of this saturated resistivity on the density of neutral defects in the inversion layer. A number of issues deserve further investigation, including the influence of disorder on the valley-wave spectrum, the effects of nonlinear interactions as the population of valley waves builds up, and the manner in which heat is finally removed from the valley-waves mode.[12]

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[12]If the amplitude of the valley waves builds up sufficiently, the configuration of $\mathbf{s}(\mathbf{r})$ will contain "instantons" and "anti-instantons" [see R. Rajaraman, *Solitons and Instantons* (North-Holland, Amsterdam, 1982), pp. 48–58]. We find that each instanton carries an electric charge $\pm e$, and motion of the instantons enables the system to extract energy continuously from the electric field $\mathbf{E}$.