Complex numbers & variables

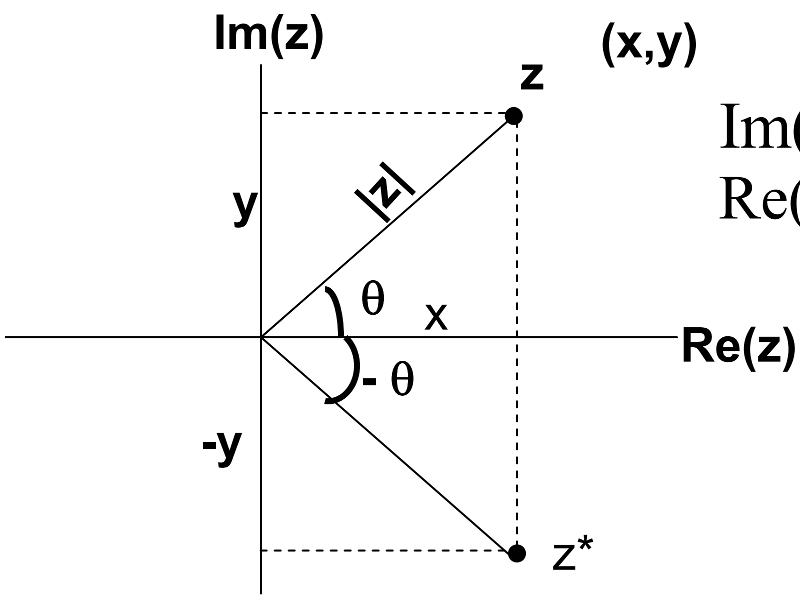
want solutions to equations like $x^2+1=0$ $\quad x^2=-1$

$$i=\sqrt{-1} \qquad i^2=-1$$

Complex variable $z = x+iy$

$$z^* = x-iy$$

Complex conjugate of z



$$\mathrm{Im}(z) = y$$
$$\mathrm{Re}(z) = x$$

$$|z|^2 = z\,z^* = x^2 + y^2$$

note

$$z=|z|e^{i\theta} \qquad z^*=|z|e^{-i\theta}$$

$$e^{i\theta}=\cos(\theta)+ i\sin(\theta)$$

$$e^{\pm i\theta}=\cos(\theta)\pm i\sin(\theta)$$

$$\sin(\theta)=\frac{e^{i\theta}-e^{-i\theta}}{2\,i}$$

$$\cos(\theta)=\frac{e^{i\theta}+e^{-i\theta}}{2}$$

Complex z-plane

cv-im1

Proof of $e^{i\theta} = \cos(\theta) + i\sin(\theta)$

$$i^3 = ii^2 = -i$$

$$i^4 = i^2\ i^2 = (-1)(-1) = 1$$

$$i^{n(odd)} = \{\ i;\ n=1,5,9\ldots\ |\ -i;\ n=3,7,11\ldots\}$$

$$i^{n(even)} = \{\ -1;\ n=2,6,10\ldots\ |\ 1;\ n=4,8,12\ldots\}$$

$$e^{i\theta} = \sum_{n-0}^{n=\infty} (i\theta)^n$$

$$e^{i\theta} = \sum_{n=0}^{n=\infty} (i\theta)^n = \sum_{n(even)=0}^{n=\infty} (i\theta)^n + \sum_{n(odd)=1}^{n=\infty} (i\theta)^n$$

$$e^{i\theta} = 1 + i\theta + (i\theta)^2 + (i\theta)^3 + (i\theta)^4 + (i\theta)^5 + (i\theta)^6 + (i\theta)^7 + (i\theta)^8 + (i\theta)^9 \ldots$$

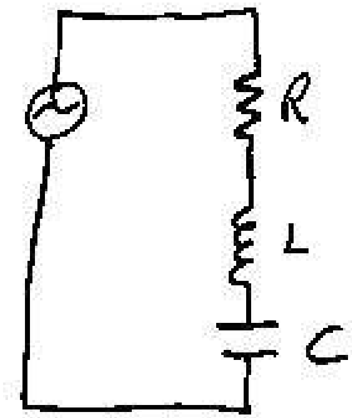
$$e^{i\theta} = \left\{1 + (i\theta)^2 + (i\theta)^4 + (i\theta)^6 + (i\theta)^8 + (i\theta)^{10} + (i\theta)^{12} \ldots\right\} + \left\{(i\theta)^1 + (i\theta)^3 + (i\theta)^5 + (i\theta)^7 + (i\theta)^9 \ldots\right\}$$

$$e^{i\theta} = \left\{1 - (\theta)^2 + (\theta)^4 - (\theta)^6 + (\theta)^8 - (\theta)^{10} + (\theta)^{12} \ldots\right\} + i\left\{(\theta)^1 - (\theta)^3 + (\theta)^5 - (\theta)^7 + (\theta)^9 \ldots\right\}$$

$$e^{i\theta} = \{\cos(\theta)\} + i\{\sin(\theta)\}$$

cv-im2

Complex impedance application



$$Z_R = R$$

$$Z_L = i\omega L$$

$$Z_C = \frac{1}{i\omega C} = -i\frac{1}{\omega C}$$

assume $I = I_0 e^{i\omega t} \quad V = V_0 e^{i\omega t \pm \varphi}$

Real currents and voltages obtained by taking Re or Im part of equations.

$$V = I(Z)$$

$$V = V_R + V_L + V_C = I(Z_R + Z_L + Z_C)$$

$$Z = R + i\omega L + \frac{1}{i\omega C}$$

$$Z = R + i[\omega L - \frac{1}{\omega C}]$$

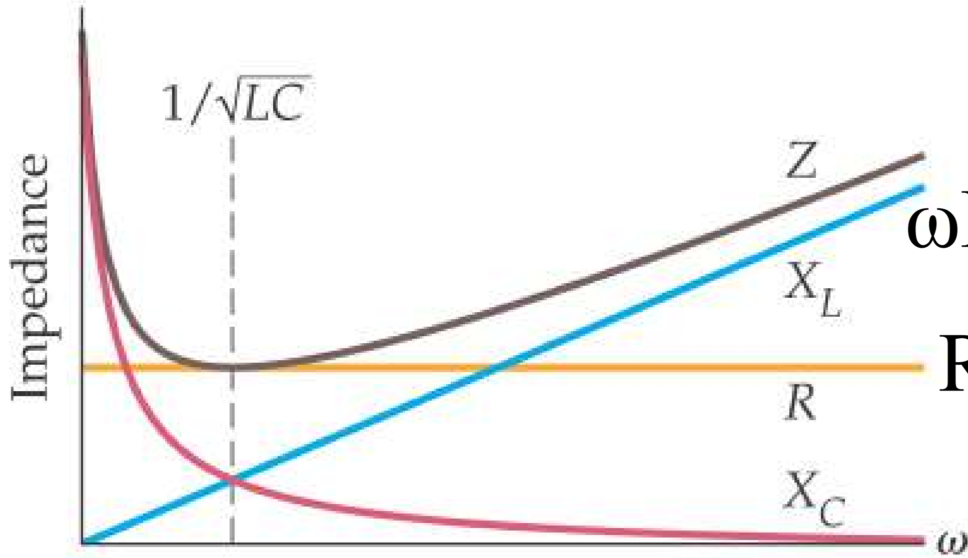
$$|Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

Resonance where $(\omega L - \frac{1}{\omega C}) \Rightarrow \mathbf{0}$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$|I| = \frac{|V|}{|Z|}$$

cv-im3



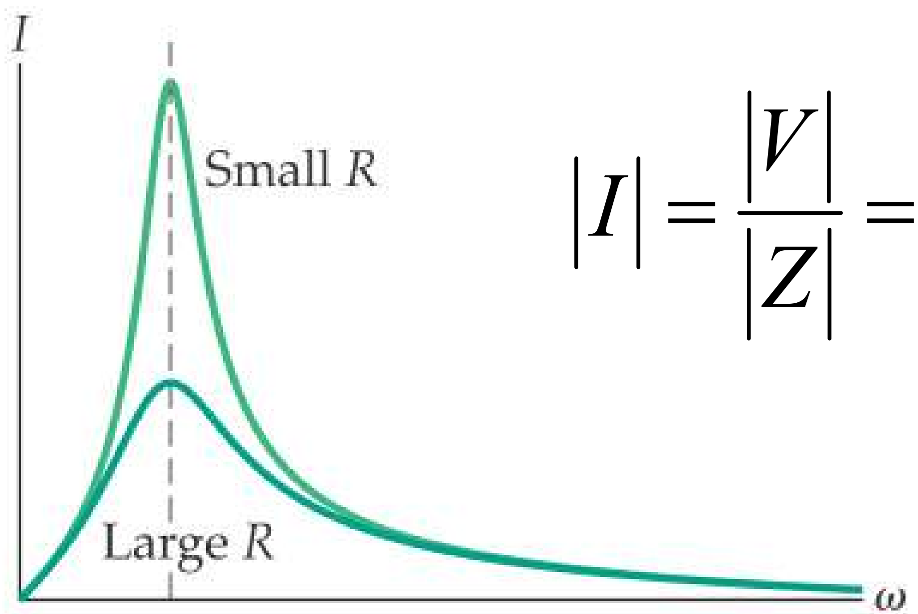
$$|Z| = \sqrt{R^2 + [\omega L - \frac{1}{\omega L}]^2}$$

ωL

R

$\frac{1}{\omega L}$

(a)



$$|I| = \frac{|V|}{|Z|} = \frac{|V|}{\sqrt{R^2 + [\omega L - \frac{1}{\omega L}]^2}}$$

(b)

cv-im4