CONVENTIONAL DEFINITIONS FOR CORRELATION FUNCTIONS

Definition:

$$G(\mathbf{r},t) \;=\; \frac{1}{N}\int d\mathbf{r}'\,\langle 0|\,\rho(\mathbf{r}'+\mathbf{r},t)\,\rho(\mathbf{r}',0)\,|0\rangle$$

$\mathbf{r} \leftrightarrow \mathbf{q}$:

$$\begin{aligned} S(\mathbf{q},t) &= N\int d\mathbf{r}\,e^{-i\mathbf{q}\cdot\mathbf{r}}\,G(\mathbf{r},t) \\ G(\mathbf{r},t) &= \frac{1}{N\Omega}\sum_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}}\,S(\mathbf{q},t) \end{aligned}$$

$t \leftrightarrow \omega$:

$$\begin{aligned} S(\mathbf{q},\omega) &= \frac{1}{2\pi}\int dt\,S(\mathbf{q},t)\,e^{i\omega t} \\ S(\mathbf{q},t) &= \int d\omega\,S(\mathbf{q},\omega)\,e^{-i\omega t} \end{aligned}$$

$t = 0^{\,*}$:

$$\begin{aligned} g(\mathbf{r}) &= G(\mathbf{r},t=0) \\ S(\mathbf{q}) &= \frac{1}{N}\,S(\mathbf{q},t=0) \end{aligned}$$

Therefore,

$$\begin{aligned} S(\mathbf{q}) &= \int d\mathbf{r}\,e^{-i\mathbf{q}\cdot\mathbf{r}}\,g(\mathbf{r}) \\ g(\mathbf{r}) &= \frac{1}{\Omega}\sum_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{r}}\,S(\mathbf{q}) \end{aligned}$$

Useful formulas:

$$\begin{aligned} S(\mathbf{q},t) &= \langle 0|\,\rho_{\mathbf{q}}(t)\,\rho^{\dagger}_{\mathbf{q}}(0)\,|0\rangle \\ S(\mathbf{q}) &= \frac{1}{N}\,\langle 0|\,\rho^{\dagger}_{\mathbf{q}}\rho_{\mathbf{q}}\,|0\rangle \\ S(\mathbf{q},\omega) &= \sum_{n} |\langle n|\rho^{\dagger}_{\mathbf{q}}|0\rangle|^2\,\delta(\omega-\omega_{n0}) \end{aligned}$$

* You may see alternative definition:

$$\begin{aligned} \overline{g}(\mathbf{r}) &= g(\mathbf{r}) - \delta(\mathbf{r}) \\ \overline{S}(\mathbf{q}) &= S(\mathbf{q}) - 1 \end{aligned}$$