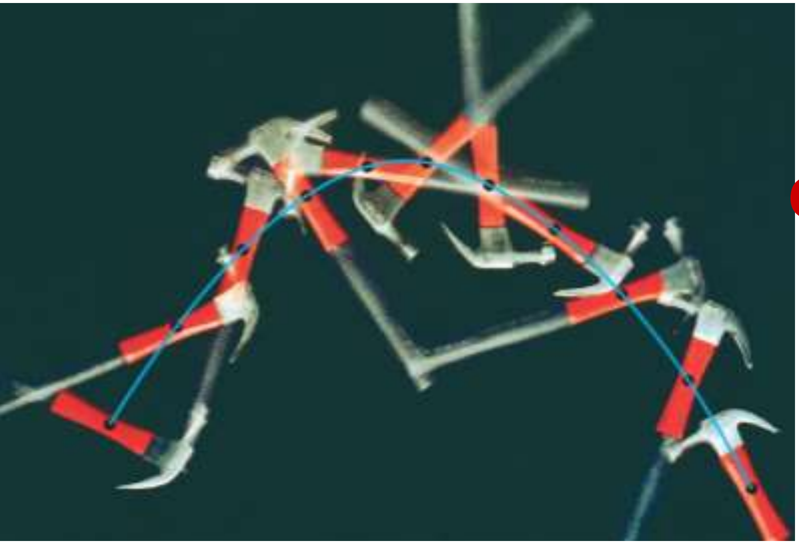
	Rotational motion		Translational Motion	
	θ	[radians]	x	[m]
direction + ccw - cw	$\bar{\omega}=\frac{\Delta\theta}{\Delta t}$	$[\frac{rad}{sec}=\frac{1}{s}]$	$\bar{v}=\frac{\Delta x}{\Delta t}$	[m/s]
	$\bar{\alpha}=\frac{\Delta\omega}{\Delta t}$	$\frac{rad}{s^2}$	$\bar{a}=\frac{\Delta v}{\Delta t}$	[m/s²]
const. acceleration	$\omega=\omega_o+\alpha t$		$v=v_o+at$	
	$\theta=\theta_o+\omega_o t+\frac{1}{2}\alpha t^2$		$x=x_o+v_o t+\frac{1}{2}at^2$	
	$\omega^2-\omega_o^2=2\alpha(\theta-\theta_o)$		$v^2-v_o^2=2a(x-x_o)$	
instantaneous quantities	$\omega=\lim_{\Delta t\to 0}[\frac{\Delta\theta}{\Delta t}]$		$v=\lim_{\Delta t\to 0}[\frac{\Delta x}{\Delta t}]$	
	$\alpha=\lim_{\Delta t\to 0}[\frac{\Delta\omega}{\Delta t}]$		$a=\lim_{\Delta t\to 0}[\frac{\Delta v}{\Delta t}]$	

$$f=\frac{1}{T} \quad [\frac{rev}{sec}=\frac{1}{sec}=Hz] \qquad \omega=2\pi f$$

in general rotation + translation every where

CM moves at constant v along dotted line.

Rotation about CM at constant angular speed ω.



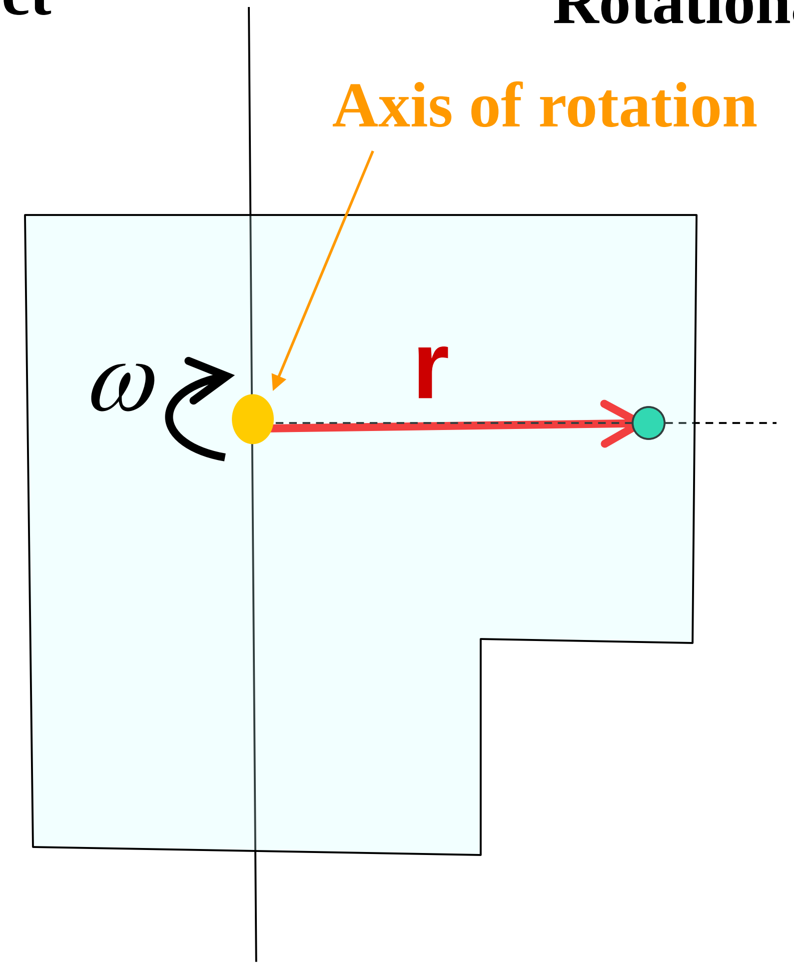
CM moves with parabolic trajectory.

Rotation about CM at constant angular speed ω.

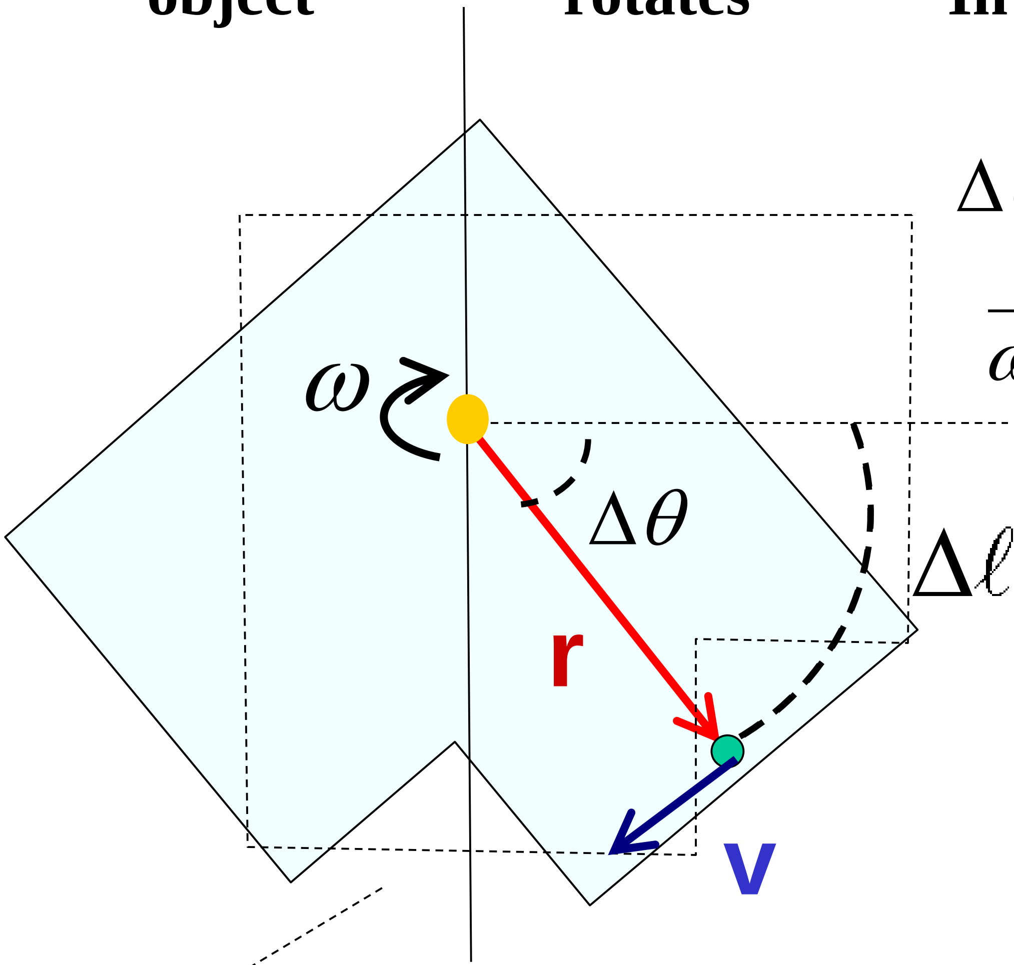
consider just rotation about an axis

Rotational Kinematics

object



object rotates In time Δt



$$\Delta\theta = \frac{\Delta\ell}{r} \quad \textbf{radians}$$

$\bar{\omega} =$ ave. angular vel. (rad./s)

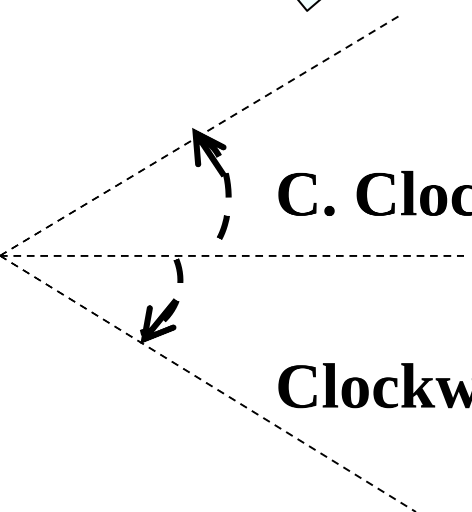
$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{1}{r}\frac{\Delta\ell}{\Delta t}$$

$$\Delta t \to 0$$

$\omega =$ instant. angular vel.

$$\omega = \frac{d\theta}{dt} = \frac{1}{r}\frac{d\ell}{dt} = \frac{v}{r}$$

$$v = \frac{d\ell}{dt}$$

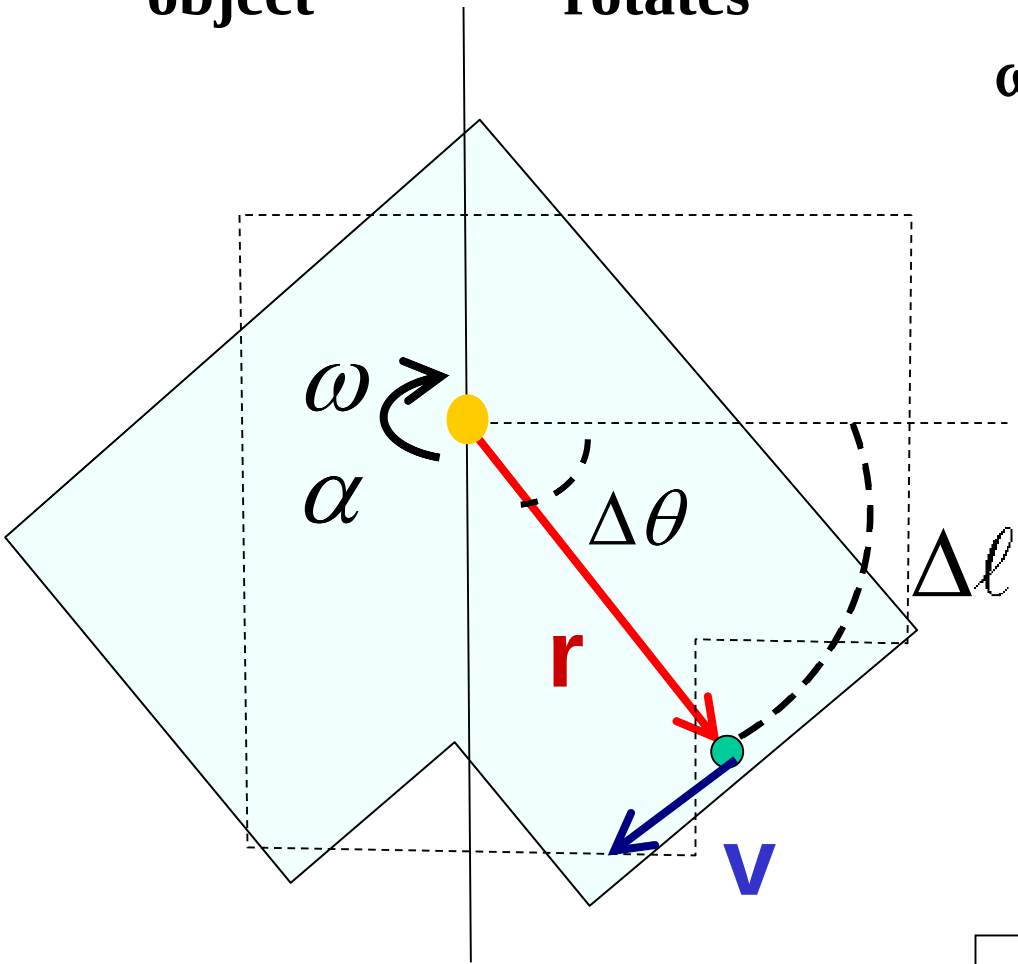


C. Clockwise (+)

Clockwise (-)

$$\omega = \frac{1}{r}v \qquad v = \omega\, r$$

object rotates



ω=angular velocity (rad./sec)

$$v = \omega\, r$$

ave. angular acceleration (rad/s²)

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{\Delta t}$$

instantaneous angular acceleration (rad/s²)

$$\alpha = \frac{d\omega}{dt}$$

$$a = \alpha r$$

$$v = \omega\, r$$

$$\Delta\ell = r\, \Delta\theta$$

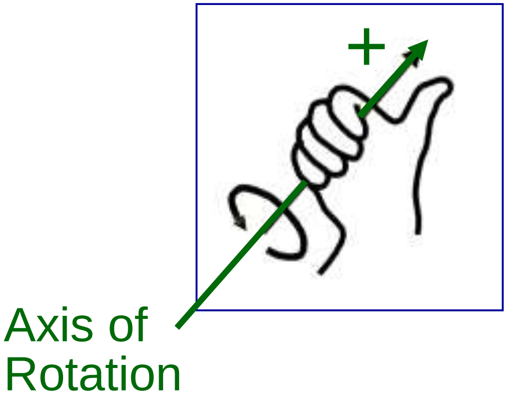
$$\Delta\theta = \frac{\Delta\ell}{r}$$

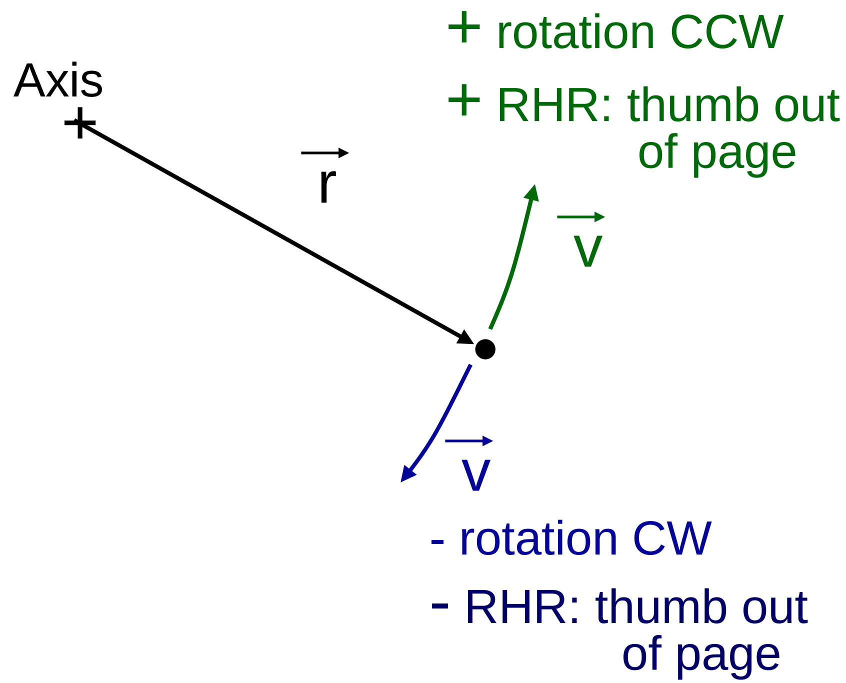
Rotational motion		Translational Motion	
	θ **[radians]**	x **[m]**	
direction **+ ccw** **- cw**	$\bar{\omega}=\frac{\Delta\theta}{\Delta t}$ $[\frac{rad}{sec}=\frac{1}{s}]$	$\bar{v}=\frac{\Delta x}{\Delta t}$ **[m/s]**	
	$\bar{\alpha}=\frac{\Delta\omega}{\Delta t}$ $\frac{rad}{s^2}$	$\bar{a}=\frac{\Delta v}{\Delta t}$ **[m/s²]**	
const. acceleration	$\omega=\omega_o+\alpha t$	$v=v_o+at$	
	$\theta=\theta_o+\omega_o t+\frac{1}{2}\alpha t^2$	$x=x_o+v_o t+\frac{1}{2}at^2$	
	$\omega^2-\omega_o^2=2\alpha(\theta-\theta_o)$	$v^2-v_o^2=2a(x-x_o)$	
instantaneous quantities	$\omega=\lim_{\Delta t\to 0}[\frac{\Delta\theta}{\Delta t}]$	$v=\lim_{\Delta t\to 0}[\frac{\Delta x}{\Delta t}]$	
	$\alpha=\lim_{\Delta t\to 0}[\frac{\Delta\omega}{\Delta t}]$	$a=\lim_{\Delta t\to 0}[\frac{\Delta v}{\Delta t}]$	

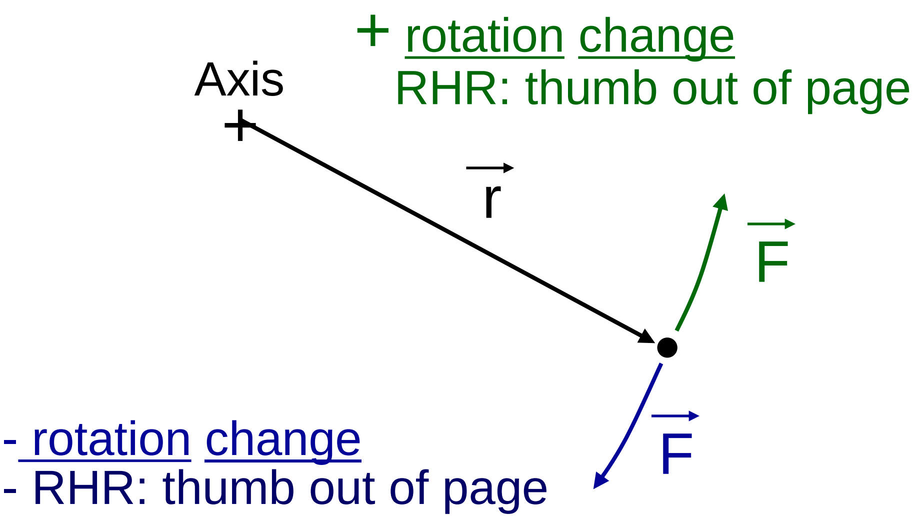
$$f=\frac{1}{T} \qquad [\frac{rev}{sec}=\frac{1}{sec}=Hz] \qquad \omega=2\pi f$$

Right Hand Rule Convention

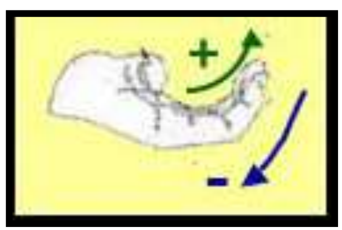
Direction Matters







Axis Out of Page



Fan demo/example

fan at full speed (stroboscope observation)

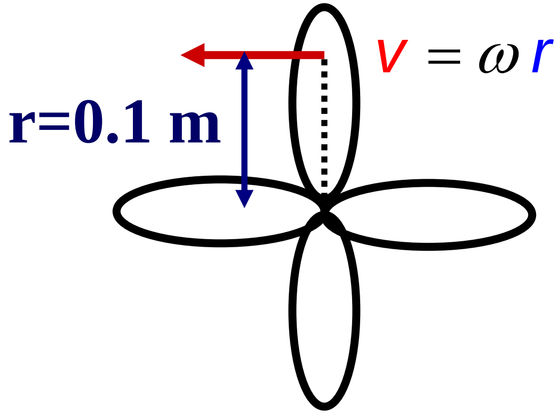
$$1340\,\frac{\text{rev}}{\text{min}} = \frac{1340}{60}\,\frac{\text{rev}}{\text{min}}\,\frac{\text{min}}{\text{sec}} \Rightarrow f = 22\,\frac{\text{rev}}{\text{sec}}$$

frequency

$$f = \frac{1}{T}$$

angular velocity $\omega = 2\pi f = 2\pi(22)\dfrac{rad}{\text{sec}} = 44\pi\dfrac{1}{s}$

linear velocity (at radius r)



$$v = \omega r = 44\pi(0.1)\frac{m}{s} = 4.4\pi\frac{m}{s}$$

Initially fan at rest $\Rightarrow$ $\omega_i = 0$

observe fan reaches final speed in ~ 3 sec.

$$\omega_f = 44\pi$$

Angular acceleration (ave.) $\bar{\alpha} = \dfrac{\omega_f - \omega_i}{\Delta t} = \dfrac{44\pi - 0}{3} = 14.7\pi \dfrac{rad}{s^2}$

assume acceleration constant $\Rightarrow$ $\alpha = \bar{\alpha}$

then $\omega = \omega_i + \alpha t = 0 + (14.7\pi)t$

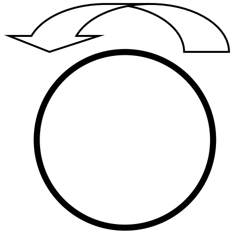
$$\theta = \theta_i + \omega_i t + \frac{\alpha}{2} t^2 = 0 + 0 + \frac{(14.7\pi)}{2} t^2$$

θ turned through in 3 sec. $\theta = \dfrac{(14.7\pi)}{2}(3)^2 = 66\pi$

$$\#\ revolutions = \frac{(\theta)}{2\pi} = 33\ rev.$$

Another example

accelerating in opposite direction



$$\omega_i = +12\pi \frac{rad}{s} \quad \alpha = -4\pi \frac{1}{s^2}$$

CCW **CW**

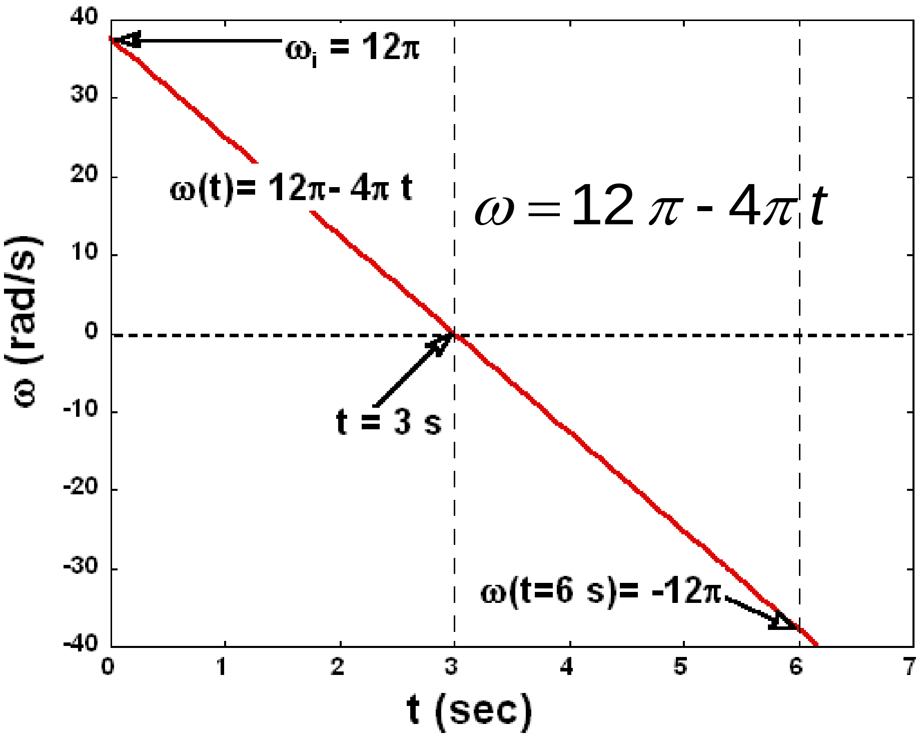
$$\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \qquad \omega = \omega_i + \alpha t \qquad \omega^2 - \omega_i^2 = 2\alpha(\theta - \theta_i)$$

Q. time for coming to rest

$$\omega = \omega_i + \alpha t$$

$$0 = 12\pi - 4\pi t$$

$$t = 3\,s$$



Q. time for $\Delta\theta = 5(2\pi)$

$$\theta = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta = (12\pi)\, t + \frac{1}{2}(-4\pi)\, t^2$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$5(2\pi) = (12\pi)\, t + \frac{1}{2}(-4\pi)\, t^2$$

$$-4\, t^2 + 24\, t - 20 = 0$$

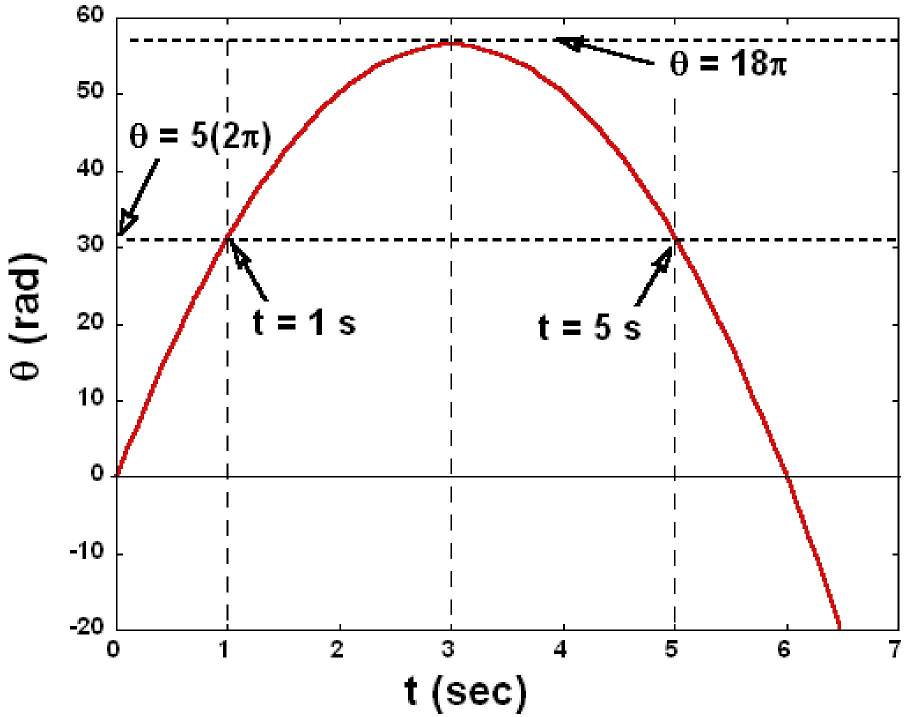
$$t^2 - 6\, t + 5 = 0$$

$$t = \frac{+6 \pm \sqrt{6^2 - 4(5)}}{2}$$

$$t = 3 \pm 2$$

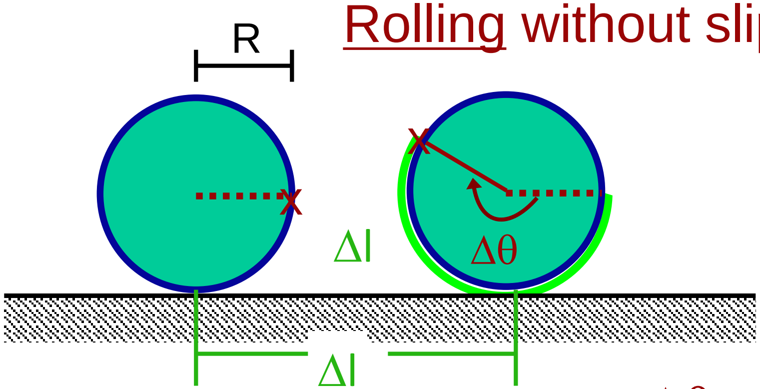
$t = 1s$ **first**

$t = 5s$ **second**

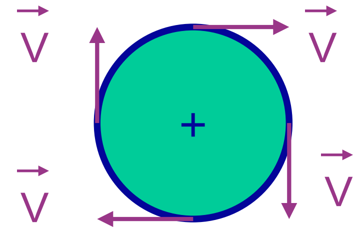


Motions of a Wheel

Rolling without slipping



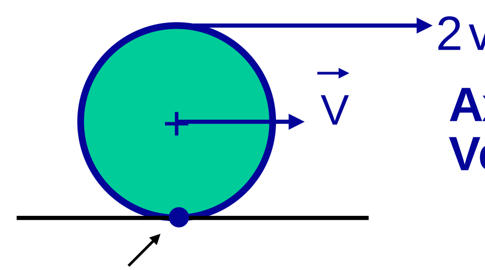
$$\Delta\theta = \frac{\Delta l}{R}$$



Tangential Velocities

with respect to axle,
but axle is moving

with respect to ground



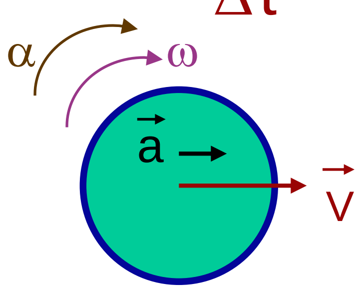
Axle Velocity

This point on wheel is momentarily at rest

Accelerating/Decelerating without slipping

$$\frac{\Delta\theta}{\Delta t} \quad \frac{1}{R}\,\frac{\Delta l}{\Delta t}$$

$$\omega = \frac{v}{R}$$

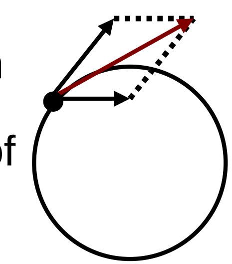


$$v = \omega R$$

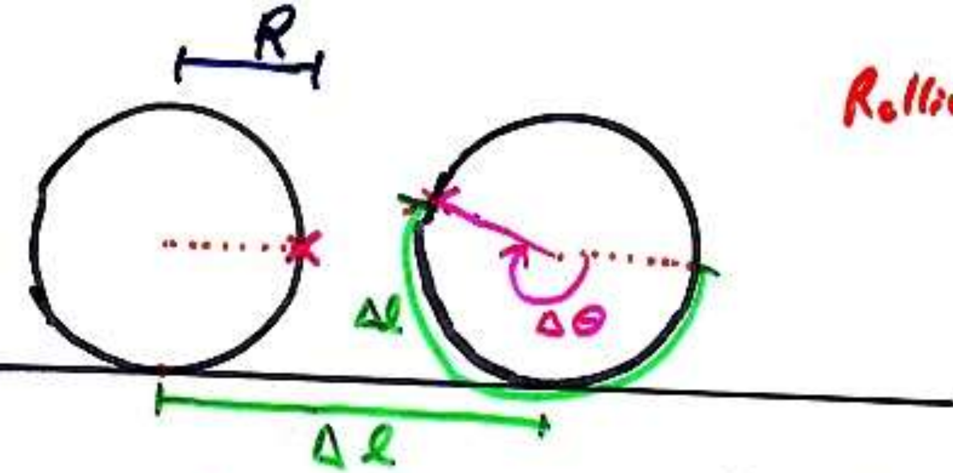
$$a = \alpha R$$

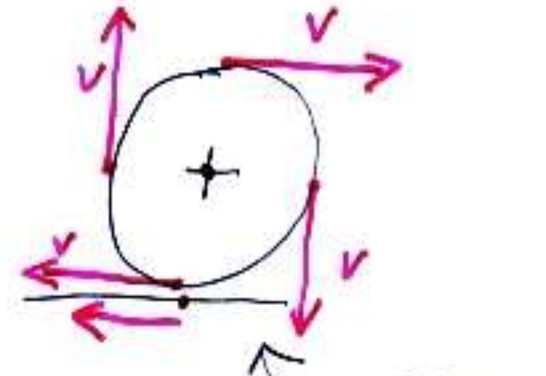
Note: α = const., but ω is not

The velocity of a spot on the edge of the wheel is just the vector addition of Tangential and Axle velocities.

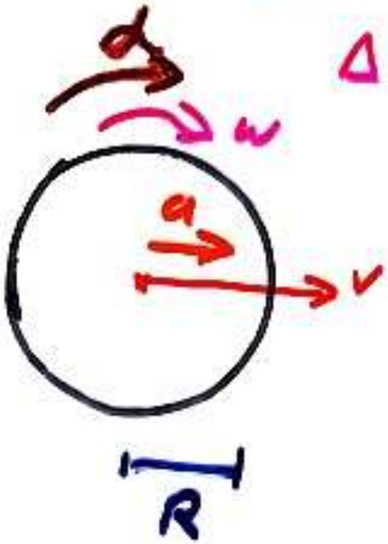


Rolling without Slipping





With respect to axile



$$\Delta\theta = \frac{\Delta \ell}{R}$$

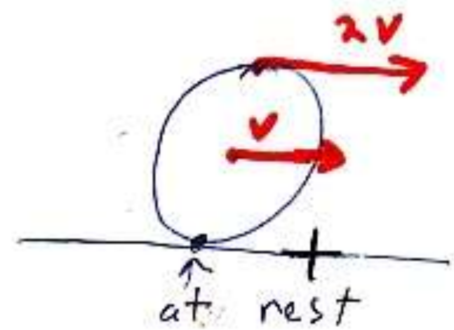
$$\frac{\Delta\theta}{\Delta t} = \frac{1}{R}\frac{\Delta \ell}{\Delta t}$$

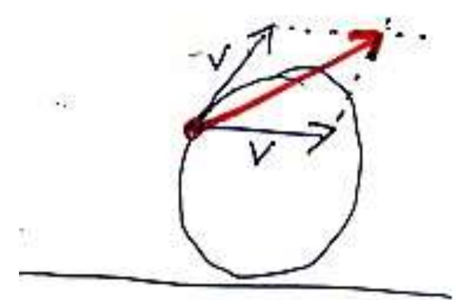
$$\omega = \frac{v}{R}$$

$$v = \omega R$$

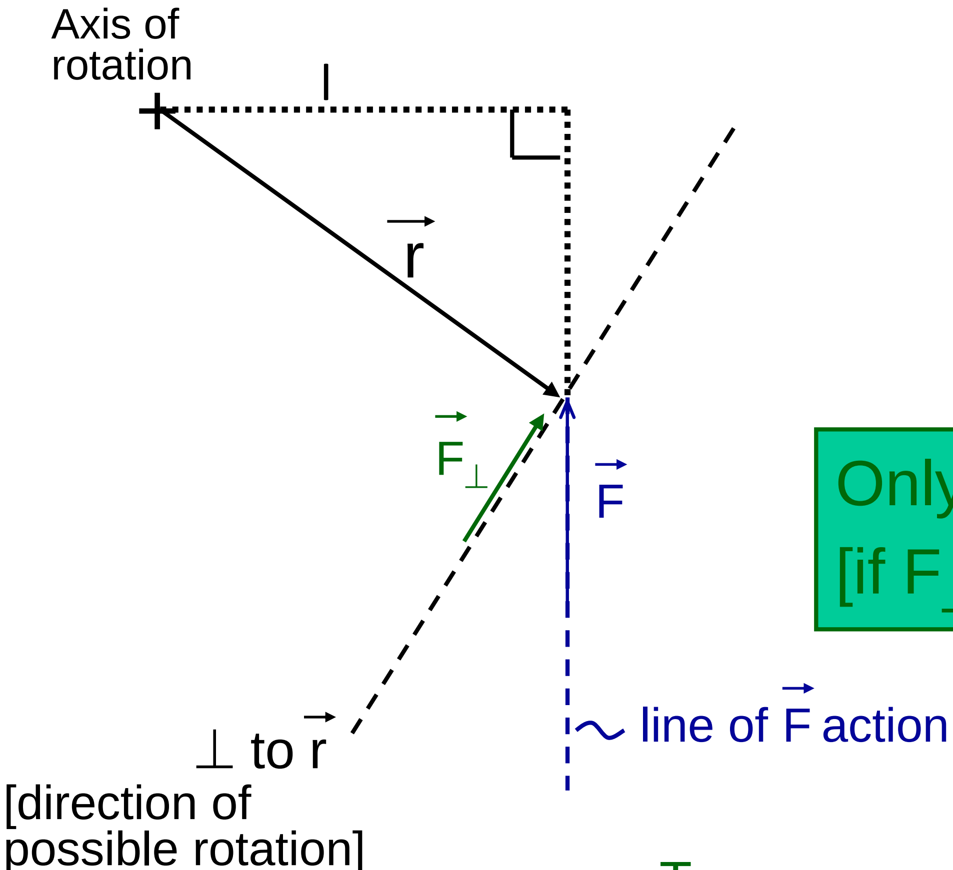
$$a = \alpha R$$

with respect to ground





Torque: τ



Def. of Lever Arm

$l \equiv r_{\perp \text{ to } F}$

$\tau = Fl = Fr_{\perp}$

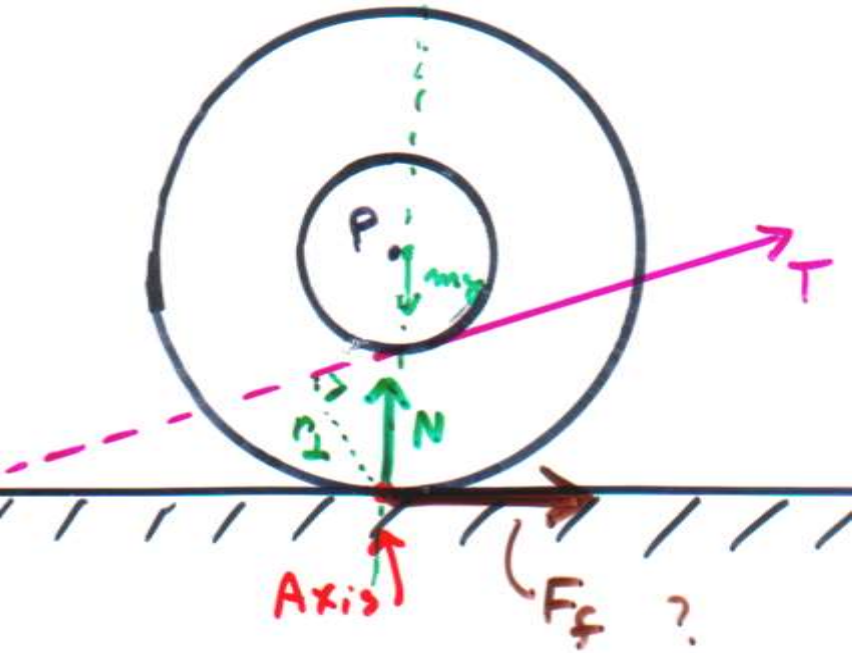
Only $F_{\perp}$ can change rotation!!
[if $F_{\perp} = 0$, rotation unchanged]

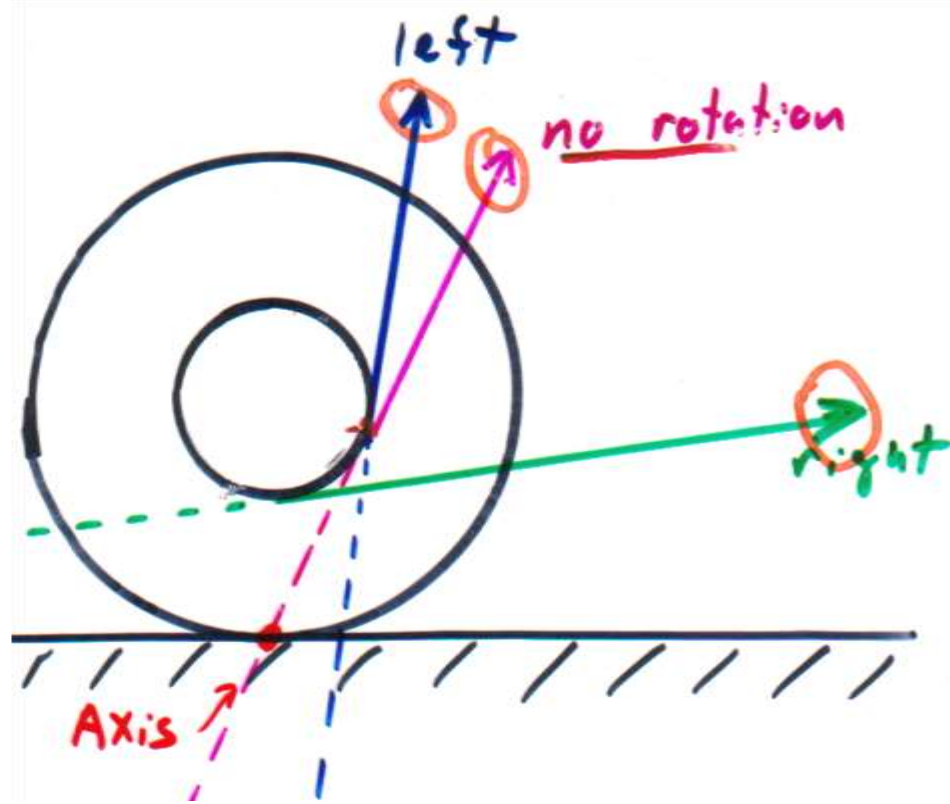
(+ CCW)

Torque

$\tau = F_{\perp} r$ [N·m]

Note: $F_{//}$ increases velocity but not rotation.





For f_s and N $\quad r_\perp = 0$

For T $\quad r_\perp \neq 0$

∴ P wants to rotate counter clockwise about Axis.

∴ Object moves right