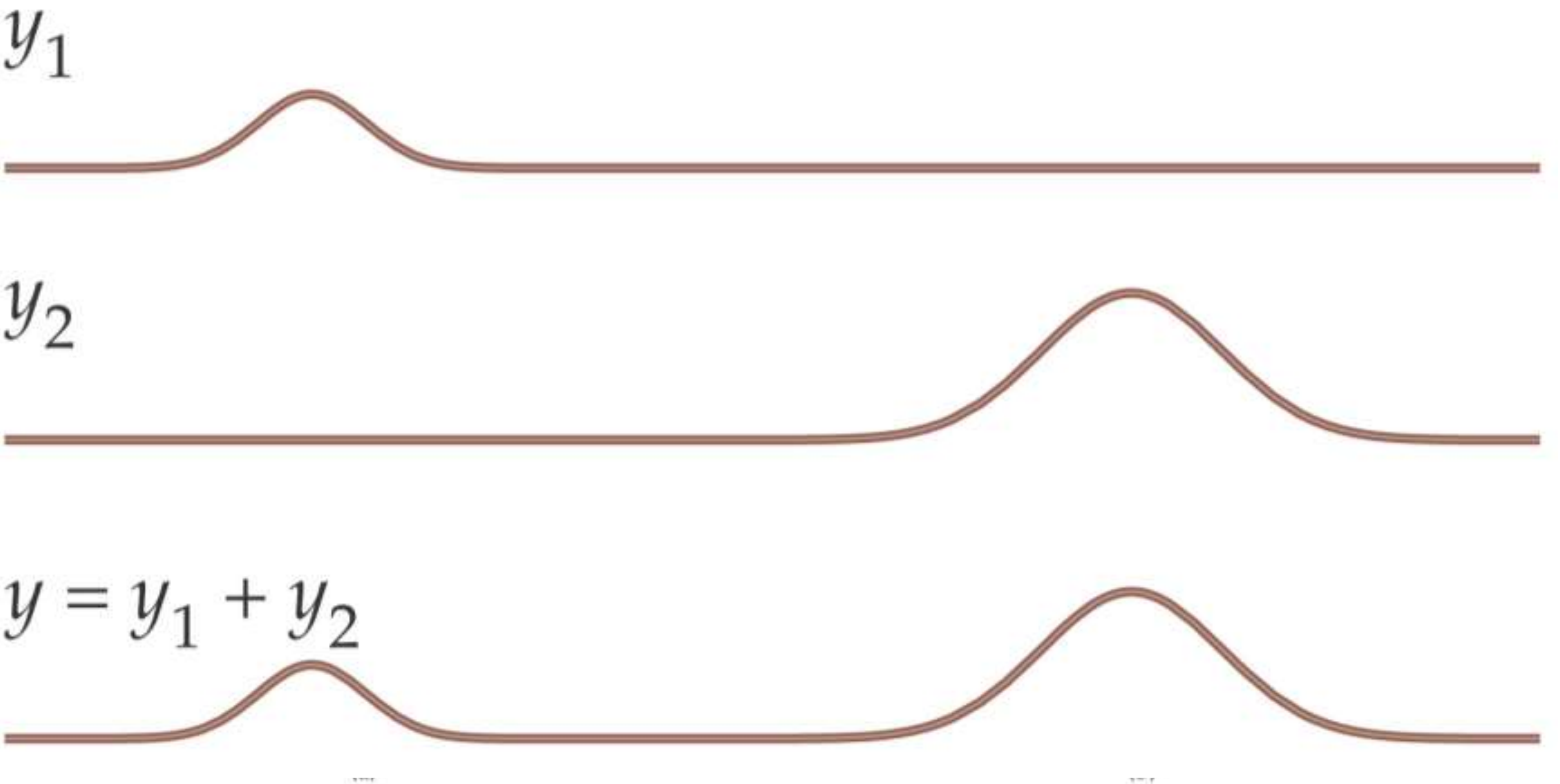
Wave Superposition

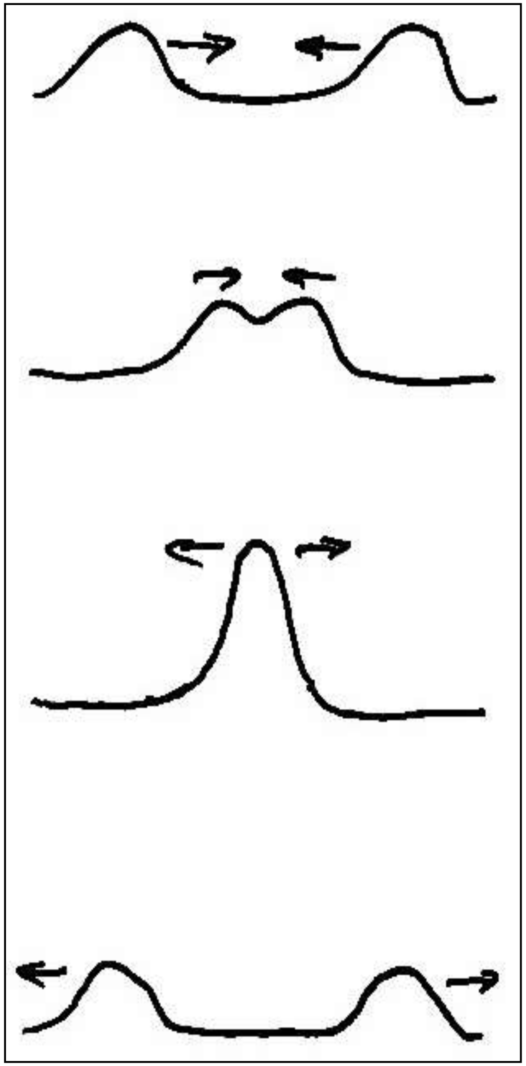
Waves of small amplitude traveling through the same medium combine, or superpose, by simple addition.

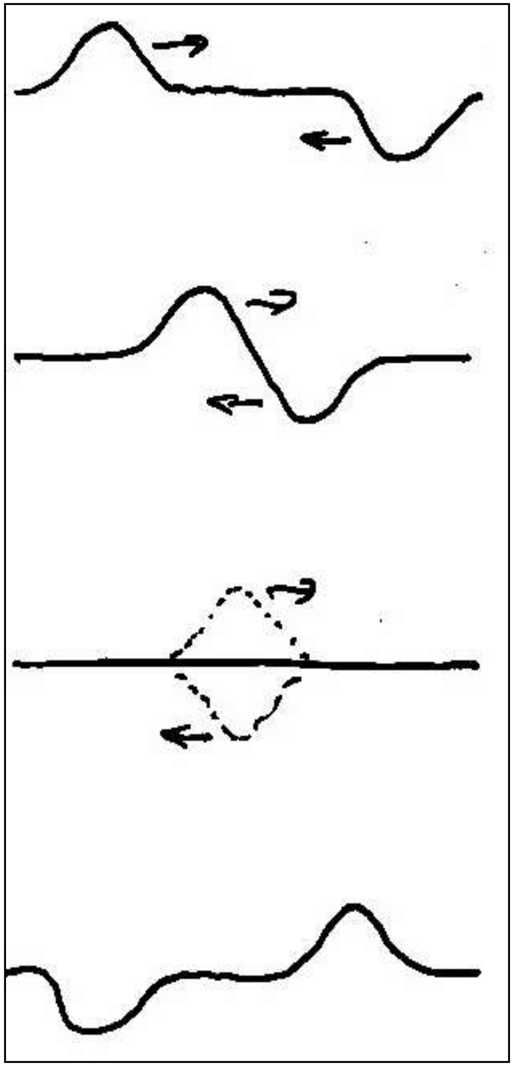


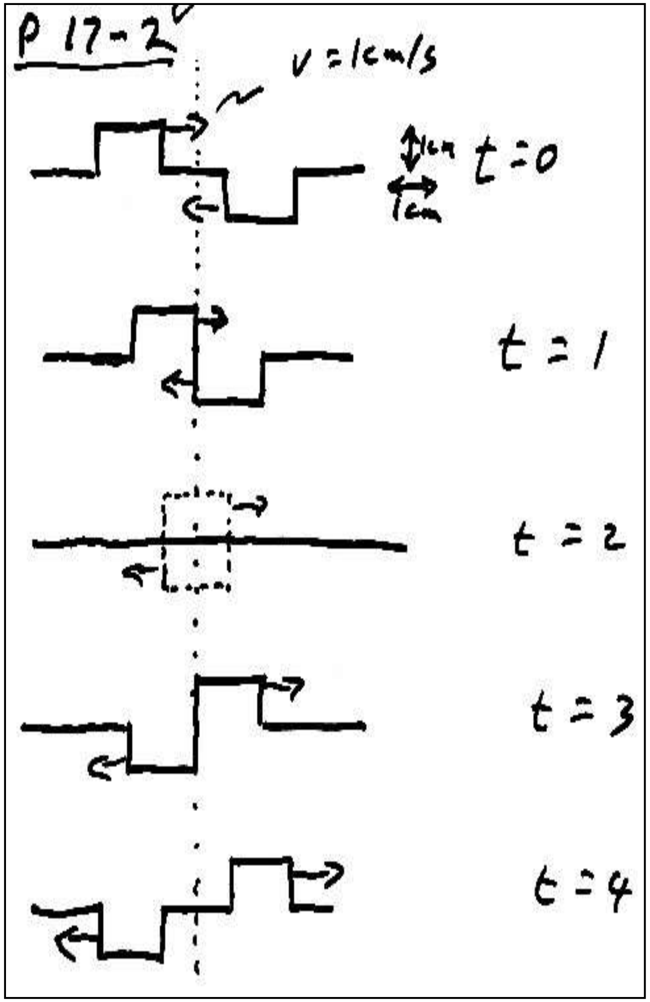
Wave Interference

Principle of Linear Superposition

$$f_{Tot} = f_1(x \pm vt) + f_2(x \pm vt)$$

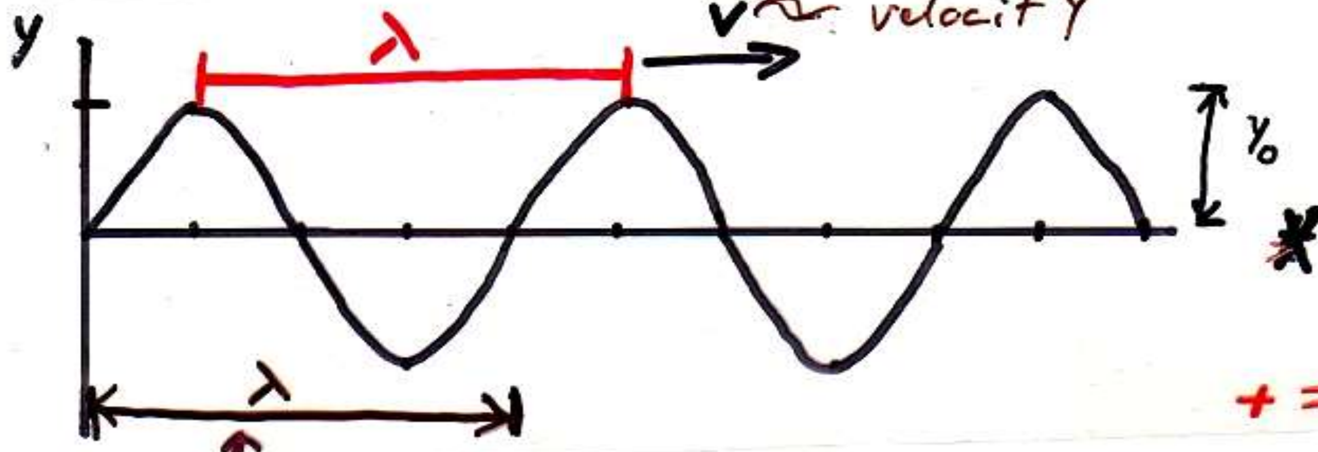






Simple Harmonic Wave

recall



+ ⇒ moves in −x dir.

$$y = y_0 \sin\left[2\pi\left(\frac{x}{\lambda} \pm \frac{t}{T}\right)\right]$$

moves in
x dir

ie/ $t=0 \Rightarrow y = y_0 \sin\left[2\pi\left(\frac{x}{\lambda}\right)\right]$

$x=0 \Rightarrow y = y_0 \sin\left[2\pi\left(\pm\frac{t}{T}\right)\right]$

Sine wave in space and time

$t=0 \Rightarrow y = y_0 \sin\left[2\pi\left(\frac{x}{\lambda}\right)\right]$

$x=0 \Rightarrow y = y_0 \sin\left[2\pi\left(\pm\frac{t}{T}\right)\right]$

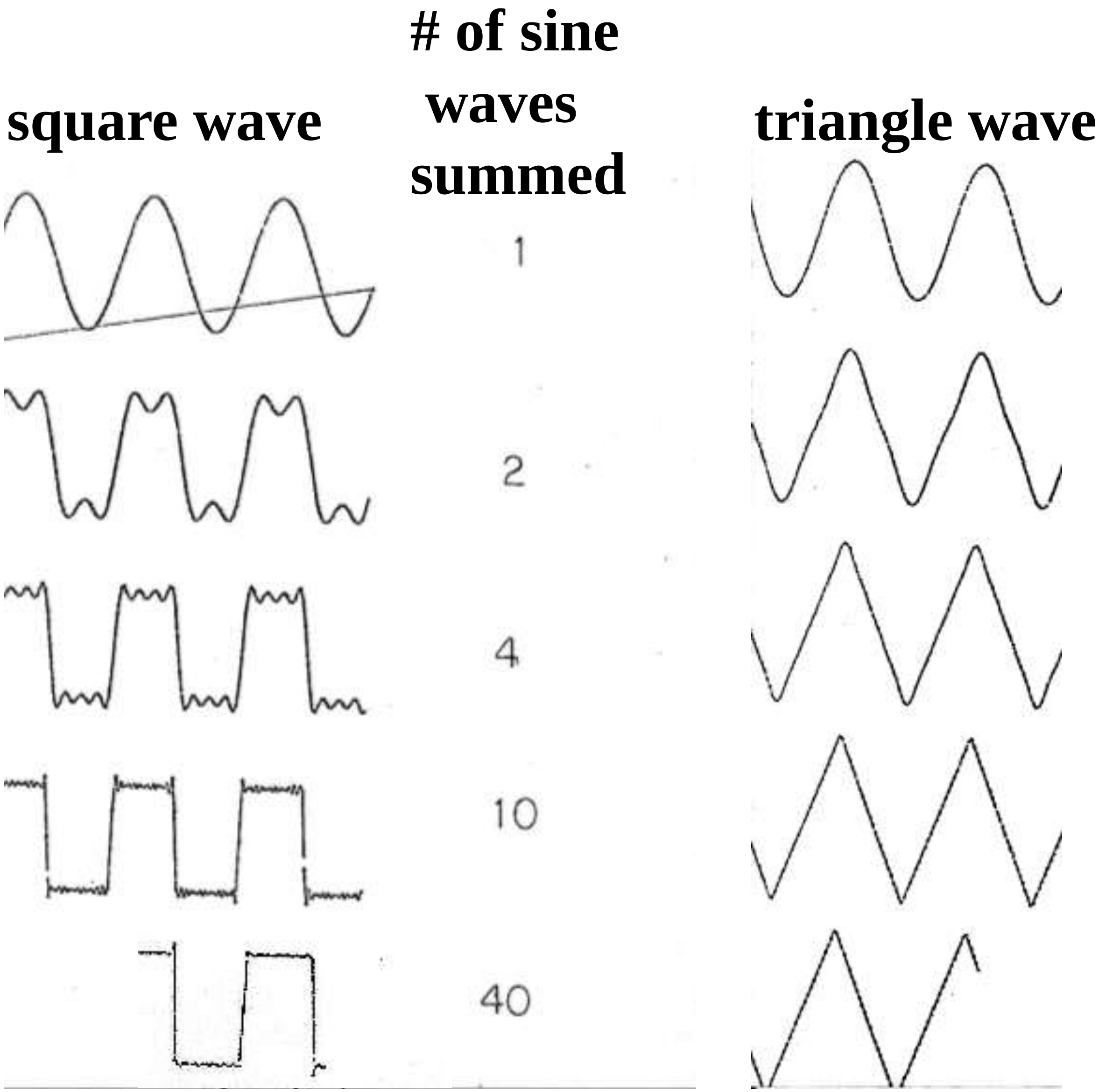
$v = \lambda \frac{1}{T} = \lambda f$

λf

$f = \frac{1}{T}$

f = frequency $\frac{1}{s} = Hz$

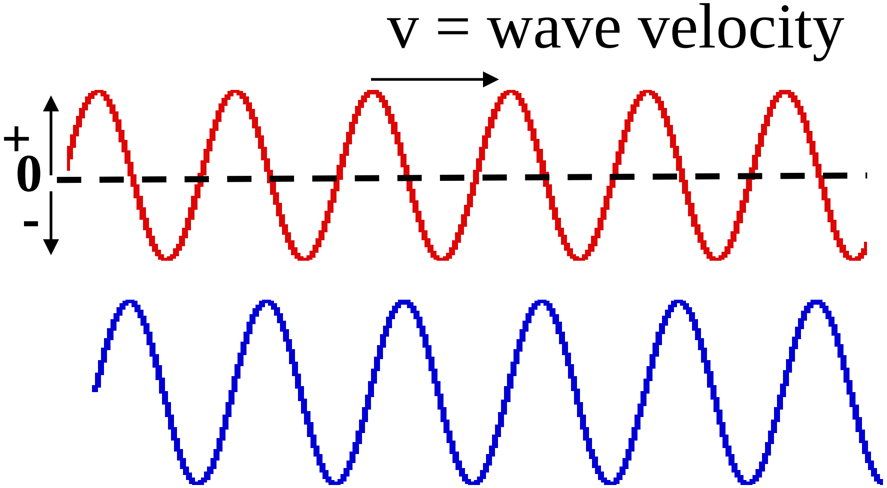
Aside: M. Fourier proved that any (well behaved) wave* can be written as a sum of simple harmonic waves.



SH waves are all we need

can handle "anything" just add up enough SH waves

Flip-side: one can break down any wave into Fourier SH components

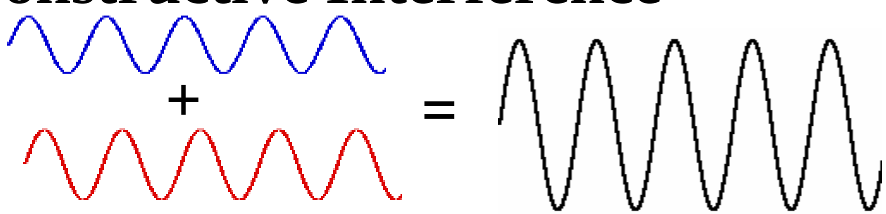


Interference: simple harmonic waves

Add up 2 waves point by point

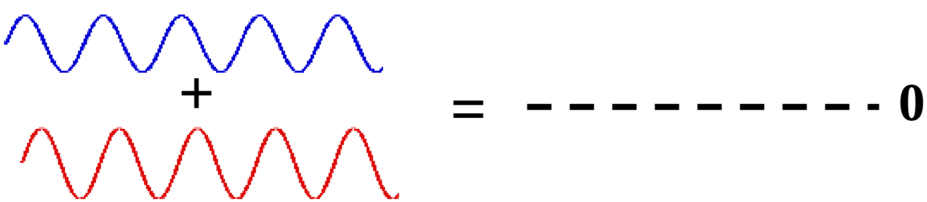
Interference – partial/total cancelation/increase

Constructive Interference

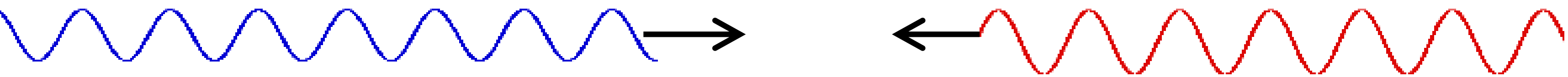


anything in-between

Destructive Interference



2 identical waves traveling in opposite directions



+ vs − only difference

$$\text{Sum} = A\sin\left(2\pi\frac{t}{T} + 2\pi\frac{x}{\lambda}\right) + A\sin\left(2\pi\frac{t}{T} - 2\pi\frac{x}{\lambda}\right)$$

θ θ'

trig. identity

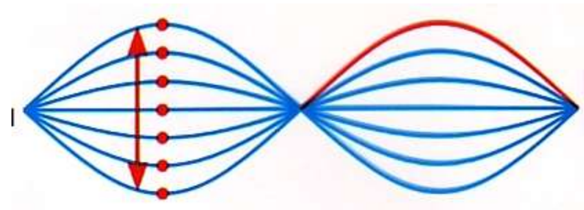
$$\sin\theta + \sin\theta' = 2\sin\left(\frac{\theta+\theta'}{2}\right)\cos\left(\frac{\theta-\theta'}{2}\right)$$

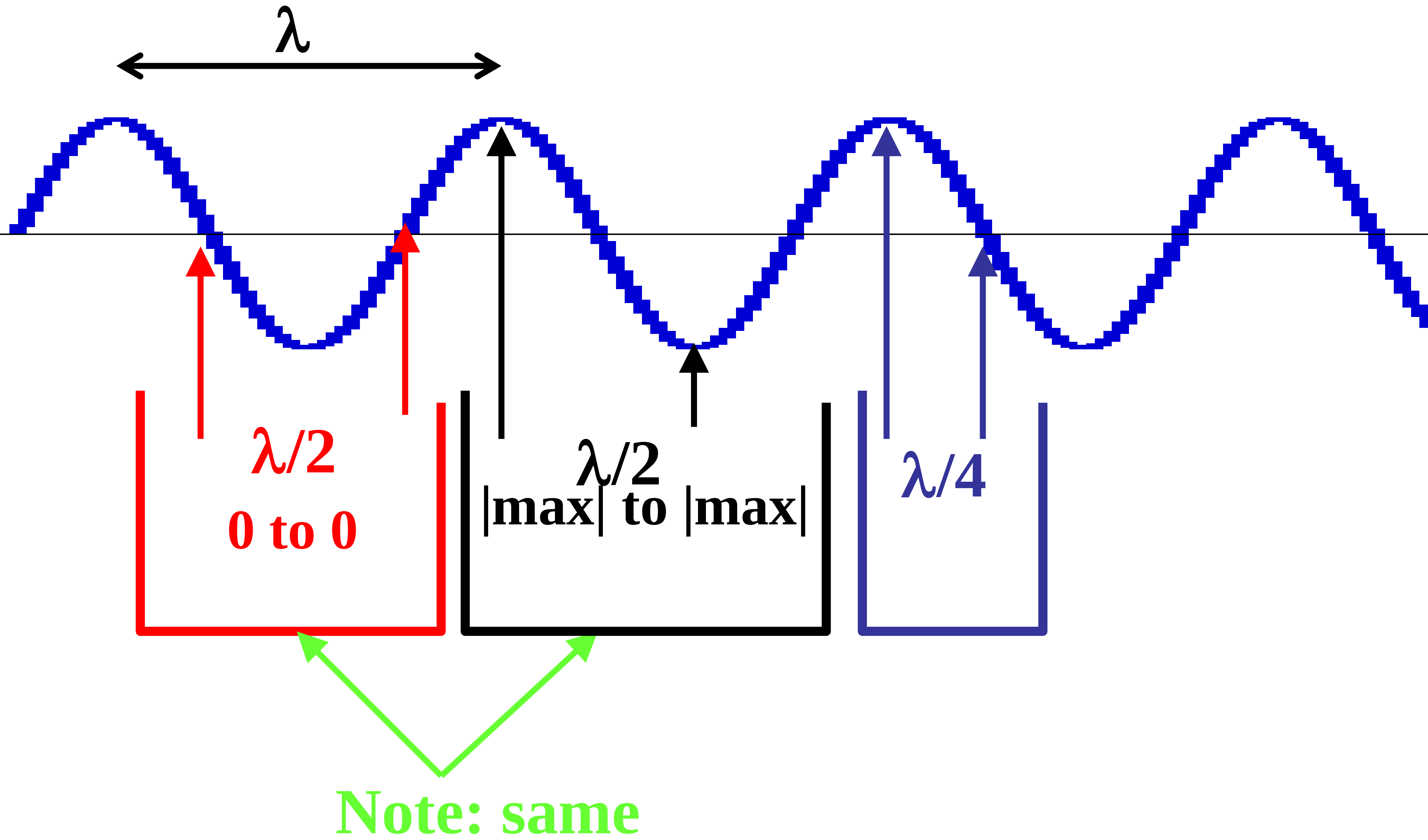
$\Downarrow$

$$\text{Sum} = 2A\sin\left(2\pi\frac{t}{T}\right)\cos\left(\frac{-2\pi x}{\lambda}\right)$$

time only **space only**

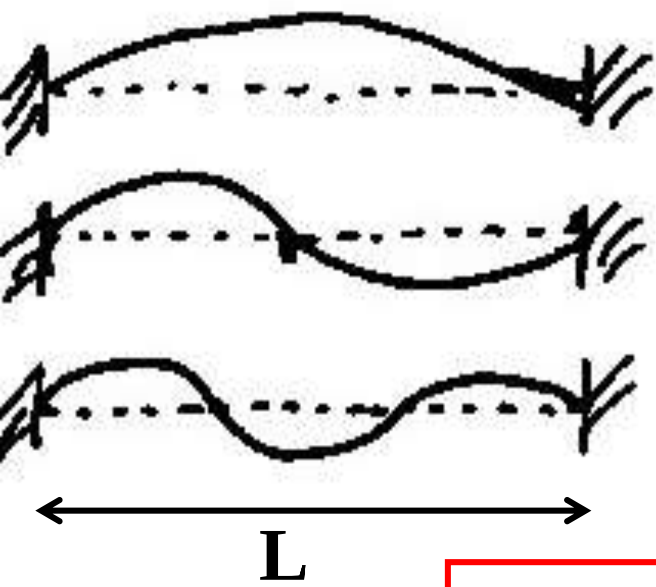
Standing wave




λ
λ/2
0 to 0
λ/2
|max| to |max|
λ/4
Note: same

Standing Waves: e.g. string with fixed end points

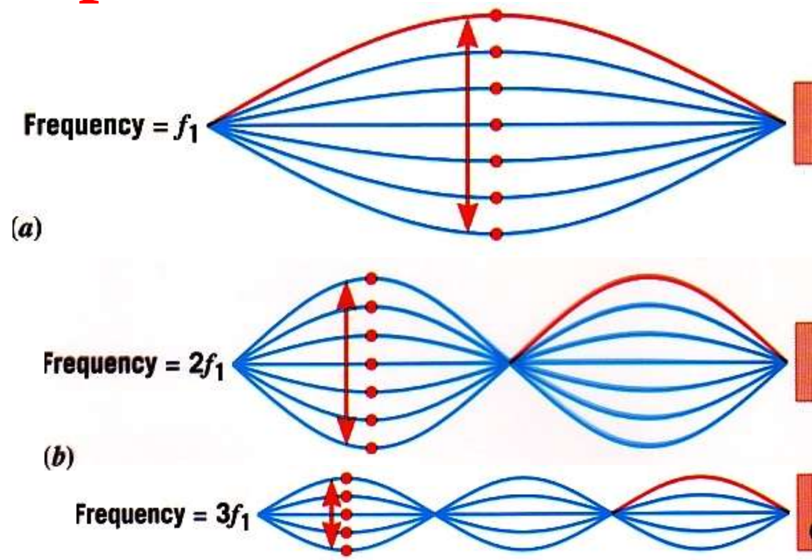
BOUNDARY CONDITIONS: no amplitude at ends



$$L=\frac{\lambda_1}{2}1$$

$$L=\frac{\lambda_2}{2}2$$

$$L=\frac{\lambda_3}{2}3$$



$$L=\frac{\lambda_n}{2}\ n$$

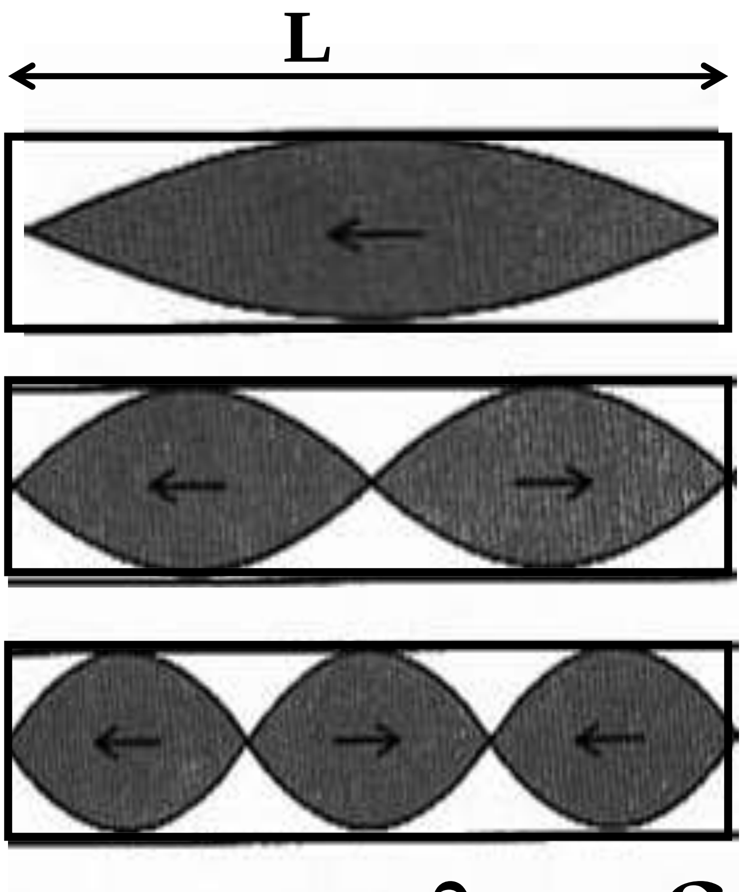
$$\lambda_n=\frac{2L}{n}\quad :n=1,2,3,4\ldots$$

$$v=\lambda_n\ f_n$$

$$f_n=\frac{v}{\lambda_n}=n\frac{v}{2L}=nf_1$$

Standing Sound Waves: e.g. pipe with 2 closed ends

BOUNDARY CONDITIONS: no amplitude at ends (again)



$$L = \frac{\lambda_1}{2} 1$$

$$L = \frac{\lambda_2}{2} 2$$

$$L = \frac{\lambda_3}{2} 3$$

$$L = \frac{\lambda_n}{2} n$$

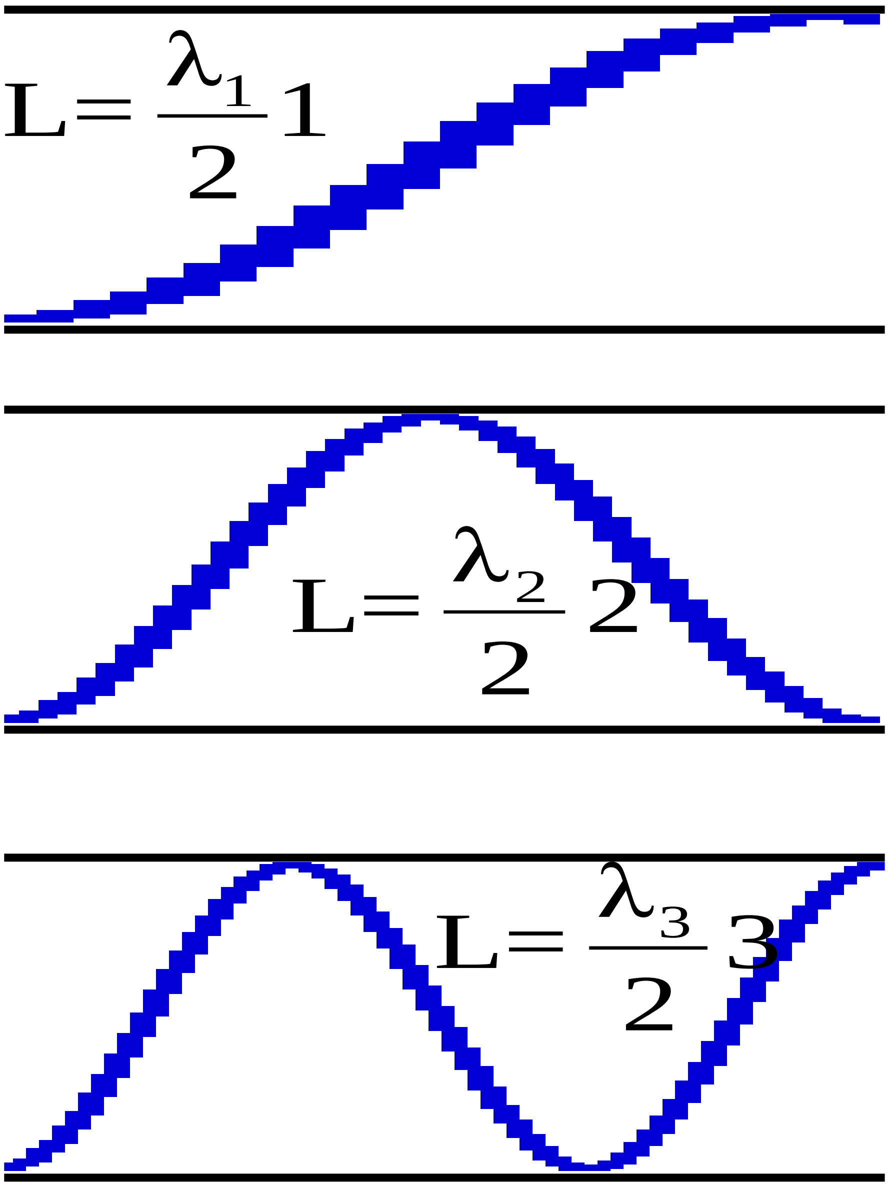
$$: n = 1, 2, 3, 4 \ldots$$

$$\lambda_n = \frac{2L}{n}$$

$$v = \lambda_n f_n$$

$$f_n = \frac{v}{\lambda_n} = n \frac{v}{2L} = n f_1$$

Open-Open end organ pipe



$$L=\frac{\lambda_n}{2}\;n$$

$$:n = 1,2,3,4\ldots$$

$$\lambda_n=\frac{2L}{n}$$

$$v=\lambda_n\;f_n$$

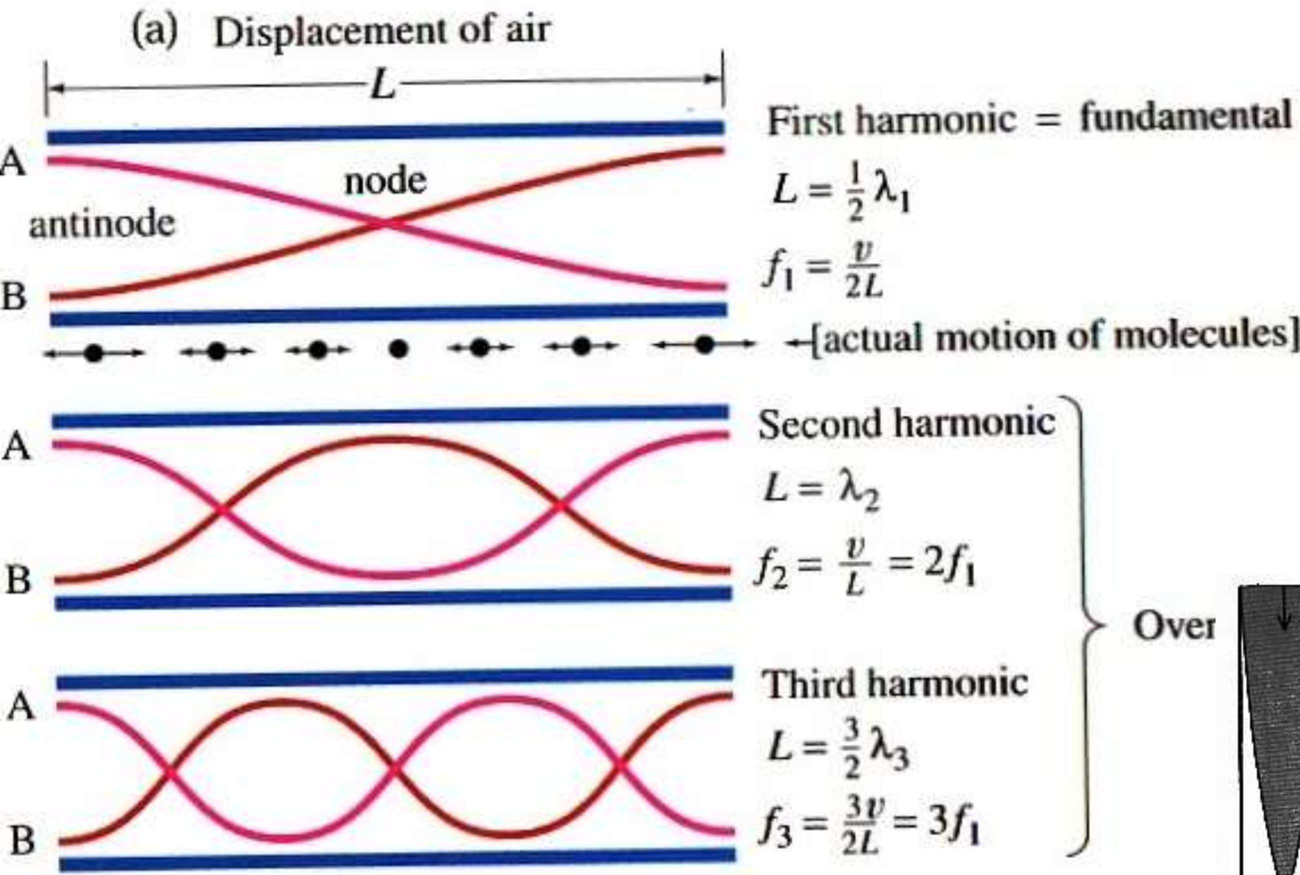
$$f_n=\frac{v}{\lambda_n}=n\frac{v}{2L}=nf_1$$

Note: open-open

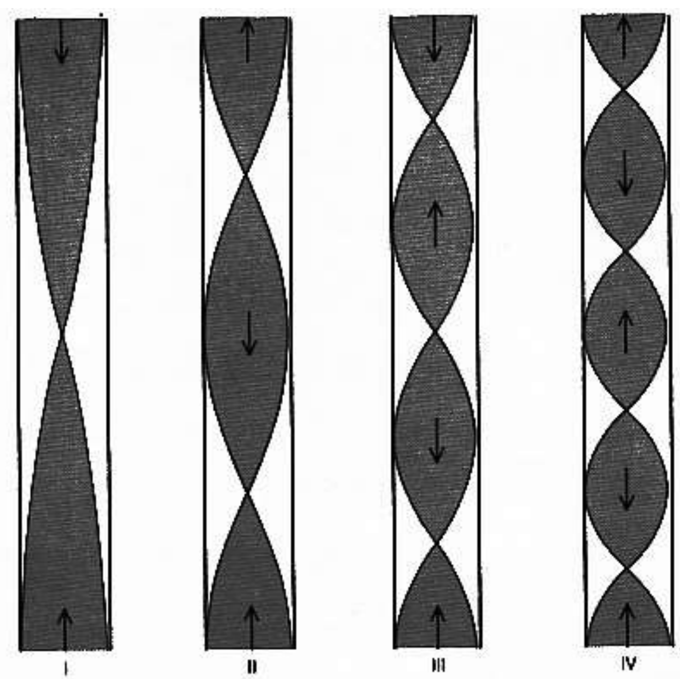
Same L-f as closed-closed

Standing Sound Waves: e.g. pipe with 2 open ends

BOUNDARY CONDITIONS: max amplitude at both ends



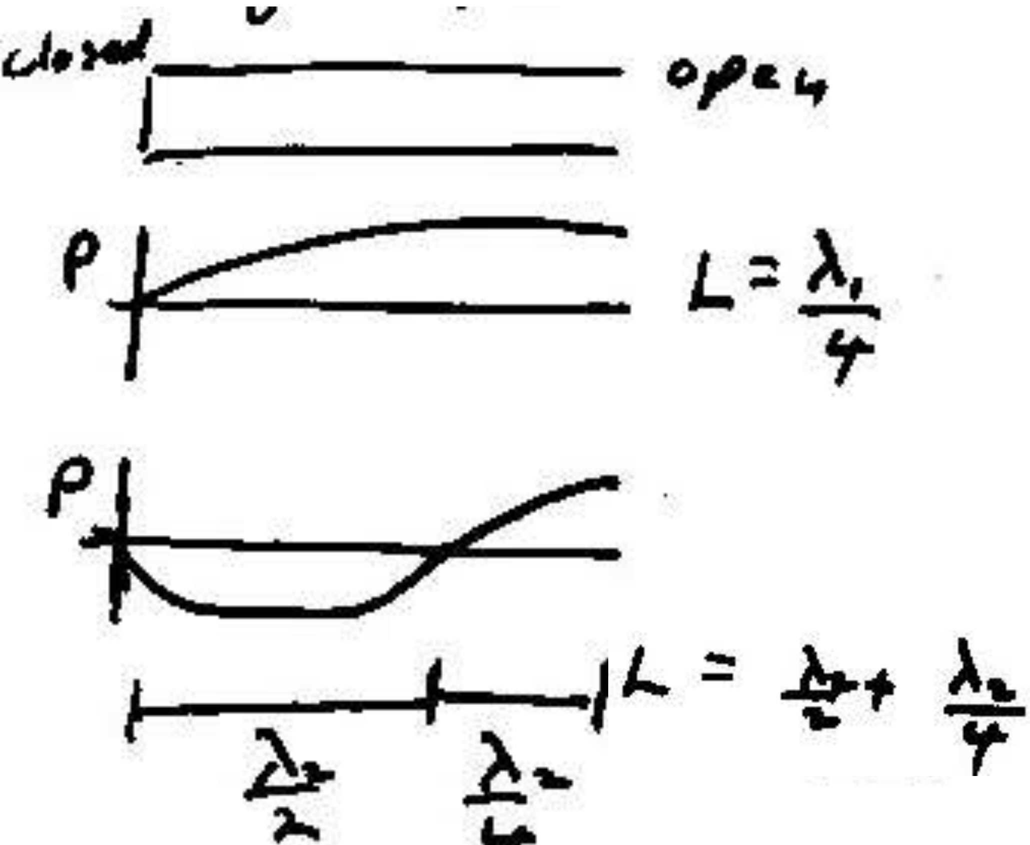
Same situation
as on last slide



12a-10a

Standing Sound Waves: e.g. pipe with 1 closed , 1 open end

BOUNDARY CONDITIONS: 0 amplitude one end: max at other



$$L = \frac{\lambda_n}{2}(n-1) + \frac{\lambda_n}{4}$$

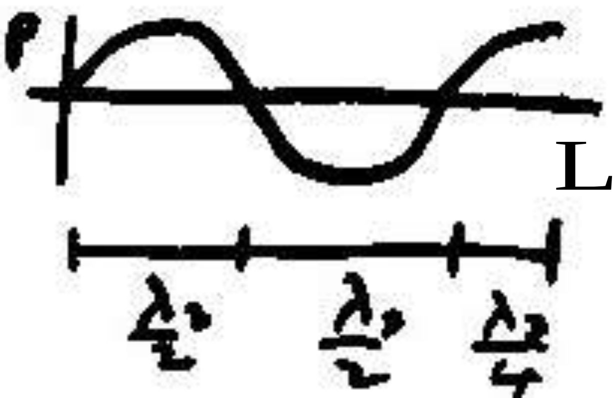
$$:n = 1,2,3,4\ldots$$

$$L = \frac{\lambda_n}{4}(2n-1)$$

$$\lambda_n = \frac{4L}{(2n-1)}$$

$$f_n = \frac{(2n-1)v}{4L}$$

$$f_n = \frac{v}{\lambda_n}$$

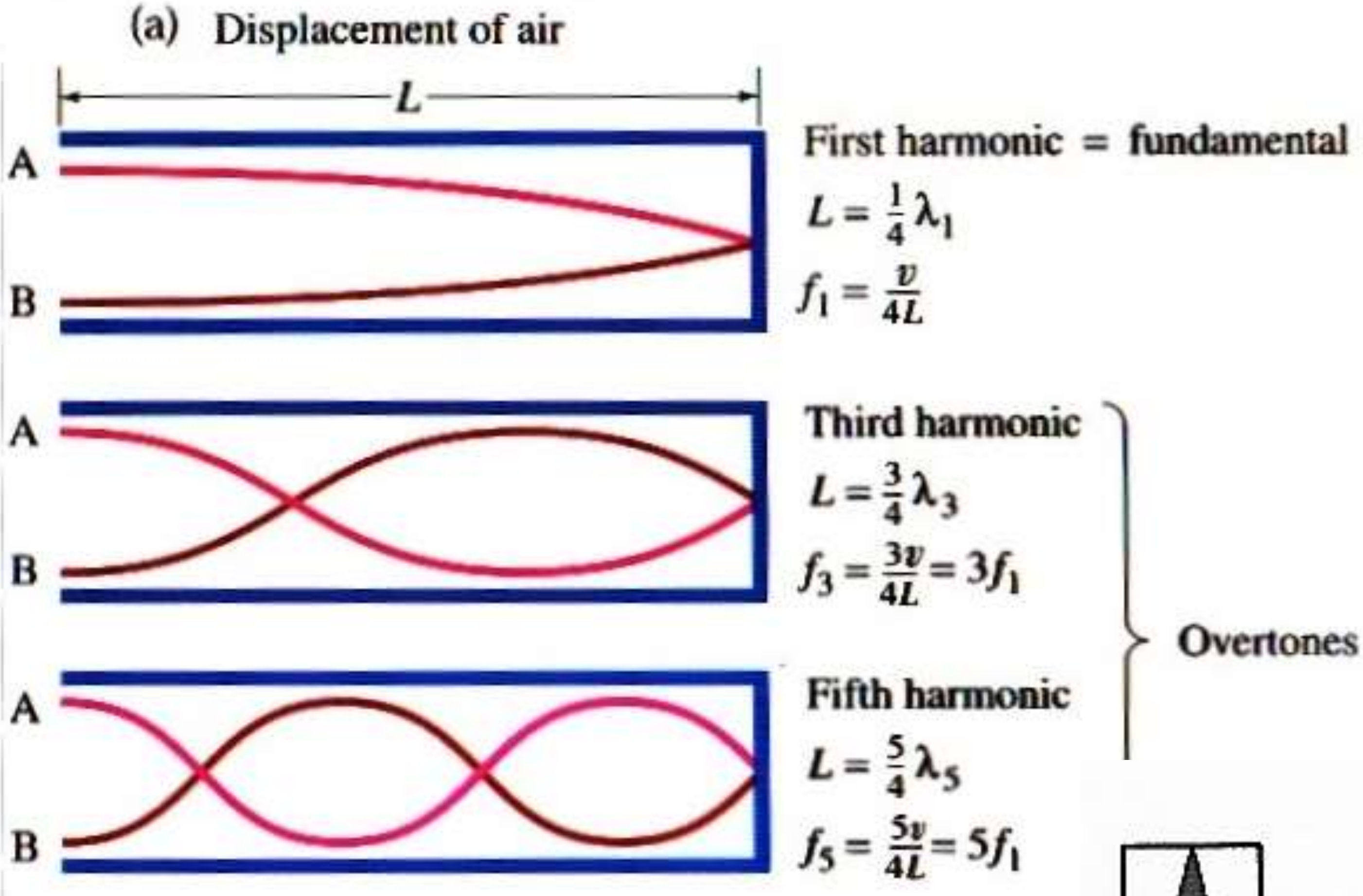


$$L = \frac{\lambda_3}{2}2 + \frac{\lambda_3}{4}$$

or $$f_m = \frac{mv}{4L}$$

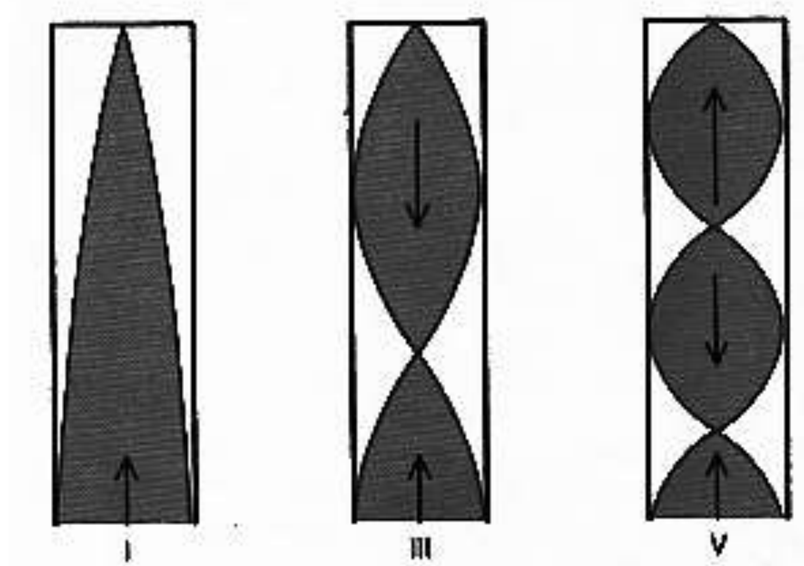
$$:m = 1,3,5,7,9\ldots$$

Same situation as on last slide

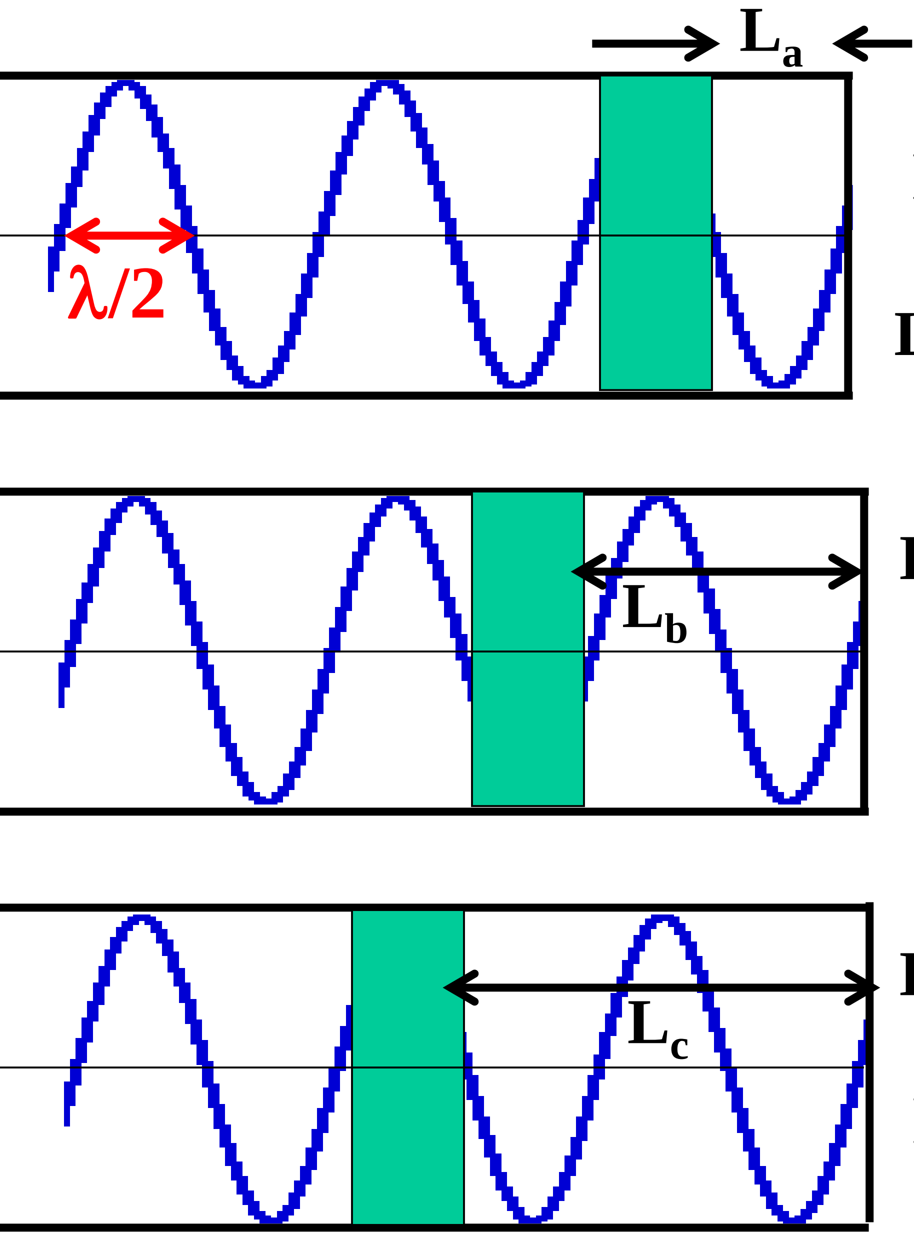


0pen end
– max. density
oscillation !!

Closed end
– density node !!



Demonstration- tube with movable speaker-closed-closed



$$L_a = \frac{\lambda}{2}$$

L_a=0.24m

L_b=0.48m

$$L_b = 2\frac{\lambda}{2}$$

L_a=72m

$$L_c = 3\frac{\lambda}{2}$$

f=700 Hz

λ/2=0.24m

λ=0.48m

$$v = \lambda f$$

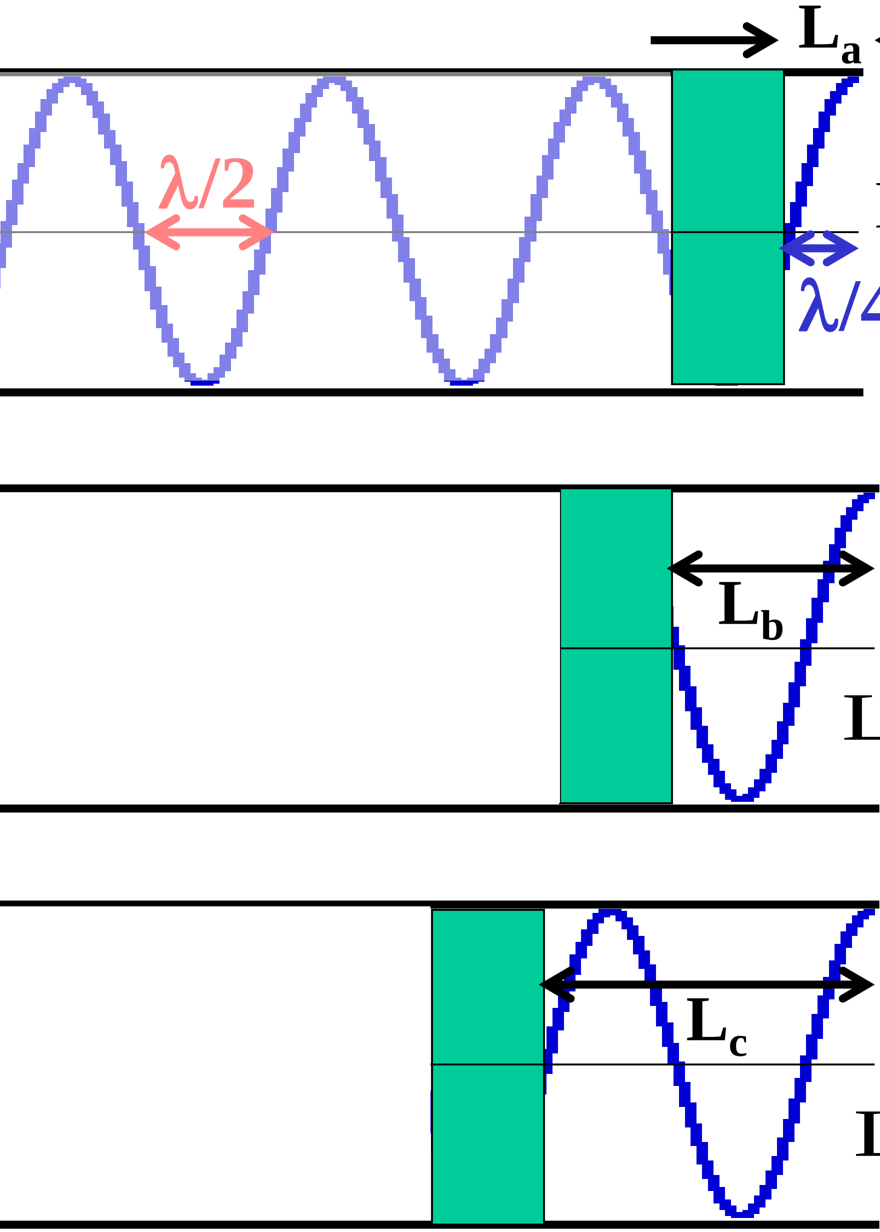
=(0.48m)(700 1/s)

v=336 m/s **!!!**

sound speed in air

Literature v=340 m/s

Demonstration- tube with movable speaker-closed-open



$$L_a = \frac{\lambda}{4}$$

L_a=0.12m

L_b=0.36m

$$L_b = \frac{\lambda}{2} + \frac{\lambda}{4}$$

L_c=0.6m

$$L_c = 2\frac{\lambda}{2} + \frac{\lambda}{4}$$

f=700 Hz

λ/4=0.12m

λ/2=0.24m

λ=0.48m

$$v = \lambda f$$

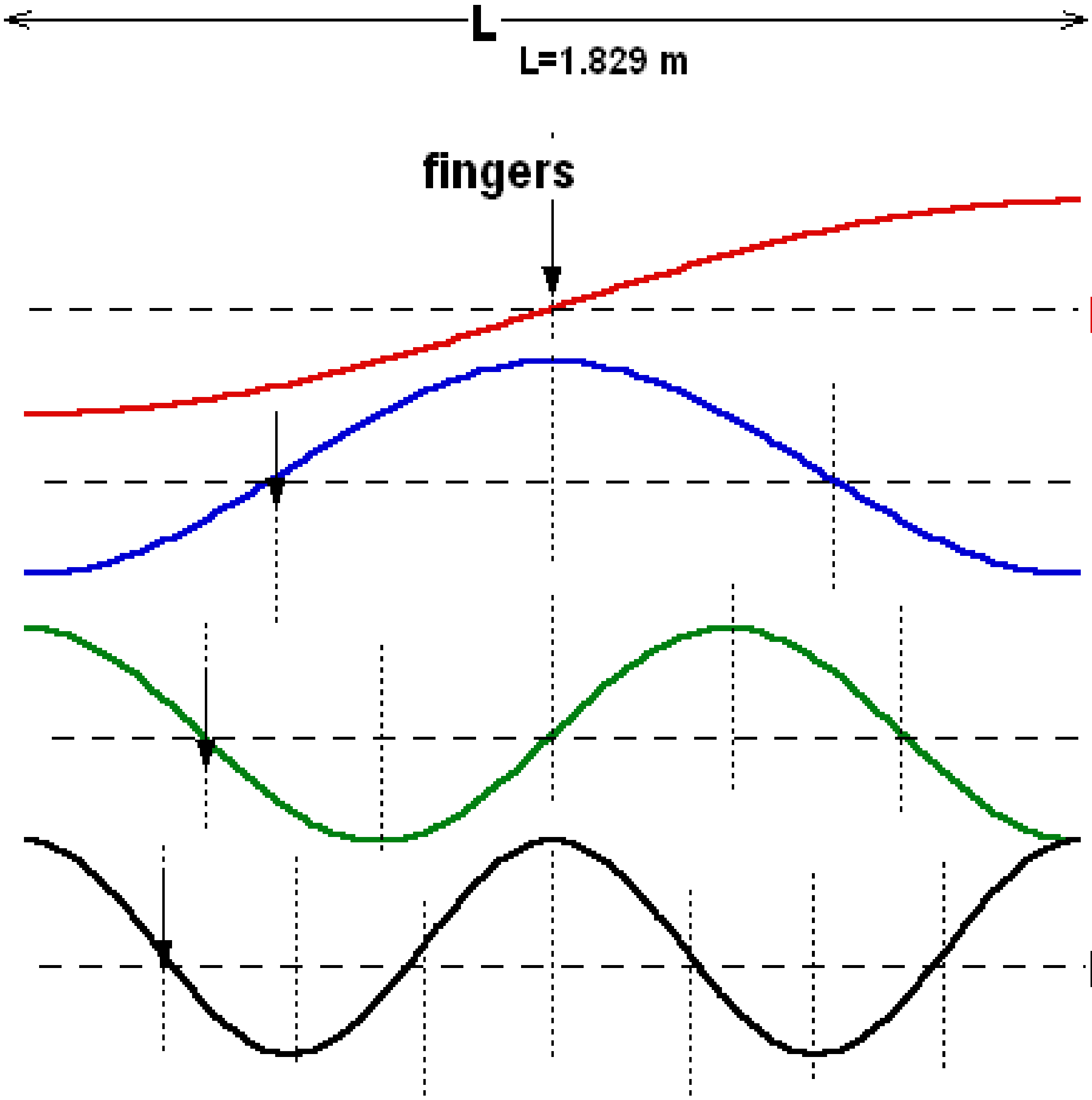
=(0.48m)(700 1/s)

v=336 m/s **!!!**

sound speed in air

Literature v=340 m/s

singing rod standing waves



condition	wave length	freq. (Hz)
$L = 2\,(\lambda_1/4)$	$\lambda_1 = 2L$	$f_1 = 1420$
$L = 4\,(\lambda_2/4)$	$\lambda_2 = L = \lambda_1/2$	$f_2 = 2f_1 = 2840$
$L = 6\,(\lambda_3/4)$	$\lambda_3 = 2L/3 = \lambda_1/3$	$f_3 = 3f_1 = 4260$
$L = 8\,(\lambda_4/4)$	$\lambda_4 = L/2 = \lambda_1/4$	$f_4 = 4f_1 = 5690$

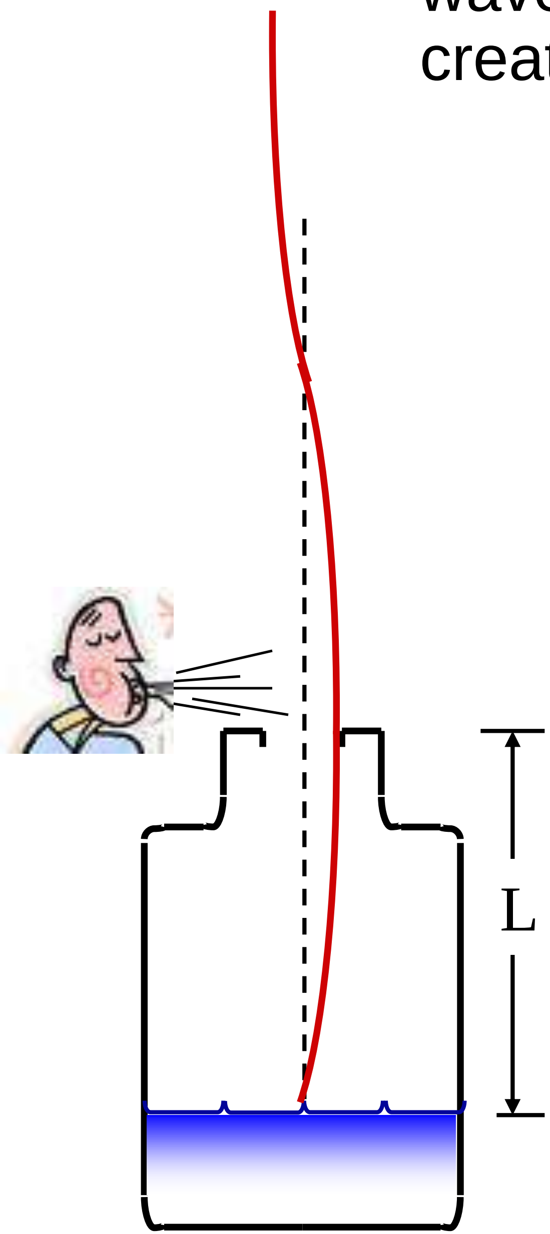
$f = v/\lambda \rightarrow v = f\lambda \rightarrow v = 1420(1/s)\,(2)(1.829\,m) = 5194\ m/s$

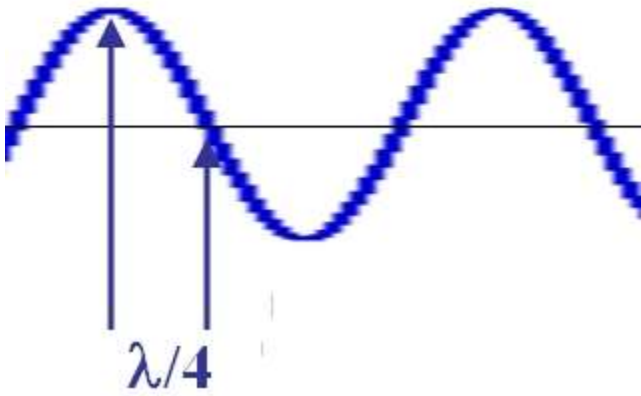
Book says vel of sound in Al about 5000 m/s

Creating Tones with Bottles

tone produced by blowing across a bottle opening

wavelength of the fundamental wave (1st harmonic) created in terms of L?





$$L = \frac{\lambda}{4} \qquad \lambda f = v$$

$$f = \frac{v}{\lambda} = \frac{v}{4L}$$

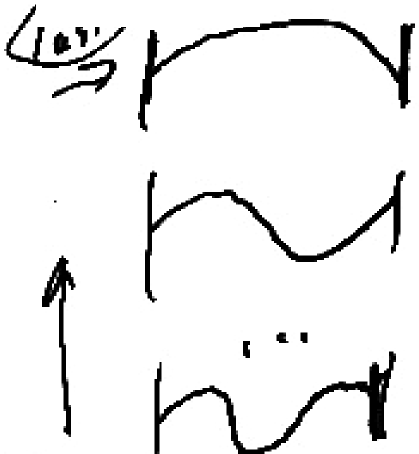
L smaller $\rightarrow$ f higher

For a rope of length L show that as the tension is increased that there is a maximum tension above which no standing waves are possible.

CQ 17-9

$$n\left(\frac{\lambda_n}{2}\right) = L$$

$$f_n \lambda_n = v = \sqrt{\frac{T}{\rho}}$$



T increases

fixed here = f

$$\lambda_n = \left(\frac{2L}{n}\right)$$

$$\lambda_n f_* = \sqrt{\frac{T}{\rho}}$$

$$\frac{2L}{n} f = \sqrt{\frac{T}{\rho}}$$

$$\frac{2Lf\sqrt{\rho}}{\sqrt{T}} = n$$

$$\sqrt{T} = \frac{2Lf\sqrt{\rho}}{n}$$

T increases n decreases

n = 1 Lowest possible

$$\sqrt{T_{max}} = \frac{2Lf\sqrt{\rho}}{1}$$

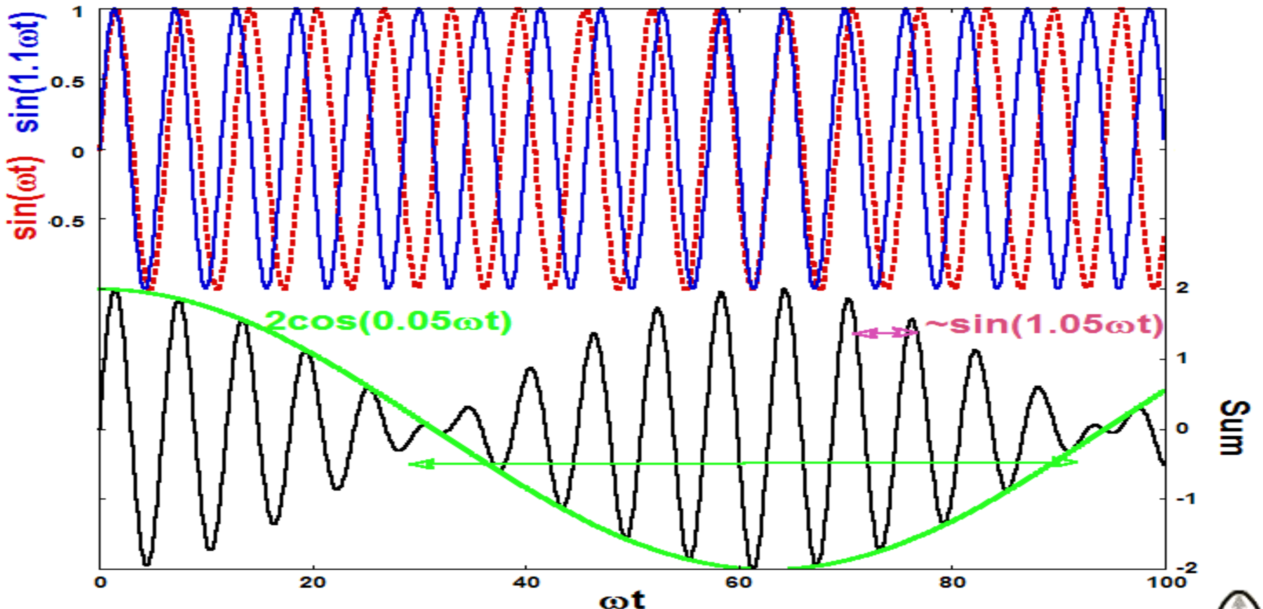
$T > T_{max}$ $n < 1$ no standing wave

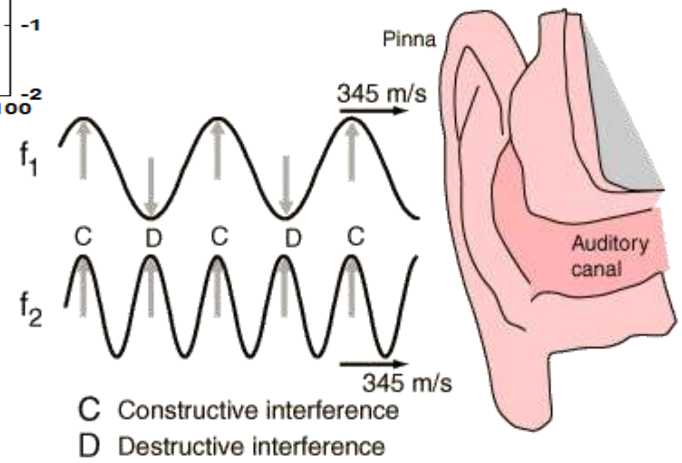
Beats **Not required**

$$\sin(\theta)+\sin(\theta')=2\sin[\frac{\theta+\theta'}{2}]\ \cos[\frac{\theta-\theta'}{2}]$$

$$\sin(\omega t)+\sin(\omega' t)=2\sin([\frac{\omega+\omega'}{2}]t)\ \cos([\frac{\omega-\omega'}{2}]t)$$

Average frequency — Long period modulation





$$f_{\text{beat}} = |f_1 - f_2|$$

Doppler Effect

Not required: will discuss next semester

Relation preferred by Croft.

$$\frac{\Delta f}{f_0} = \pm \frac{v}{v_{wave}}$$

+ coming at you (blue shift)

- going away (red shift)

The new frequency is:

$$f' = \frac{v + u}{(v/f)} = \left(\frac{v + u}{v}\right)f = (1 + u/v)f$$

where u is source velocity

General case (both source and receiver moving):

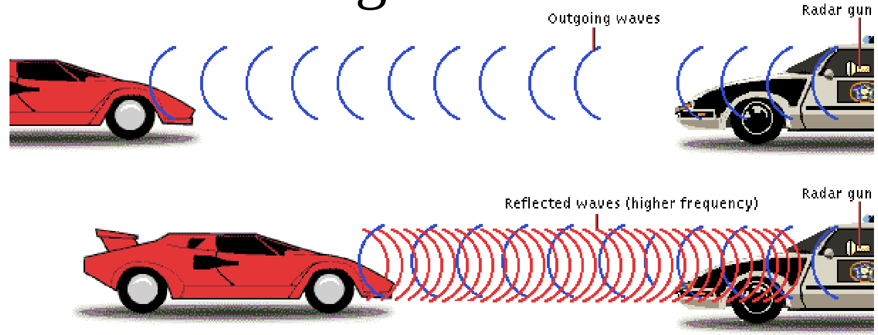
$$f' = \left(\frac{1 \pm u_o/v}{1 \mp u_s/v}\right)f$$

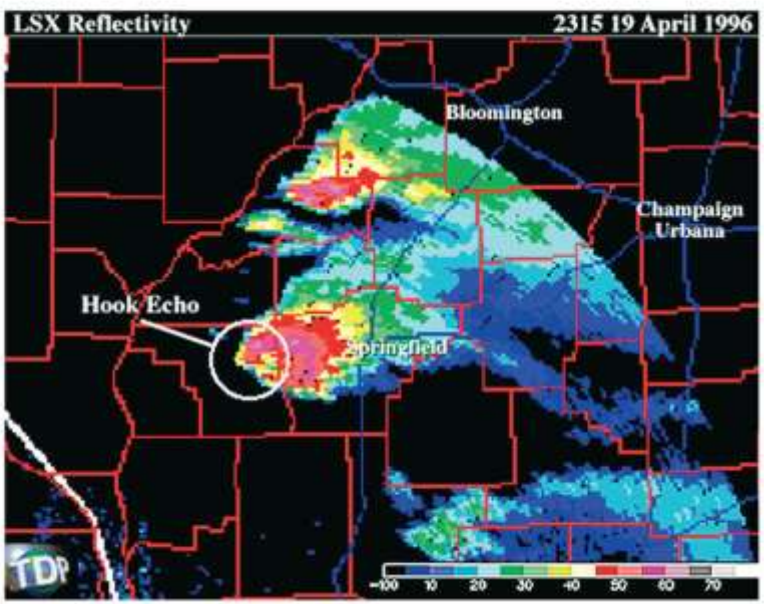
Applications of Doppler

Not required: will discuss next semester

Electro magnetic waves

Electro magnetic waves





Detecting Tornadoes

Electro magnetic waves

Doppler Blood Flow Meter

sound waves