$$F = \frac{GMm}{R^2}$$

Satellite Motion

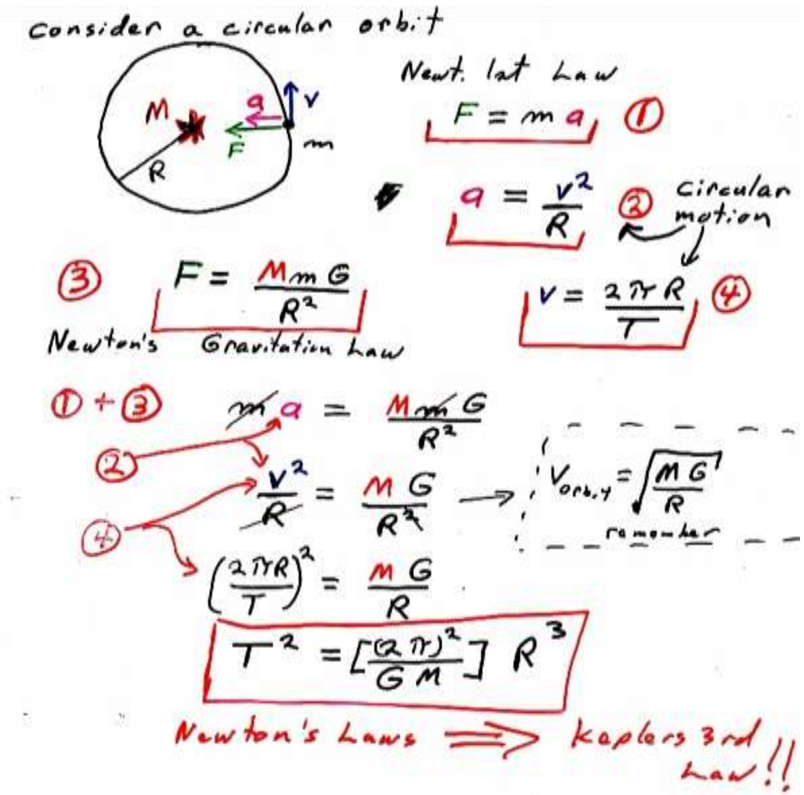
$$U(r) = -\frac{GMm}{r} \qquad U(r = \infty) = 0$$

$$E_i = E_f$$

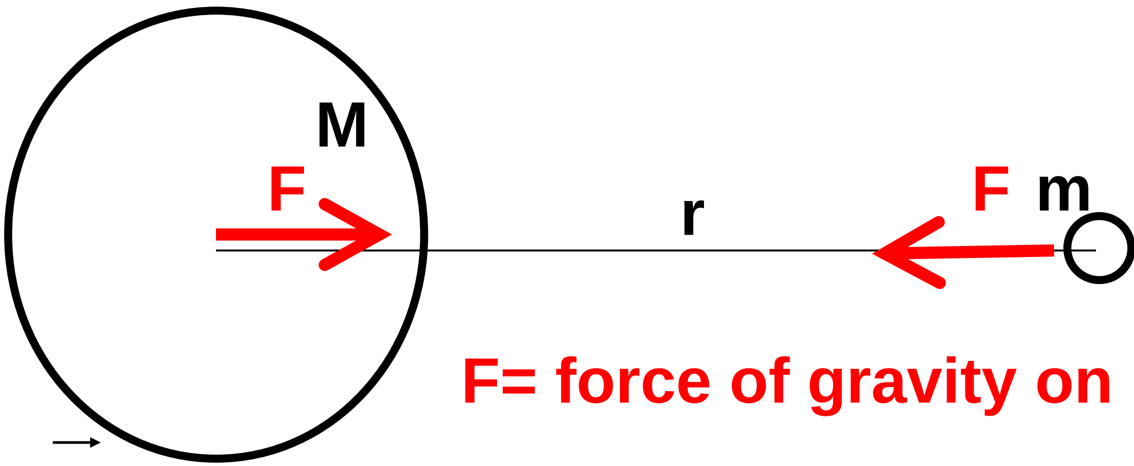
how high does it go?

Escape velocity

Kepler's 2nd Law ::= Areas ⇔ Angular Mom. Conservation !!!!



Newton's Universal Law of Gravity



F= force of gravity on m (M) due to M (m)

1) $\vec{F}$ acts along the line connecting the centers of objects
   "Central Force" (for spherical objects)

2) $\vec{F}$ points toward M (attractive force)

3) Newton says:

$$\vec{F} = \frac{G\,M\,m}{r^2}$$

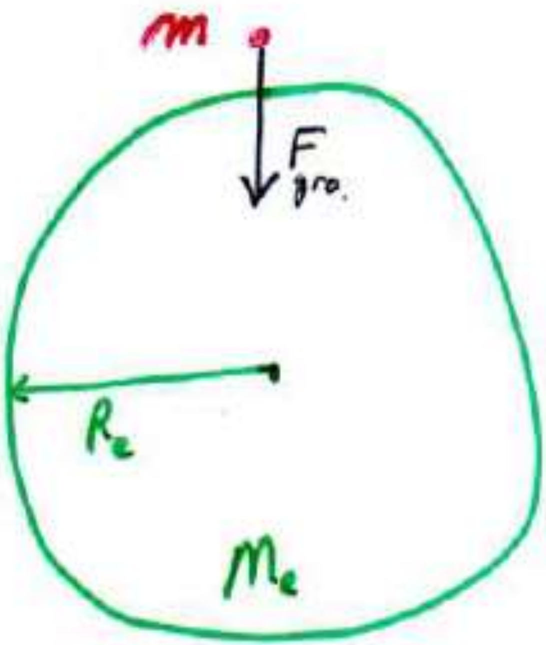
$G=6.67\text{x}10^{-11}\ [m^3/Kg\ s^2]$

for special case on the Earth's surface,
this acceleration is: $g = 9.81\ m/s^2$

G is a <u>very</u> small number!

Recall

Object of mass m at surface of Earth



$a \downarrow$

$$F_{gra} = \frac{G M_e m}{R_e^2}$$ N. U. L. G.

$$F_{2nd} = m a$$ N 2nd Law

$$F_{gra} = F_{2nd}$$

$$\frac{G M_e \cancel{m}}{R_e^2} = \cancel{m} a$$

mass of object cancels ! !

$$a = g = \frac{G M_e}{R_e^2}$$

all objects accelerate near earth's surface with $g = 9.8\ m/s^2$ $(32\ ft/s^2)$

$$G = 6.67\,(10)^{-11}\,\frac{Nm^2}{kg^2}$$
$$M_e = 5.97\,(10)^{24}\,kg$$
$$R_e = 6378\ km = 6378\,(10)^3\,m$$

$$g = \frac{\{G\}M_e}{R_e^2}$$

$$= \left\{[6.67]\,(10)^{-11}\,\frac{Nm^2}{kg^2}\right\}\frac{5.97\,(10)^{24}\,kg}{[6378\,(10)^3\,m]^2}$$

$$= \frac{[6.67]\,(5.97)\,(10)^{-11}(10)^{24}}{[6.378\,(10)^3(10)^3\,]^2}\quad\frac{N\,m^2\,kg}{kg^2\,m^2}$$

$$= \frac{(6.67)\,(5.97)}{(6.38)^2\,[10^6]^2}\,10^{24-11}\quad\frac{N}{kg}$$

$$N = \frac{kg\,m}{s^2}$$

$$= \left\{\frac{(6.67)(5.97)}{(6.38)^2}\right\}\frac{10^{24-11}}{10^{12}}\quad\frac{\left[\frac{kg\,m}{s^2}\right]}{kg}$$

$$= \{.978\}\quad 10^{24-11-12}\quad\frac{m}{s^2}$$

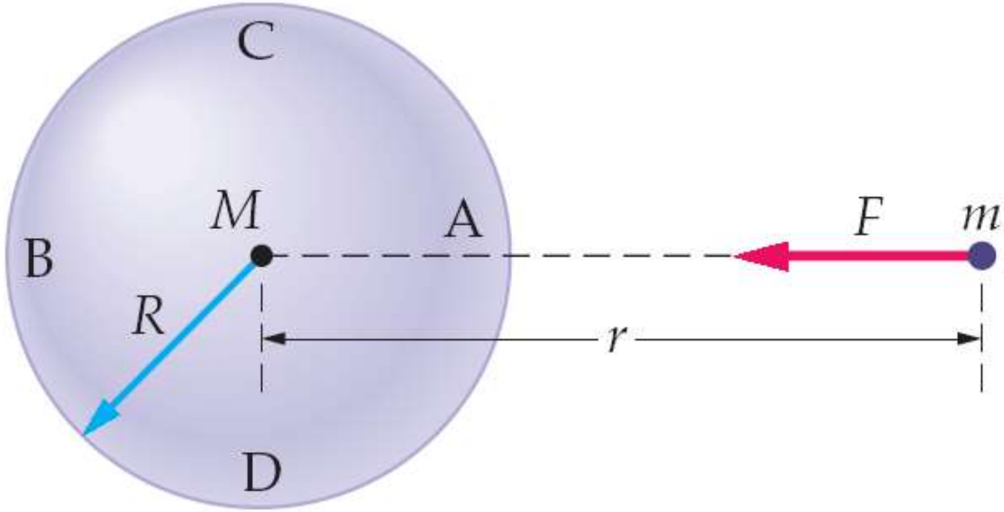
$$g = \{.978\}\quad 10^1\ m/s^2$$

$$\boxed{g = 9.78\ m/s^2}$$

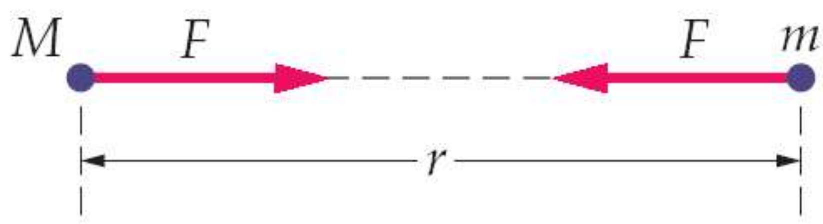
for all objects near surface of the earth

Properties of Gravity

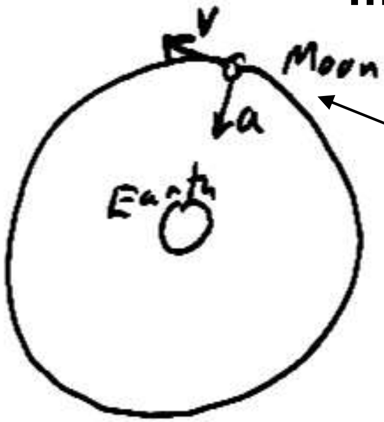
Gravitation from a Sphere



Gravitational force between an object and a sphere is the same as if all the mass of the sphere was concentrated at its center.



Calculate $a_{moon\text{-}in\text{-}orbit}$ for later



(circular orbit)

$$a = \frac{v^2}{R}$$

$$v = \frac{2\pi R}{T}$$

$$a = \frac{v^2}{R} = \left(\frac{2\pi R}{T}\right)^2 \frac{1}{R} = \boxed{\frac{4\pi^2 R}{T^2} = a}$$

$T_{\text{period of moons orbit}}$ = 27 days

$$= 27 \text{ days} \left(\frac{24 \text{ hr.}}{\text{day}}\right)\left(\frac{60 \text{ min}}{\text{hr}}\right)\left(\frac{60 \text{ sec}}{\text{min}}\right)$$

= 2,332,800 sec

$$T = 2.3(10)^6 \text{ sec}$$

$$R = 384{,}401 \text{ km} \left(\frac{10^3 \text{ m}}{\text{Km}}\right) \Rightarrow R = 3.84(10)^8 m$$

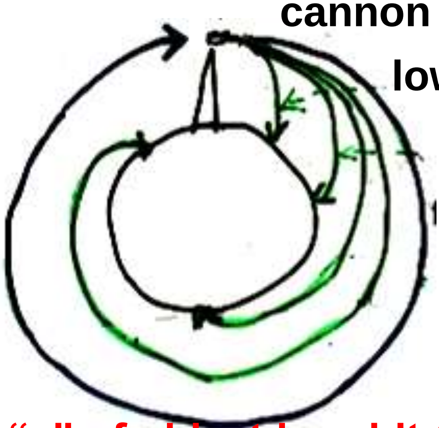
What is a_{moon} in orbit ?

$$a = \frac{(2\pi)^2 R}{T^2}$$

$$= \frac{(2[3.14])^2 3.84(10)^8 m}{[2.3(10)^6 s]^2}$$

$$= 28.6 \frac{(10)^8}{(10)^{12}} \frac{m}{s^2}$$

$$a_{Moon} = 0.00286 \frac{m}{s^2}$$



cannon fired horizontally

low velocity- ~parabolic path

higher velocity
- earth curves away under falling object

For special "orbital" velocity the ball falls at the same rate that the earth curves away

"a" of object in orbit (at surface) is same as dropped object

Newton's connection $a_{falling\text{-}apple} = a_{orbiting\text{-}apple} = g$!!!!!!

$$\frac{R_e}{R_{Moon-dist}} = \frac{6.378(10)^6 m}{3.84(10)^8 m}$$

$$\frac{R_e}{R_{Moon-dist}} = .0166 = \frac{1}{60.2}$$

Same idea for moon in orbit- moon "falling" toward Earth

earth moon

$$F = \frac{GMm}{R^2} = ma$$

$$\frac{GM}{R^2} = a_{moon}$$

$$\frac{a_{moon}}{a_{apple}} = \frac{\frac{GM}{R^2}}{\frac{GM}{R_e^2}} = \frac{R_e^2}{R^2}$$

$$a_{apple} = g$$

$$a_{moon} = a_{apple}\frac{R_e^2}{R^2}$$

$$a_{moon} = g\frac{R_e^2}{R^2}$$

$$a_{Moon} = \left(\frac{R_e}{R_{Moon-dist}}\right)^2 a_{apple}$$

$$a_{Moon} = \left(\frac{1}{60.2}\right)^2 9.8\frac{m}{s^2}$$

$$a_{Moon} = 0.0027\frac{m}{s^2}$$

dead right Newt. nailed it !!!

apples-moon –everything ruled by $1/r^2$ gravity

10a-4

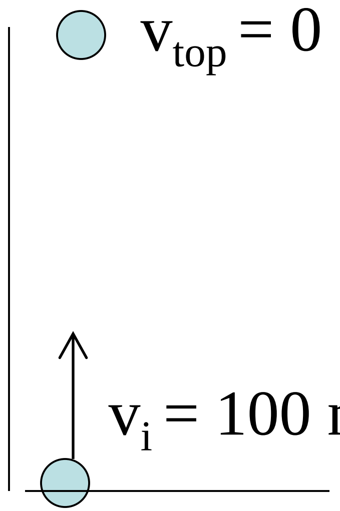
http://galileo.phys.virginia.edu/classes/109N/more_stuff/Applets/newt/newtmtn.html

Recall energy conservation near earth's surface

Throw an object up (very slow) – how high does it go?

$$E_i = E_f \Longrightarrow \frac{1}{2}mv^2 = mgh \Longrightarrow \boxed{h = \frac{v_i^{\,2}}{2g}}$$

$$h = \frac{100^2}{2(9.8)} \quad \frac{\left(\frac{m}{s}\right)^2}{\left(\frac{m}{s^2}\right)}$$



$$E_f = 0 + mgh$$

$$h = 510 \text{ m}$$

$$E_i = \frac{1}{2}mv_i^{\,2} + 0$$

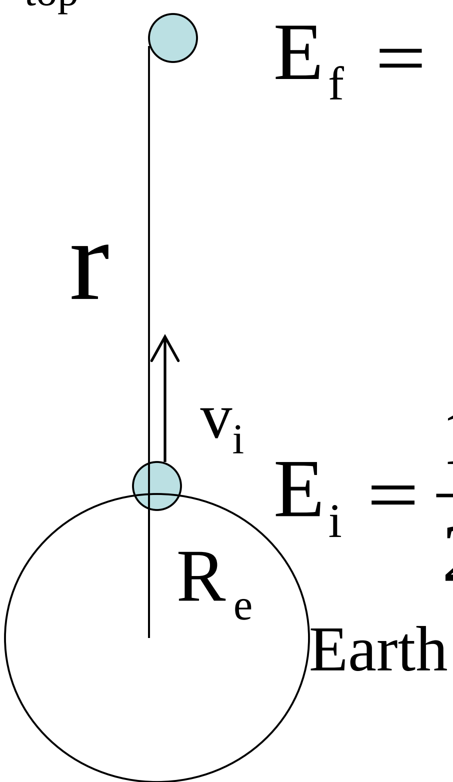
Universal Gravitational Potential Energy

$$U(r) = -\frac{GMm}{r} \quad :::: \quad U(r=\infty) = 0$$

$$E_i = E_f \Longrightarrow \frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e} = -\frac{GM_e m}{r}$$

$v_{top} = 0$: know

$$E_f = -\frac{GM_e m}{r}$$



$$E_i = \frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e}$$

3 cases

- $E < 0$ can find r_{max} and object falls back
- $E = 0$ can just get to $r = \infty$ and stops
- $E > 0$ gets to $r = \infty$ And keeps going

Universal Gravitational Potential Energy

$$U(r) = -\frac{GMm}{r} \quad :::: \quad U(r=\infty) = 0$$

recall $mg = m\frac{GM_e}{R_e^2}$

so $gR_e = \frac{GM_e}{R_e}$

Throw an object up (very fast)– how high does it go?

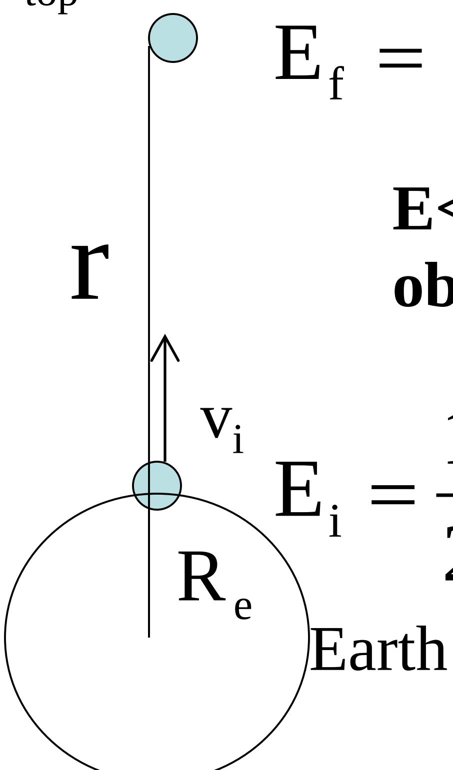
10a-6a

$$E_i = E_f \Rightarrow \frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e} = -\frac{GM_e m}{r}$$

$v_{top} = 0$: know

$$E_f = -\frac{GM_e m}{r}$$

E< 0 can find r_{max} and object falls back



$$E_i = \frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e}$$

$$r = -\frac{GM_e m}{\frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e}}$$

$$r = \frac{R_e}{1 - \frac{R_e}{2GM_e}v_i^2}$$

$$r_{max} = \frac{R_e}{\{1 - \frac{v_i^2}{2gR_e}\}}$$

note $\{1 - \frac{v_i^2}{2gR_e}\} \rightarrow 0 \Rightarrow r_{max} \rightarrow \infty$

Universal Gravitational Potential Energy

$$U(r) = -\frac{GMm}{r} \;::::\; U(r=\infty) = 0$$

recall

$$mg = m\frac{GM_e}{R_e^2}$$

so

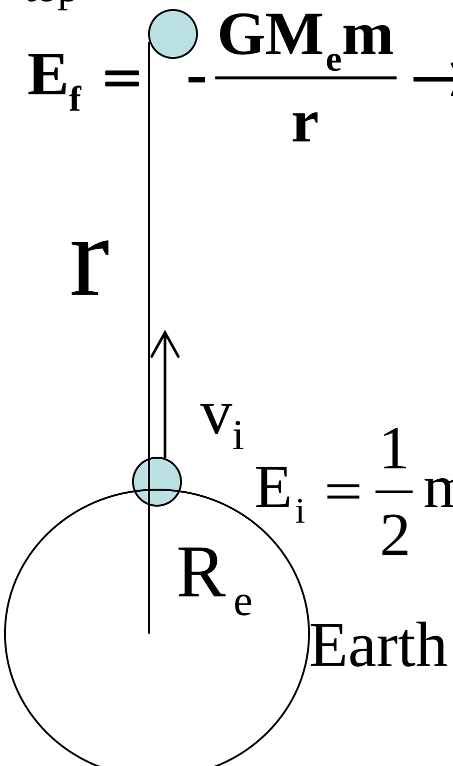
$$gR_e = \frac{GM_e}{R_e}$$

Escape velocity : $v_f=0$ and $r_{max}=\infty \Rightarrow E=0$

$$E_i = E_f \rightarrow \boxed{\frac{1}{2}mv_e^2 - \frac{GM_e m}{R_e} = 0}$$

$v_{top} = 0$: know

$$E_f = -\frac{GM_e m}{r} \rightarrow 0 (\text{at } r=\infty)$$



$$E_i = \frac{1}{2}mv_i^2 - \frac{GM_e m}{R_e}$$

Escape velocity $r=\infty$

$$\frac{1}{2}mv_{esc}^2 - \frac{GM_e m}{R_e} = 0$$

$$v_{esc}^2 = \frac{2GM_e}{R_e}$$

$$v_{esc} = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e}$$

recall $gR_e = \frac{GM_e}{R_e}$

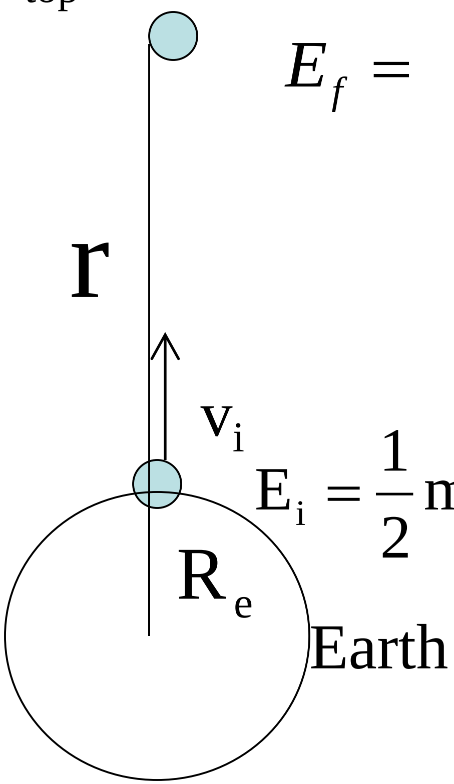
Escape velocity : $v_f=0$ and $r_{max}=\infty \Rightarrow E=0$

E=0 critical escape condition

$$E_i = E_f = 0 \implies \frac{1}{2} m v_i^2 - \frac{G M_e m}{R_e} = 0 \implies v_{esc}^2 = \frac{2GM_e}{R_e}$$

$v_{top} = 0$: know and $U(r=\infty)=0$

$$E_f = 0$$



$$E_i = \frac{1}{2} m v_i^2 - \frac{GM_e m}{R_e}$$

Escape velocity

$$v_{esc} = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e}$$

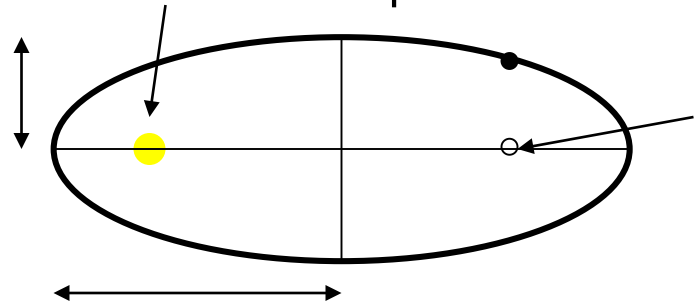
$$v_{esc} = \sqrt{2(9.8\frac{m}{s^2})(6.378(10)^6 m)}$$

$$v_{esc} = 11.1(10)^3 \frac{m}{s}$$

Kepler's 1st Law

(all objects bound to sun- Kepler figured out with Mars)

1. The orbital motion of the planets about the sun is an ellipse with the sun at one focus.



ellipse = squashed circle

eccentricity ~ r_{min}/r_{maj}

$r_{min} \Rightarrow r_{maj}$ get circle

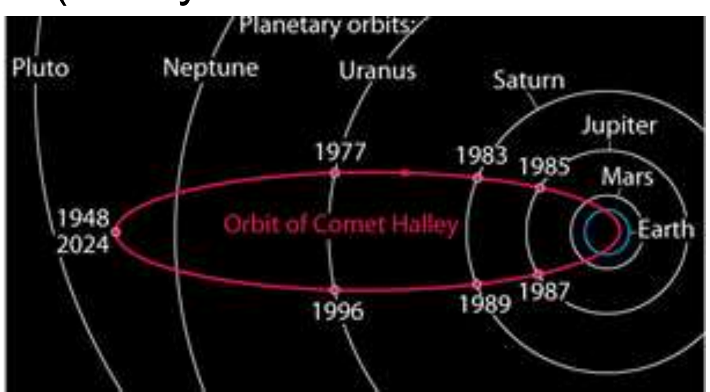
Note in reality both M_{sun} and $m_{planets\text{-}etc}$ orbit common CM

Newton!!

- all motion in $1/r^2$ force law (like gravity) follows **conic sections (slices of a cone)**
- bound motion ellipses – unbound motion paranoia or hyperbola
- note one assumes for this $M_{sun} >>> m_{planets\text{-}etc.}$

-Halley's Comet coming back every 76 years nailed this

(Halley didn't live to see it return but used Newton's Laws to predict exactly the return)



Your chance to see Halley's Comet
1986+76= 2062

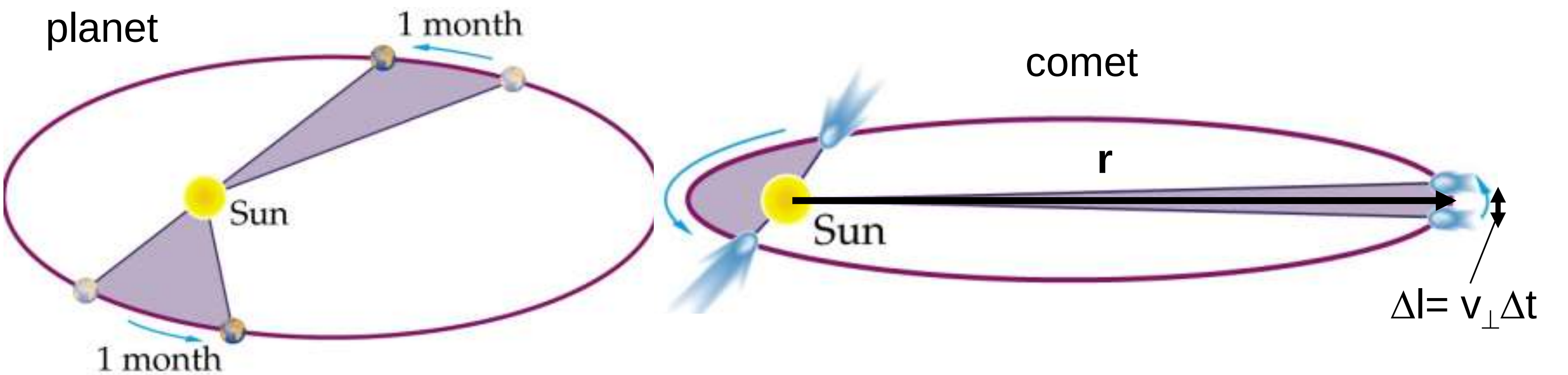
(my father saw in 1910 with his grandfather)

(I showed my children, in 1986)

Kepler's 2nd Law

(all objects bound to sun- Kepler figured out with Mars)

2. As the planet moves in its orbits it sweeps out equal areas in an equal times
planet moves faster (slower) when closer to (farther from) to the sun $M_{sun} >>> m_{planets\text{-}etc.}$



Newton!!

-Gravity = a central force (acts between centers of ~spherical objects)

$\Rightarrow$ no torque on planet (comet…etc.) $\Rightarrow$ angular momentum conserved

Area triangle= ΔA

$$\Delta A = \frac{1}{2} r\,(\Delta l) = \frac{1}{2} r\,(v_\perp \Delta t)$$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2} r\, v_\perp = [\frac{1}{2m}]\,(mr\, v_\perp) = [\frac{1}{2m}]\,(L)$$

Recall !!! Ang. Mom.

$$L = mr\, v_\perp$$

Kepler's 2nd Law ::= Areas $\Leftrightarrow$ Angular Mom. Conservation !!!!

1612 Kepler moves to Linz (Austria)

1618 "Harmony of the World" -- much nonsense about music of the spheres
but also

Kepler's 3'rd Law **[P squared -- a cubed]**

$$P^2 = k\, a^3$$

k=constant = 1 yr^2/AU^3

P= period of the planets orbit (in years)

a= semimajor axis of planets orbit in AU (astronomical units)

e.g. for Mars a=1.524 AU and P=1.881 yr

$$(1.881)^2 = 3.54 = (1.524)^3$$

1618-1621 worked on text "Epitome of Copernican Astronomy"

1627 "Rudolphine Tables of Planetary Positions"(
Tycho's observations + Kepler's Laws

10a-9

Simplicity - Beauty

100 times more accurate than before

No thinking person could deny the superiority Copernican-Keplerian system !!

Kepler's 3rd Law

$$P^2 = R^3$$

P in years; R in AU; AU = R(earth) = 1 AU

$$P^2 = k\ R^3$$

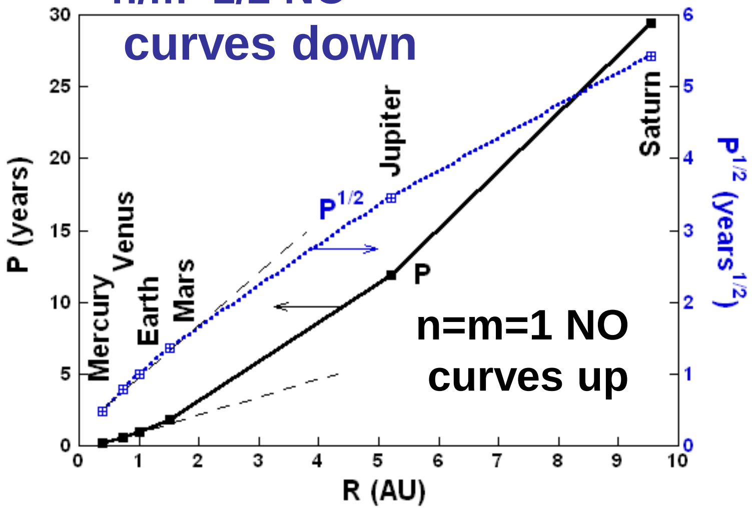
P in sec; k in $\frac{sec^2}{m^3}$; R in m

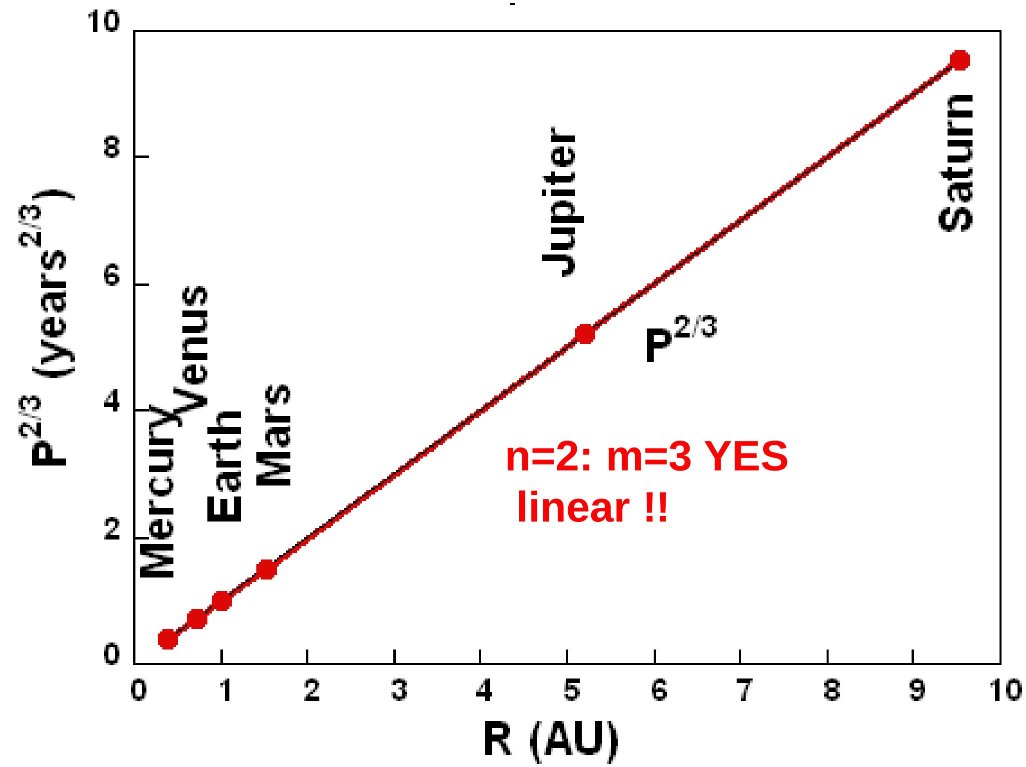
Kepler probably tried

$$P^n = R^m$$

$$R = P^{n/m}$$

n/m=1/2 NO

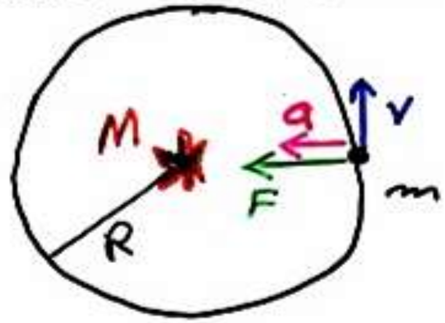




Kepler's 3rd Law ($T^2 \sim R^3$) Newton's Law connection

how "Newton" knew he'd nailed it

Consider a circular orbit



Newt. 1st Law

$$F = m a \quad (1)$$

$$a = \frac{v^2}{R} \quad (2)$$

circular motion

$$(3) \quad F = \frac{MmG}{R^2}$$

Newton's Gravitation Law

$$v = \frac{2\pi R}{T} \quad (4)$$

(1) + (3)

$$m a = \frac{M m G}{R^2}$$

(2) →

$$\frac{v^2}{R} = \frac{M G}{R^2} \rightarrow V_{orbit} = \sqrt{\frac{MG}{R}}$$

remember

(4) →

$$\left(\frac{2\pi R}{T}\right)^2 = \frac{MG}{R}$$

$$T^2 = \left[\frac{(2\pi)^2}{GM}\right] R^3$$

Mass of Sun !!!

Newton's Laws ⟹ Keplers 3rd Law!!

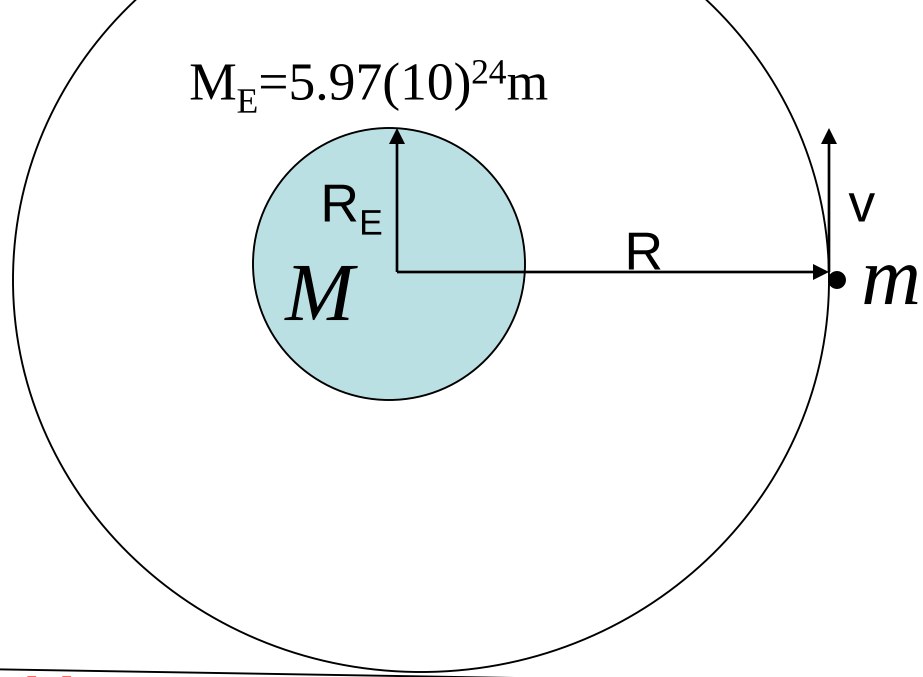
Satellite Motion

$$F = ma$$

$$G\frac{Mm}{R^2} = m\frac{v^2}{R}$$

$$v^2 = \frac{GM}{R}$$

$$v = \sqrt{\frac{GM}{R}}$$



Satellite at surface (of Earth)

$R = R_E = 6378(10)^3$m

$$\frac{GMm}{R_E^2} = mg \qquad \frac{GM}{R_E} = gR_E$$

$$v = \sqrt{\frac{GM}{R_E}} = \sqrt{gR_E} = \sqrt{9.8(6.378)(10)^6\,\frac{m^2}{s^2}}$$

$v = 7.906(10)^3$ m/s

$$v = \frac{2\pi R}{T}$$

$$T = \frac{2\pi R}{v} = \frac{2(3.14159)6.378(10)^6}{7.906(10)^3} = 5.07(10)^3 s = 84.5\,\text{min}$$

Satellite Motion

F=ma

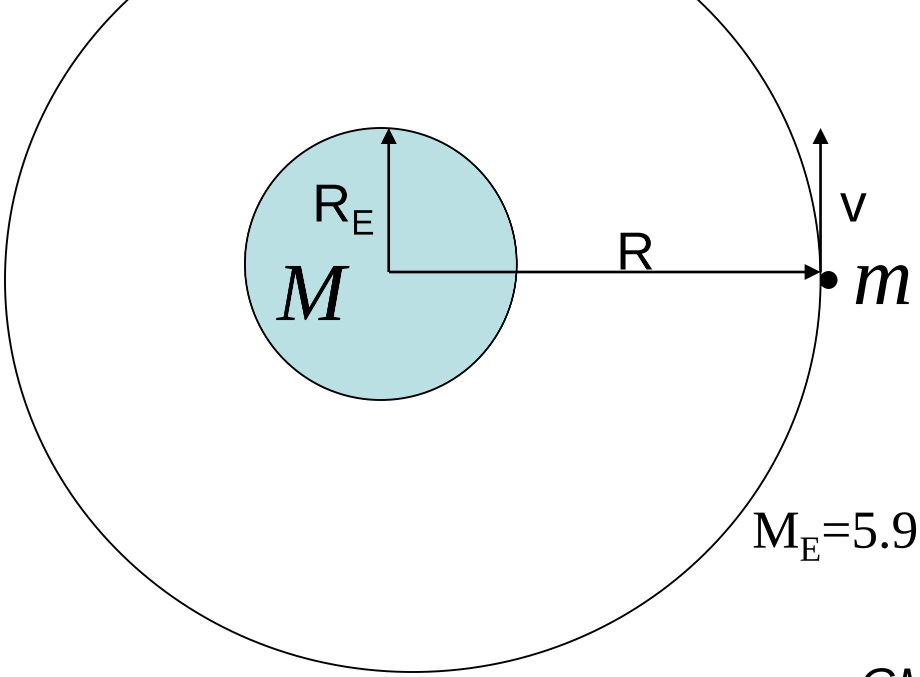
$$G\frac{Mm}{R^2} = m\frac{v^2}{R}$$

$$v^2 = \frac{GM}{R}$$

$$v = \sqrt{\frac{GM}{R}}$$

$$v = \frac{2\pi R}{T}$$

$$T^2 = \frac{(2\pi)^2}{GM}R^3$$



$M_E=5.97(10)^{24}$m

$$\frac{GM}{R_E} = gR_E$$

Geocentric Satellite

$$R^3 = \frac{GM}{(2\pi)^2}T^2$$

$$T = 24h \approx 8.64\times10^4 s$$

$$R = \sqrt[3]{\frac{GM}{(2\pi)^2}T^2}$$

R=9598 $(10)^3$m

Geocentric satellites

- **Find the height (altitude) of a satellite above the Earth surface so that it is always above the same point on the Earth's surface.**

$$G\frac{M_E m_S}{(R_E+h)^2} = m_S\frac{v_S^2}{(R_E+h)}$$

$$v_S = \frac{2\pi(R_E+h)}{T}$$

$$G\frac{M_E}{(R_E+h)^2} = \frac{\left(\frac{2\pi(R_E+h)}{T}\right)^2}{(R_E+h)}$$

$$(R_E+h)^3 = \frac{GM_E T^2}{4\pi^2}$$

$$(R_E+h) = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}}$$

$$T = 24h \approx 8.64\times10^4 s$$

$$h = 3.58\times10^6 m$$

10a-12a

Notice, in order for the satellite to hang above the same spot on the Earth it has to have T = 24 hours