Physics 602 **Homework Set 1** **Spring, 2008**

Due date: Monday Feb. 4

Reading: Skim TH Ch. 1 and read Ch. 2.1-6

This is a short "mini homework" designed to warm you up on the second quantized notation.

1) (10 points)
   a) Show that $[a, bc] = [a, b]c + b[a, c]$ and $[a, bc] = \{a, b\}c - b\{a, c\}$.
   b) Find similar expressions for $[a, bcde]$ and $[ab, cd]$.
   c) Show that the number operator commutes with any one-body or two-body operator, and thus, that the eigenstates of our electron Hamiltonian $H = T + U + V$ can be chosen to have definite particule number.

2) (10 points)
   a) Show $\rho_\sigma^\dagger(\mathbf{r}) = \rho_\sigma(\mathbf{r})$ and $[\rho_\sigma(\mathbf{r}), \rho_{\sigma'}(\mathbf{r'})] = 0$.
   b) Show $[\rho_{\mathbf{k}\sigma}\,, \rho_{\mathbf{k}'\sigma'}\,] = 0$.
   c) Show $\rho_{\mathbf{k}\sigma}^\dagger = \rho_{-\mathbf{k},\sigma}$ (from which it also follows that $[\rho_{\mathbf{k}\sigma}{}^\dagger,\, \rho_{\mathbf{k}'\sigma'}\,] = 0$).
   d) Verify that $\langle\Psi|\rho_\sigma(\mathbf{r})|\Psi\rangle = \sum_{i=1}^{N} |\phi_i(\mathbf{r}, \sigma)|^2$ in the case where Ψ is a Slater determinant, $|\Psi\rangle = c_1^\dagger c_2^\dagger ... c_N^\dagger|0\rangle$ (here $c_i^\dagger$ creates an electron in state ϕ_i).

3) (10 points) Do TH Problem 2.4. (This will be useful later for superconductivity.)

4) (10 points) Do TH Problem 2.6 [to verify Eq. (2.7.1)]. (This will be useful later for the derivation of the RPA.)