EM Waves in Dielectric Media

No free charges

Maxwell:

$$\nabla \cdot \underline{D} = 0 \qquad \nabla \times \underline{E} + \frac{1}{c}\frac{\partial \underline{B}}{\partial t} = 0$$

$$\nabla \cdot \underline{B} = 0 \qquad \nabla \times \underline{B} - \frac{1}{c}\frac{\partial \underline{D}}{\partial t} = 0$$

Try $\underline{E} = \underline{E}_0 e^{i\underline{q}\cdot\underline{r}} e^{-i\omega t}$

$$\underline{D} = \epsilon(\underline{q}, \omega)\, \underline{E}_0 e^{i\underline{q}\cdot\underline{r}} e^{-i\omega t}$$

$\nabla \cdot \underline{D} = 0 \Rightarrow$ $\boxed{\epsilon(\underline{q}, \omega)\, \underline{q} \cdot \underline{E}_0 = 0}$

Case (i) $\epsilon(\underline{q}, \omega) = 0$

$\therefore \underline{D} = 0 \Rightarrow \left.\begin{matrix} \nabla \cdot \underline{B} = 0 \\ \nabla \times \underline{B} = 0 \end{matrix}\right\} \Rightarrow \underline{B} = 0 \Rightarrow \nabla \times \underline{E} = 0$

$\Rightarrow \underline{q} \times \underline{E}_0 = 0$

$\boxed{\epsilon(\underline{q}, \omega) = 0 \text{ is condit. for Longitudinal Wave}}$ $\underline{q} \parallel \underline{E}_0$

Case (ii) $\underline{q} \cdot \underline{E}_0 = 0 \Rightarrow \underline{q} \perp \underline{E}_0$, "Transverse"

$$0 = \nabla \times \left[\nabla \times \underline{E} + \frac{1}{c}\frac{\partial \underline{B}}{\partial t} \right] = \nabla(\nabla \cdot \underline{E}) - \nabla^2 \underline{E} + \frac{1}{c^2}\frac{\partial^2 \underline{D}}{\partial t^2}$$

$$0 = -\underline{q}(\underline{q} \cdot \underline{E}_0) + q^2 \underline{E}_0 - \frac{\omega^2}{c^2}\epsilon \underline{E}_0$$

$$q^2 \underline{E}_0 = \frac{\omega^2}{c^2}\epsilon(\underline{q}, \omega)\, \underline{E}_0$$

$\boxed{q^2 c^2 = \omega^2 \epsilon(\underline{q}, \omega) \text{ is condit. for Transverse Wave}}$

$\underline{q} \perp \underline{E}_0$