Air/Fluid Resistance to Moving Object

$$F_{res} = \frac{C\rho A}{2} v_{rel}^2$$

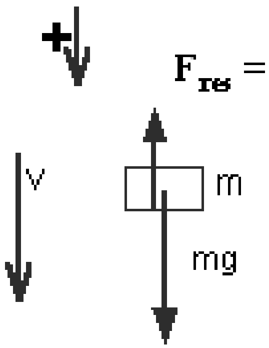
C = drag coeff. ρ = density of gas(fluid)

A = cross-sectional area of object

v_{rel} = speed relative to air

(air at rest $v_{rel}=v$)

(air moving at v_a / $v_{rel}=v+v_a$)



$$F_{res} = \frac{C\rho A}{2} v^2$$

$$ma = F$$

$$ma = mg - \frac{C\rho A}{2} v^2$$

$$a = \frac{dv}{dt} = g - \frac{C\rho A}{2m} v^2 = g - \frac{1}{L} v^2$$

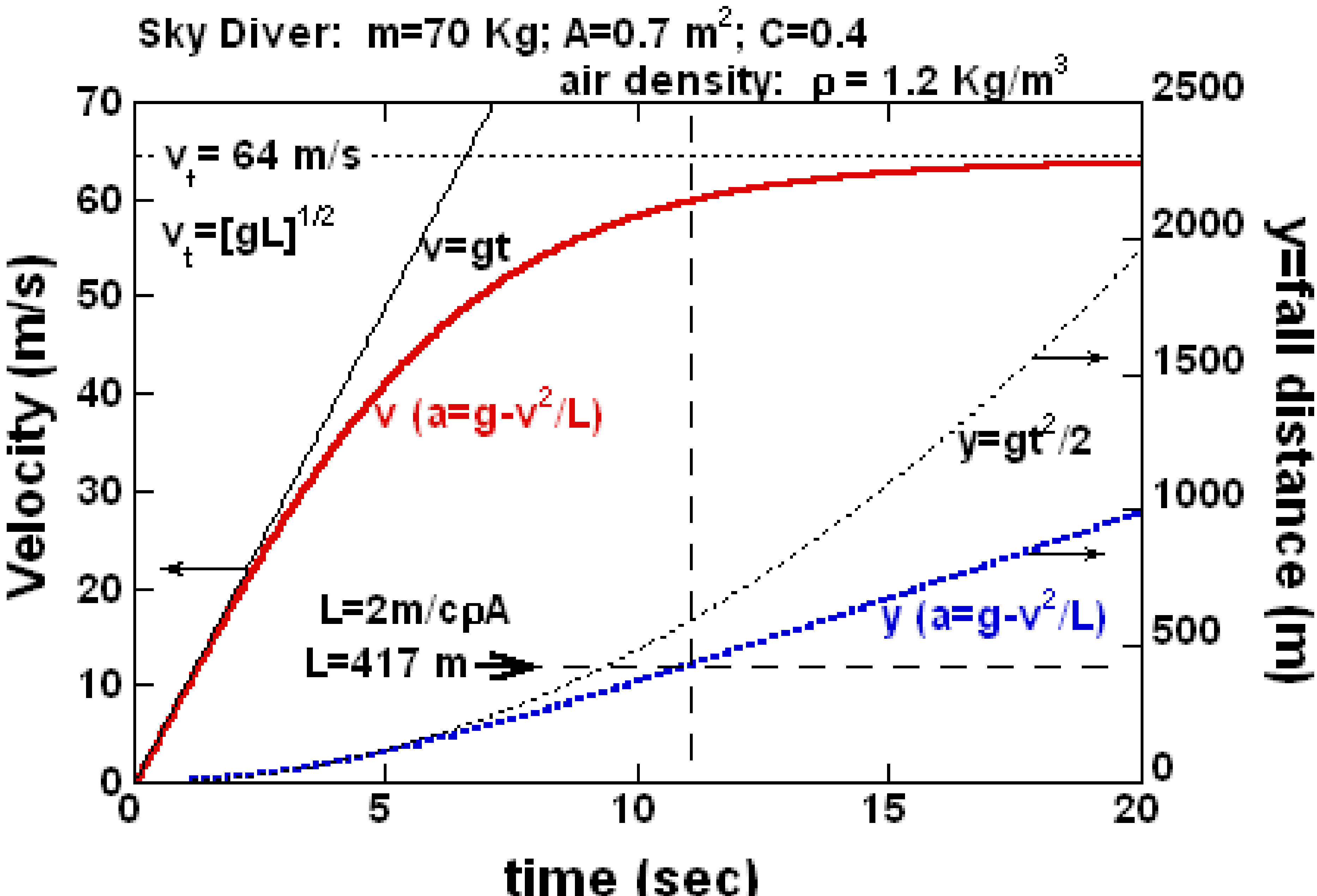
nonlinear differential equation

$$L = \frac{2m}{C\rho A}$$

$$\frac{dv}{dt} = g - \frac{v^2}{L}$$

$$\frac{dx}{dt} = v$$

Can solve with calculus with luck : can solve numerically for sure



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solution $\frac{dv}{dt}=g-\frac{v^2}{L}$ $\frac{dy}{dt}=v$ **with** $y(0)=0$, $v(0)=0$

Numerical Integration Solution to Newtons Law

$$a = g - \left(\frac{1}{k}\right) v^2$$

(falling object with air resistance example)

$$\frac{\Delta v}{\Delta t} = g - \left(\frac{1}{k}\right) v^2$$

$$\Delta v = g\,\Delta t - \left(\frac{1}{k}\right) v^2\,\Delta t$$

$$\Delta v = v(t_i + \Delta t) - v(t_i)$$

best to use $\bar{v}$ (ave)

approx. with $v(t_i)$!

$$v(t_i + \Delta t) = v(t_i) - g\,\Delta t - \left(\frac{1}{k}\right) v^2\,\Delta t$$

$$v(t_i + \Delta t) = v(t_i) - g\,\Delta t - \left(\frac{1}{k}\right) v(t_i)^2\,\Delta t$$

$$v_{i+1} = v_i - g\,\Delta t - \left(\frac{1}{k}\right) v_i^2\,\Delta t$$

!! know v_i $(v(t_i))$ set Δt

& you can find v_{i+1} $(v(t_i + \Delta t))$

anybody can solve with spreadsheet

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$g = 9.8\ m/s^2 \quad \Delta t = 0.05\ s$

$\frac{1}{L} = 417\ m/s$

$t_1 = 0 \qquad v_1 = 0$

$t_2 = t_1 + \Delta t \qquad v_2 = v_1 + g\Delta t - \left(\frac{1}{L}\right) v_1^2\ \Delta t$

$t_3 = t_2 + \Delta t \qquad v_3 = v_2 + g\Delta t - \left(\frac{1}{L}\right) v_2^2\ \Delta t$

$$\Delta x = v_{ave}\Delta t = \frac{v_1 + v_2}{2}\Delta t$$

$$x_2 = x_2 + \frac{v_1 + v_2}{2}\Delta t$$

NUMREAL06.xls

	A	B	C
1	t	v(t)	x(t)
2		B3+9.8*0.05-B3^2*0.05/417	C3+0.05*(B5+B3)/2
3	0.05	0	0
4	0.1	0.49	0.02449928
5	0.15	0.979971211	0.073495682
6	0.2	1.469856062	0.146984887
7	0.25	1.959597012	0.244959703
8	0.3	2.449136578	0.367410063
9	0.35	2.938417361	0.514323029
10	0.4	3.427382073	0.685682802
11	0.45	3.915973567	0.881470725
12	0.5	4.404134856	1.101665293
13	0.55	4.891809148	1.346242161
14	0.6	5.378939868	1.615174157
15	0.65	5.865470684	1.908431292
16	0.7	6.351345534	2.225980776
17	0.75	6.836508653	2.567787029
18	0.8	7.320904594	2.933811702