Condensed Formulas for BCS

$$H - \mu N = \sum_k \tilde{\varepsilon}_k \left(c_k^+ c_k + c_{-k}^+ c_{-k} \right) - V \sum_{kk'} c_{k'}^+ c_{-k'}^+ c_{-k} c_k$$

$$|\Psi_0\rangle = \prod_k \left(u_k + v_k c_k^+ c_{-k}^+ \right) |0\rangle$$

$$E_0 = \langle \Psi_0 | H - \mu N | \Psi_0 \rangle = 2 \sum_k \tilde{\varepsilon}_k v_k^2 - V \left[\sum_k u_k v_k \right]^2$$

Define

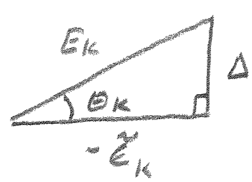
$$u_k \equiv \sin(\theta_k/2) \quad \Rightarrow \quad v_k = \cos(\theta_k/2)$$

$$\Delta \equiv V \sum_k u_k v_k$$

$$0 = \frac{dE_0}{d\theta_k} \Rightarrow \boxed{\tan\theta_k = \frac{\Delta}{-\tilde{\varepsilon}_k}} \quad \text{AND} \quad \boxed{1 = \frac{1}{2} V \sum_k \frac{1}{\sqrt{\Delta^2 + \tilde{\varepsilon}_k^2}}}$$

↑ DETERMINES θ_k ↑ DETERMINES Δ

TRIANGLE FORMULAS: DEFINE $E_k \equiv \sqrt{\Delta^2 + \tilde{\varepsilon}_k^2}$



$$\sin\theta_k = \frac{\Delta}{E_k} \qquad u_k^2 = \frac{1 - \cos\theta_k}{2}$$

$$\cos\theta_k = \frac{-\tilde{\varepsilon}_k}{E_k} \qquad v_k^2 = \frac{1 + \cos\theta_k}{2}$$

$$u_k v_k = \frac{\sin\theta_k}{2}$$

AFTER ENERGY INTEGRALS:

$$1 = \frac{1}{2} g_F V \sinh^{-1}\left(\frac{\omega_D}{\Delta}\right), \quad E_0 = \frac{g_F}{2} \omega_D^2 \left[1 - \sqrt{1 + \frac{\Delta^2}{\omega_D^2}} \right]$$

BROKEN PAIR:

$$|\Psi_k\rangle = c_k^+ \prod_{k' \neq k} \left(u_{k'} + v_{k'} c_{k'}^+ c_{-k'}^+ \right) |0\rangle$$

$$\langle \Psi_k | H - \mu N | \Psi_k \rangle - \langle \Psi_0 | H - \mu N | \Psi_0 \rangle = E_k$$