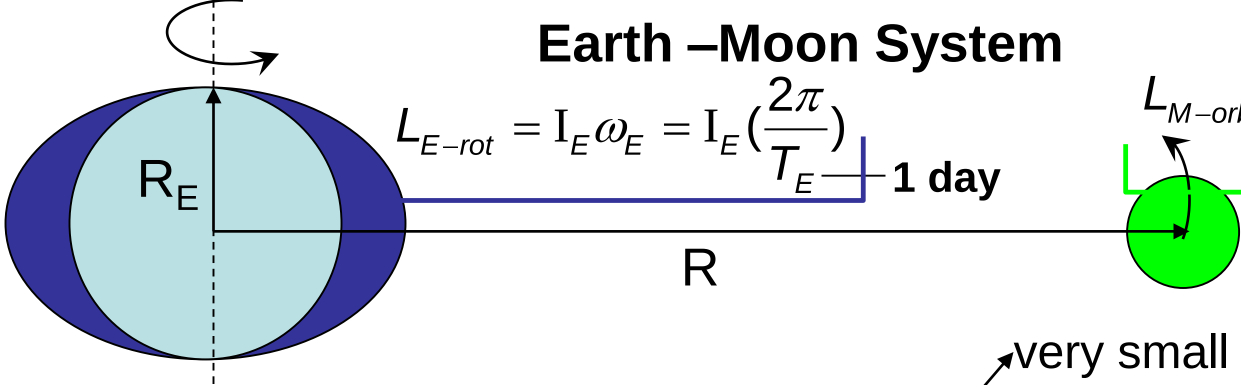
Earth –Moon System



$$L_{E-rot} = I_E \omega_E = I_E\left(\frac{2\pi}{T_E}\right)$$

$$L_{M-orb} = I_{M-orb}\omega_{M-orb} = m_M R^2\left(\frac{2\pi}{T_{M-orb}}\right)$$

T_E — 1 day; T_{M-orb} — 27 days

$$I_E = \frac{2}{5} M_E R_E^2$$

$$L_{tot} = L_{E\text{-}rot} + L_{M\text{-}orbit} + L_{M\text{-}rot}$$

$L_{M\text{-}rot}$: very small (neglect)

constant →
$$L_{tot} = I_E\left(\frac{2\pi}{T_E}\right) + m_M R^2\left(\frac{2\pi}{T_{M-orb}}\right)$$

$$K_{E-rot} = \frac{1}{2} I_E \omega_E^{\,2} = \frac{1}{2} I_E\left(\frac{2\pi}{T_E}\right)^2$$

solid Earth – rotates under ocean tidal budge
- frictional energy loss

$$\Delta E_E = \Delta K_{E\text{-}rot} = -W_{tidal\text{-}friction}$$

$$-W_{tidal} = \Delta K_{E-rot} = \left[\frac{(2\pi)^2}{2} I_E\right]\left(\frac{1}{T_E(later)^2} - \frac{1}{T_E(earlier)^2}\right)$$

Earth's rotation slows with time

$$T_E(later) > T_E(earlier)$$

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constant →
$$L_{tot} = I_E\left(\frac{2\pi}{T_E}\right) + m_M R^2\left(\frac{2\pi}{T_{M-orb}}\right)$$

decreases (first term) **increases** (second term)

L transferred to moon

$$T^2 = \frac{(2\pi)^2}{GM_E} R^3$$

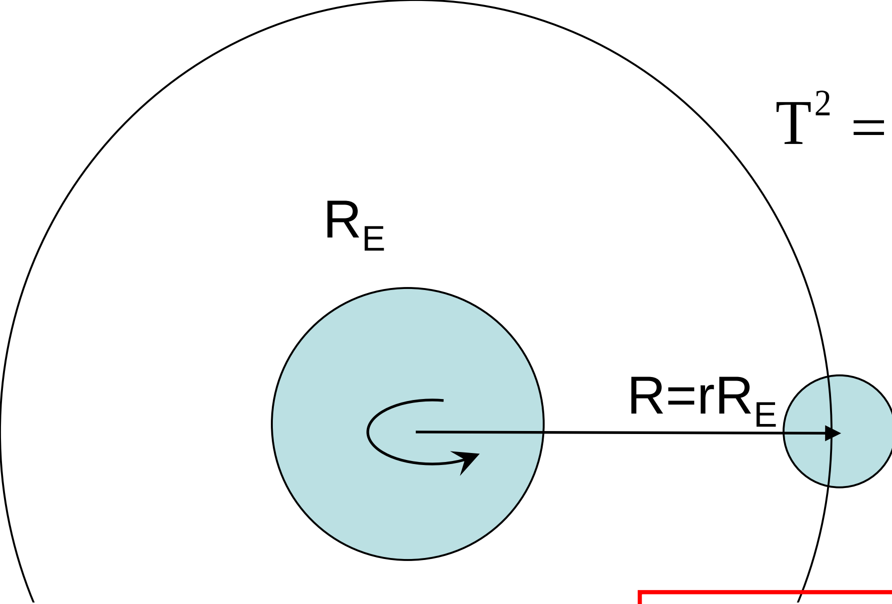
$$T^{-2} = \frac{GM_E}{(2\pi)^2 R^3}$$

$$\frac{1}{T} = \frac{\sqrt{GM_E}}{(2\pi)} \frac{1}{R^{3/2}}$$

$$\frac{m^3}{s^2 kg} = G$$

moon moves away with time

$$L_{tot} = I_E\left(2\pi / T_E\right) + m_M \sqrt{GM_E}\, R^{1/2}$$



$$T^2 = \frac{(2\pi)^2}{GM_E} R^3$$

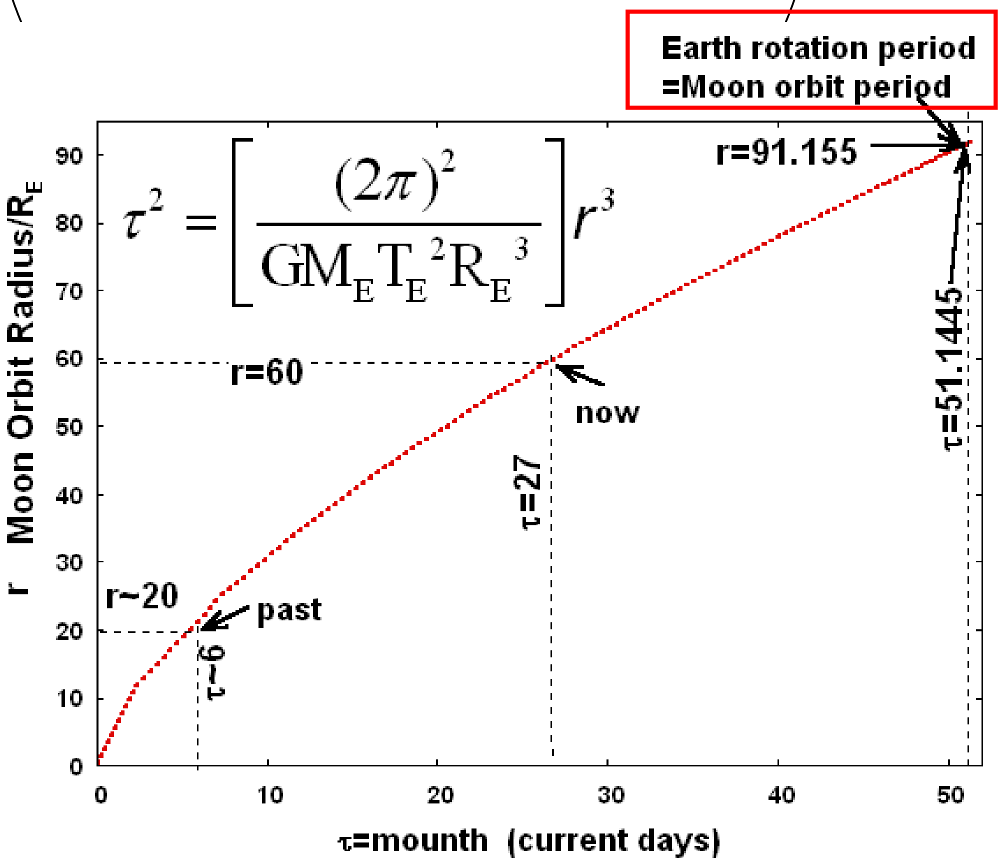
$$T_{M\text{-orbit}} = 27 \text{ days} = T_{E\text{-rot27}}(now)$$

$$T_{M\text{-orbit}} = \tau T_{E\text{-rot}}(now)$$

$R=rR_E$

$$\tau^2 = \left[\frac{(2\pi)^2 R_E^{\,3}}{GM_E T_E^{\,2}}\right] r^3$$

$$\tau^2 = [0.003584]\, r^3$$



distant future

moon-orbit at 91 R_E

$\tau^2 = [0.0033375]\, r^3$

day'=month'=51 days (current)

distant past

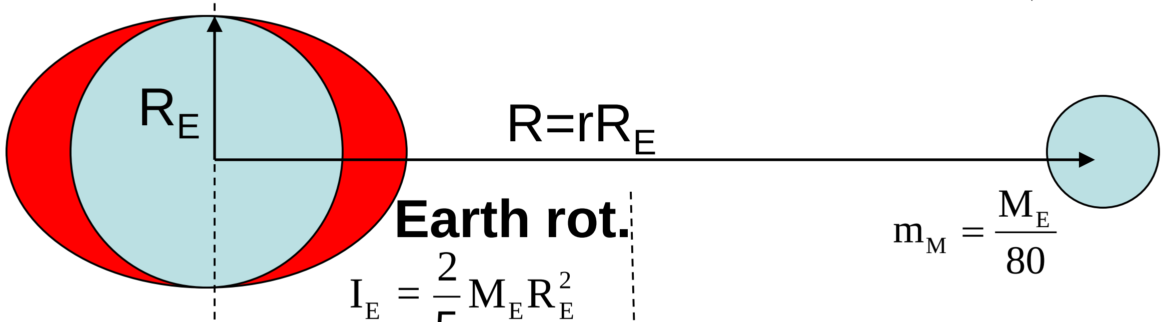
moon- much closer
day shorter

moon- formed at about 3.8 R_E

Roche limit 1.5 R_E

$$L_{tot}=L_{E\text{-rot}}+L_{M\text{-orbit}}+L_{M\text{-rot}}$$

very small (neglect)



Earth rot.

$$I_E = \frac{2}{5} M_E R_E^2$$

$$m_M = \frac{M_E}{80}$$

$$L_{E\text{-rot}} = \{I\}\frac{2\pi}{T_E}$$

$$L_{E\text{-rot}} = \frac{2}{5}\left[\frac{M_E R_E^2 2\pi}{T_E}\right]$$

$$T_{E\text{-rot}}\,(\text{at other time}) = t\,T_{E\text{-rot}}\,(now)$$

$$L_{E\text{-rot}} = \frac{2}{5t}\left[\frac{M_E R_E^2 2\pi}{T_E}\right]$$

Moon orbit

$$T_{M\text{-orbit}} = 27\text{ days} = T_{E\text{-rot}27}\,(now)$$

$$T_{M\text{-orbit}} = \tau T_{E\text{-rot}}\,(now)$$

$$\tau = 27\ (now)$$

$$L_{M\text{-orbit}} = m_M R^2 \left(\frac{2\pi}{T_{M\text{-orbit}}}\right)$$

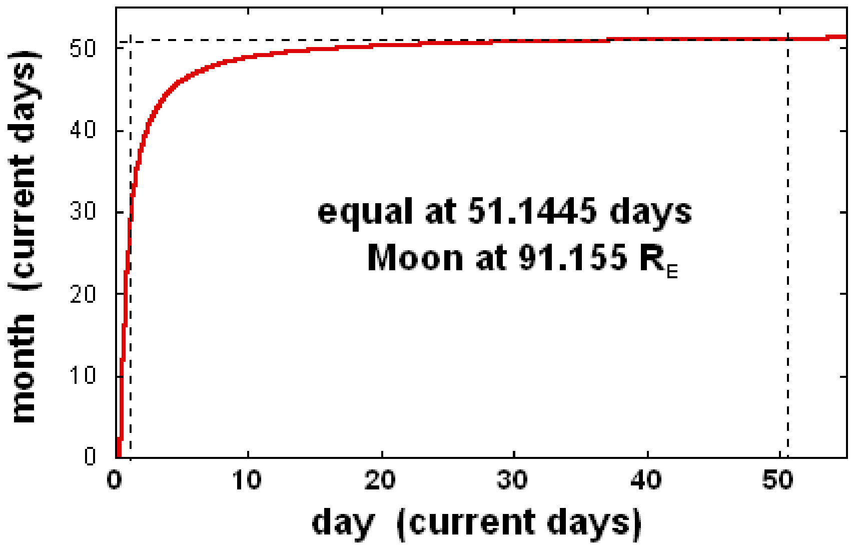
$$L_{M\text{-orbit}} = \frac{M_E}{80}(rR_E)^2\left(\frac{2\pi}{\tau T_{E\text{-rot}}}\right)$$

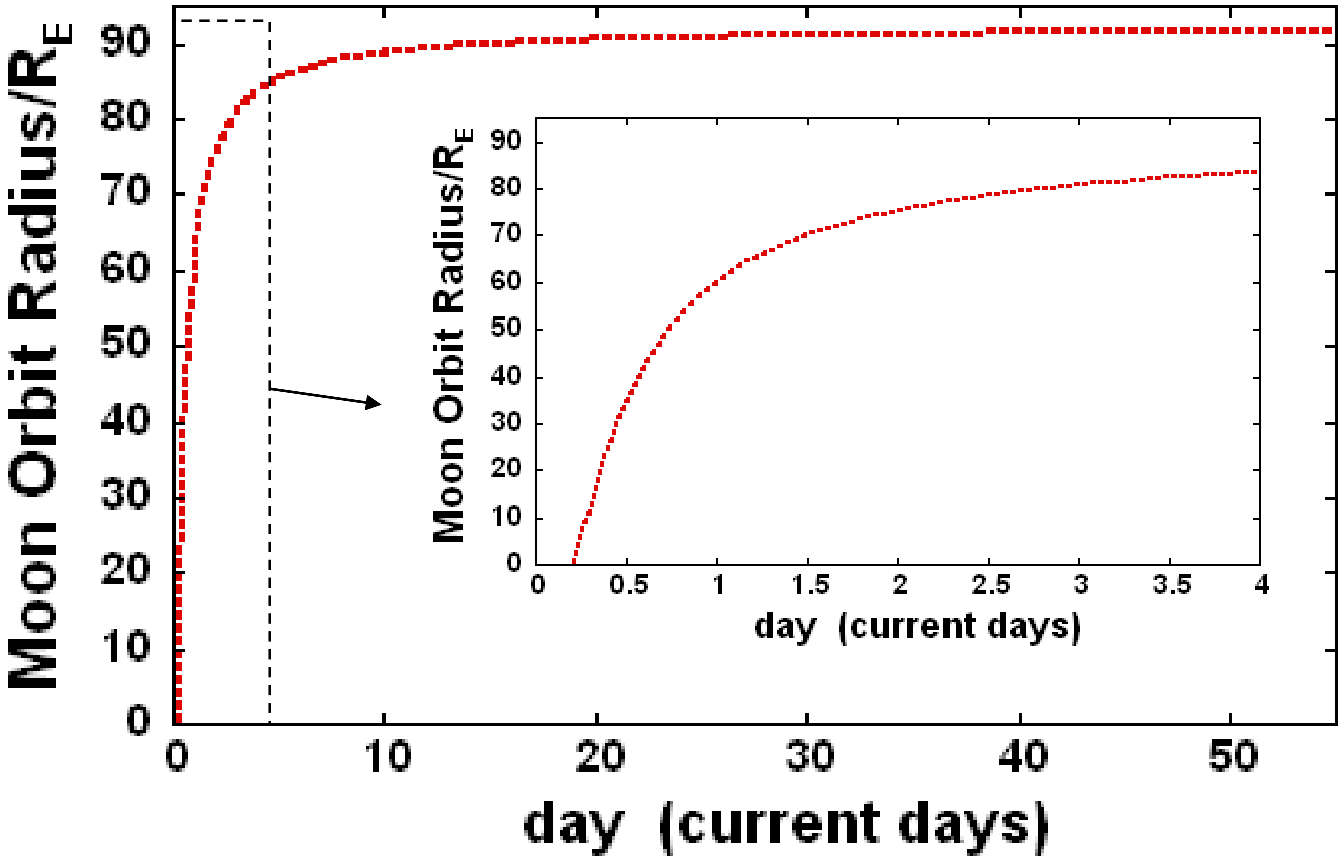
$$L_{M\text{-orbit}} = \frac{r^2}{80\tau}\left[\frac{M_E R_E^{\,2} 2\pi}{T_E}\right]$$

$$L_{totM\text{-orbit}} = L_{E\text{-rot}} + L_{M\text{-orbit}}$$

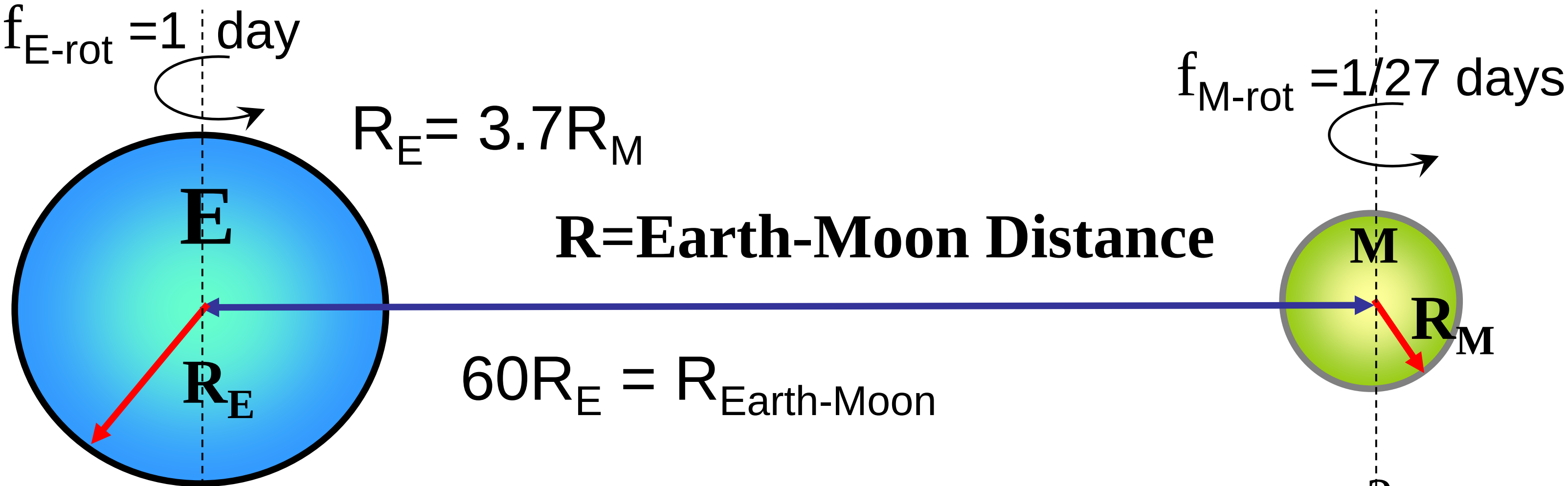
$$L_{totM\text{-orbit}} = \left\{\frac{2}{5t} + \frac{r^2}{80\tau}\right\}\left[\frac{M_E R_E^{\,2} 2\pi}{T_E}\right]$$

Evolution of Moons Orbit





$L_{M\text{-rot}} >> L_{E\text{-rot}}$



$$M_E = 80m_M$$

$$I_M = \frac{2}{5} m_M R_M^2$$

$$I_E = \frac{2}{5} M_E R_E^2$$

$$I_M = \frac{2}{5} m_M R_M^2 = \frac{2}{5} \frac{M_E}{80} \left(\frac{R_E}{3.7}\right)^2 = (0.000913) I_E$$

$$L_{E\text{-rot}} = I_E (2\pi f_{E\text{-rot}})$$

$$L_{M\text{-rot}} = I_M (2\pi f_{M\text{-rot}})$$

$$L_{M\text{-rot}} = (0.000913) I_E (2\pi f_{E\text{-rot}}) \frac{1}{27}$$

$$L_{M\text{-rot}} = (0.0000338) L_{E\text{-rot}}$$

Tidal Effect Differential Gravitations Force