Earth – Moon System

\[ L_{E-rot} = I_E \omega_E = I_E \left( \frac{2\pi}{T_E} \right) \]

1 day

\[ L_{M-orb} = I_{M-orb} \omega_{M-orb} = m_M R^2 \left( \frac{2\pi}{T_{M-orb}} \right) \] 27 days

very small (neglect)

\[ L_{tot} = L_{E-rot} + L_{M-orbit} + L_{M-rot} \]

\[ L_{tot} = I_E \left( \frac{2\pi}{T_E} \right) + m_M R^2 \left( \frac{2\pi}{T_{M-orb}} \right) \]

constant

\[ K_{E-rot} = \frac{1}{2} I_E \omega_E^2 = \frac{1}{2} I_E \left( \frac{2\pi}{T_E} \right)^2 \]

solid Earth – rotates under ocean tidal budge

- frictional energy loss

\[ \Delta E_E = \Delta K_{E-rot} = -W_{tidal-friction} \]

\[ -W_{tidal} = \Delta K_{E-rot} = \left[ \frac{(2\pi)^2}{2} I_E \right] \left[ \frac{1}{T_E(later)^2} - \frac{1}{T_E(earlier)^2} \right] \]

Earth’s rotation slows with time

\[ T_E(later) > T_E(earlier) \]

constant

\[ L_{tot} = \left( \frac{2\pi}{T_E} \right) I_E + m_M R^2 \left( \frac{2\pi}{T_{M-orb}} \right) \]

decreases

increases

L transferred to moon

moon moves away

with time

\[ L_{tot} = I_E \left( \frac{2\pi}{T_E} \right) + m_M \sqrt{GM_E} R^{1/2} \]

\[ m^3 \quad \text{s}^2 \text{kg} = G \]

\[ T^2 = \frac{(2\pi)^2}{GM_E} R^3 \]

\[ T^2 = \frac{GM_E}{(2\pi)^2 R^3} \]

\[ \frac{1}{T} = \sqrt{GM_E} \frac{1}{(2\pi)^3 R^{3/2}} \]
$T^2 = \frac{(2\pi)^2}{GM_E} R^3$

$T_{\text{M-orbit}} = 27 \text{ days} = T_{E-\text{rot27}}(\text{now})$

$T_{\text{M-orbit}} = \tau T_{E-\text{rot}}(\text{now})$

$R = r R_E$

$\tau^2 = \left[ \frac{(2\pi)^2 R_E^3}{GM_E T_E^2} \right] r^3$

$\tau^2 = [0.003584] r^3$

distant future

moon-orbit at 91 $R_E$

day’=month’=51 days (current)

distant past

moon- much closer
day shorter
\( L_{\text{tot}} = L_{E-\text{rot}} + L_{M-\text{orbit}} + L_{M-\text{rot}} \) very small (neglect)

\[ I_E = \frac{2}{5} M_E R_E^2 \]

\[ L_{E-\text{rot}} = \{I\} \frac{2\pi}{T_E} \]

\[ L_{E-\text{rot}} = \frac{2}{5t} \left[ \frac{M_E R_E^2 2\pi}{T_E} \right] \]

\[ T_{E-\text{rot}} \text{(at other time)} = t T_{E-\text{rot}} \text{(now)} \]

\[ L_{E-\text{rot}} = \frac{2}{5t} \left[ \frac{M_E R_E^2 2\pi}{T_E} \right] \]

Moon orbit

\[ T_{M-\text{orbit}} = 27 \text{ days} = T_{E-\text{rot}} \text{27 (now)} \]

\[ T_{M-\text{orbit}} = \tau T_{E-\text{rot}} \text{(now)} \]

\[ \tau = 27 \text{ (now)} \]

\[ m_M = \frac{M_E}{80} \]

\[ L_{M-\text{orbit}} = m_M R^2 \left( \frac{2\pi}{T_{M-\text{orbit}}} \right) \]

\[ L_{M-\text{orbit}} = \frac{M_E}{80} (r R_E)^2 \left( \frac{2\pi}{\tau T_{E-\text{rot}}} \right) \]

\[ L_{M-\text{orbit}} = \frac{r^2}{80\tau} \left[ \frac{M_E R_E^2 2\pi}{T_E} \right] \]

\[ L_{\text{totM-orbit}} = L_{E-\text{rot}} + L_{M-\text{orbit}} \]

\[ L_{\text{totM-orbit}} = \{ \frac{2}{5t} + \frac{r^2}{80\tau} \} \left[ \frac{M_E R_E^2 2\pi}{T_E} \right] \]
equal at 51.1445 days
Moon at 91.155 R_E
\[ R_E = 3.7R_M \]

\[ 60R_E = R_{\text{Earth-Moon}} \]

\[ I_E = \frac{2}{5} M_E R_E^2 \]

\[ L_{E-\text{rot}} = I_E (2\pi f_{E-\text{rot}}) \]

\[ f_{E-\text{rot}} = 1/27 \text{ days} \]

\[ f_{M-\text{rot}} = 1/27 \text{ days} \]

\[ I_M = \frac{2}{5} m_M R_M^2 \]

\[ I_M = \frac{2}{5} m_M R_M^2 = \frac{2}{5} \frac{M_E}{80} \left( \frac{R_E}{3.7} \right)^2 = (0.000913)I_E \]

\[ L_{M-\text{rot}} = I_M (2\pi f_{M-\text{rot}}) \]

\[ L_{M-\text{rot}} = (0.000913)I_E (2\pi f_{E-\text{rot}}) \left( \frac{1}{27} \right) \]

\[ L_{M-\text{rot}} = (0.0000338)L_{E-\text{rot}} \]
\[ F = \frac{GM_M m}{(R + R_E)^2} \]

\[ F = \frac{GM_M m}{(R - R_E)^2} \]

\[ F = \frac{GM_M m}{(R)^2} \]

Earth

Moon

\[ F = \frac{GM_M m}{R^2} \]