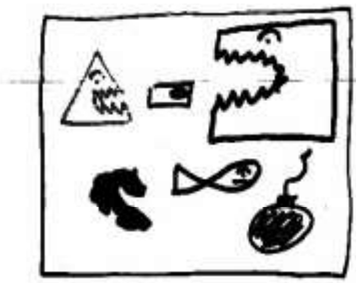
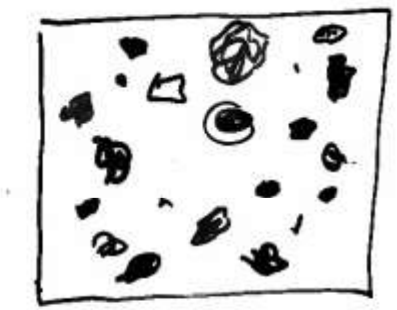
Mass Conservation



Initial

$$M_i = \sum m_k$$
$$= m_1 + m_2 + m_3 + \ldots$$



Final

$$M_f = \sum m_n$$

$$M_i = M_f$$

Conservation of Mass

Will see next semester

Einstein: $E = mc^2$ OR $m = \frac{E}{c^2}$

Conservation of (Mass + Energy)

Momentum Conservation

Recall Newton's Law:

$$\vec{F} = m\,\vec{a}$$

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\vec{F} = \frac{\Delta (m\vec{v})}{\Delta t}$$

Define momentum: $\vec{p} = m\vec{v}$

$$\therefore \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$ = Newton's Law written differently

Note: if $F_{External} = 0$ Then

$$\frac{\Delta \vec{p}}{\Delta t} = 0 \quad \text{OR} \quad \Delta \vec{p} = 0$$

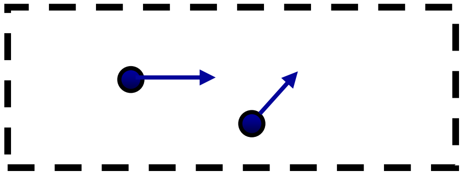
No change in $\vec{p}$

$$\therefore \vec{p}_i = \vec{p}_f \quad \text{if} \quad \vec{F} = 0$$

initial final

p conserved

Generalize 2-Particle System… N-Particle System



The System

$$\vec{F}_{ext} = \frac{\Delta(m_1\vec{v}_1)}{\Delta t} + \frac{\Delta(m_2\vec{v}_2)}{\Delta t}$$

$$= \frac{\Delta[m_1\vec{v}_1 + m_2\vec{v}_2]}{\Delta t}$$

$$\vec{F}_{ext} = \frac{\Delta(\vec{p}_1 + \vec{p}_2)}{\Delta t}$$

$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

(total momentum)

$$\vec{F}_{ext} = \frac{\Delta\vec{P}}{\Delta t}$$

If $\vec{F}_{ext} = 0$,

Then $\vec{P}$ = constant

($\vec{P}$ conserved)

Corollary: If there are outside forces and/or particles enter or leave the system, then total momentum is not conserved.

Good for 3 particles:
$m_1v_1 + m_2v_2 + m_3v_3$

Good for 12 particles:
$m_1v_1 + m_2v_2 + \ldots + m_{12}v_{12}$

Good for billions and billions particles:
$m_1v_1 + m_2v_2 + \ldots + m_Nv_N$

(You get the idea.)

$$\vec{F}_{external} = 0 \Rightarrow \Delta\vec{P} = 0$$ **Isolated system**

general

$$\vec{P}_{init} = \vec{P}_{final}$$

$$\left(\sum_{j}^{all} m_j \vec{v}_j\right)_{init} = \left(\sum_{j}^{all} m_j \vec{v}_j\right)_{final}$$

1-D, N bodies

$$P_{init} = P_{final}$$

$$\left(\sum_{j}^{all} m_j v_j\right)_{init} = \left(\sum_{j}^{all} m_j v_j\right)_{final}$$

momentum conservation

2-D or N bodies

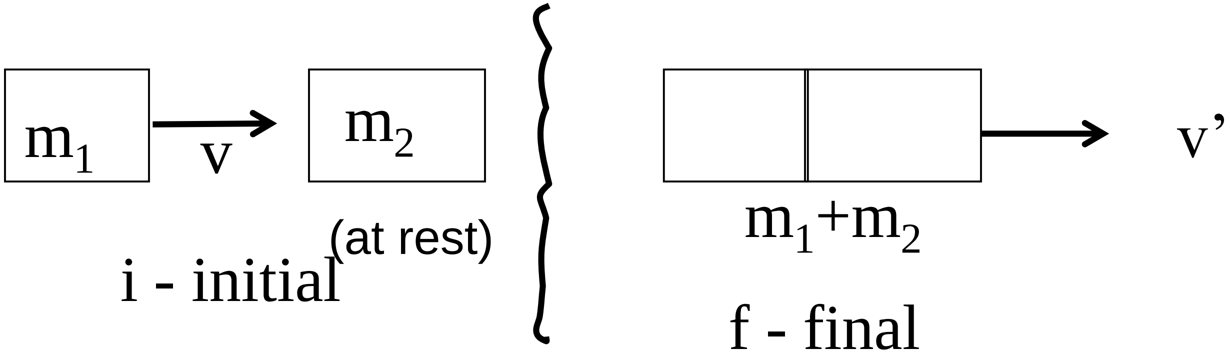
$$(P_x)_{init} = (P_x)_{final}$$

$$\left(\sum_{j}^{all} m_j (v_x)_j\right)_{init} = \left(\sum_{j}^{all} m_j (v_x)_j\right)_{final}$$

$$(P_y)_{init} = (P_y)_{final}$$

$$\left(\sum_{j}^{all} m_j (v_y)_j\right)_{init} = \left(\sum_{j}^{all} m_j (v_y)_j\right)_{final}$$

m_1 (with v) hits m_2 (rest) and sticks with new velocity, v′



$$p_i = p_f$$

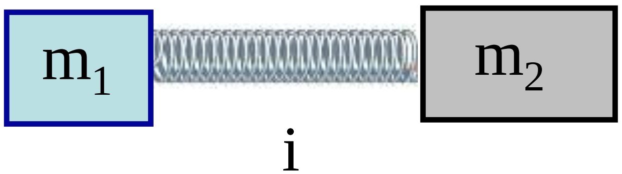
$$m_1 v + 0 = (m_1 + m_2)\, v'$$

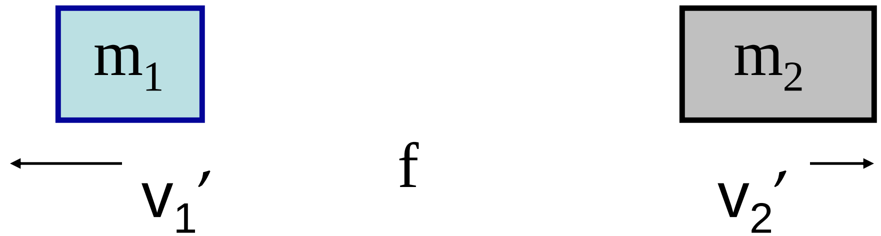
$$\mathbf{v' = \frac{m_1 v}{[m_1 + m_2]}}$$

1-D, P conservation example (explosion)

i : Two masses (rest)with compressed (mass less) spring between

f : masses moving spring drops out





$$p_i = p_f$$

$$0 = m_1 v_1' + m_2 v_2'$$

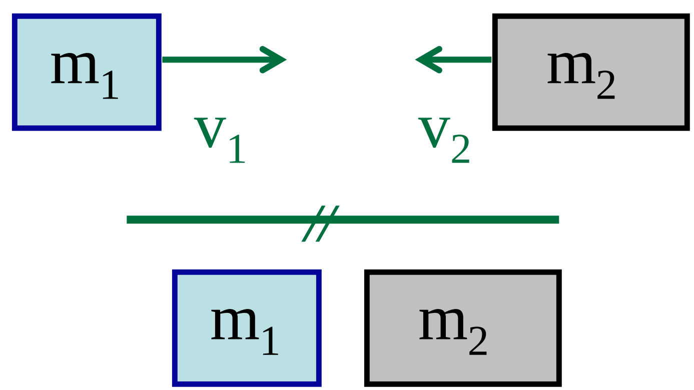
$$-m_1 v_1' = m_2 v_2'$$

opposite directions

$$v_1' = -\frac{m_2}{m_1} v_2'$$

Speed ratio = inverse mass ratio

Later - reverse the objects' v's



$p_i = 0$

$p_f = 0$

Simple 1D collision

$$P_i = m_1 v_1 + 0 \qquad P_f = m_1 v_1' + m_2 v_2'$$

$$P_i = P_f$$

$$m_1 v_1 = m_1 v_1' + m_2 v_2'$$

Example

$m_1 = 2\,kg$, $m_2 = 1\,kg$

$v_1 = 5\,\frac{m}{s}$, $v_2' = 4\,m/s$

$v_1' = ?$

$$v_1' = \frac{m_1 v_1 - m_2 v_2'}{m_1}$$

$$= \frac{2(5) - (1)(4)}{2}\ \frac{kg\,m/s}{kg} = 3\,m/s$$

Example 2

make m_2 big

$m_1 = 2\,kg$, $m_2 = 20\,kg$

$v_1 = 5\,m/s$, $v_2' = 0.7\,m/s$

again

$$v_1' = \frac{m_1 v_1 - m_2 v_2'}{m_1} = \frac{2(5) - 20(0.7)}{2}$$

$$= -2\,m/s$$

m_1 bounces backward

Consider Energy in Collisions

- $\vec{p}$ always conserved !!
- Kinetic Energy not always conserved!!

(some E can be carried away as heat)

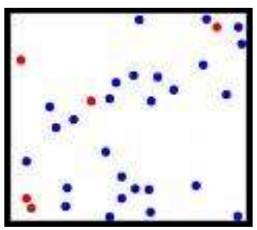
Until now,
no mention of energy
in collisions

"Elastic collisions" ⇒ **E is conserved**

"Inelastic collisions" ⇒ **some E is converted to heat (or material deformation)**

In fact in most of the real world, E is at best approximately conserved

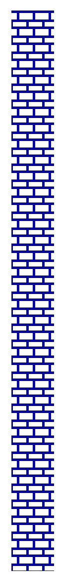
Examples



Atoms in a box

Elastic

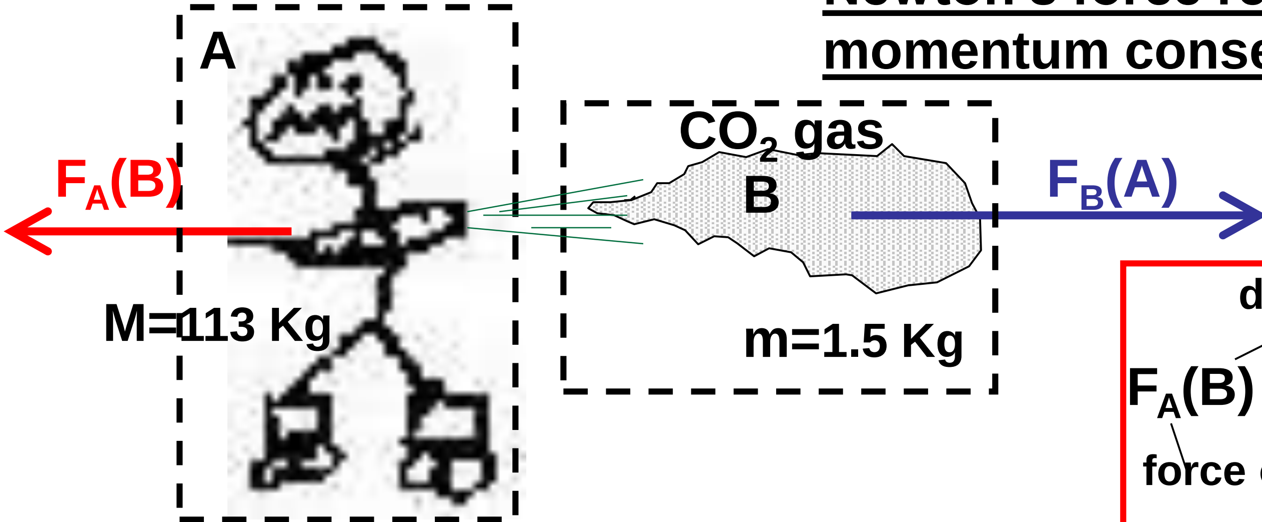
Highly Elastic



Inelastic

Inelastic

Newton's force reaction force (3^{nd}) Law & momentum conservation in rocket demo



Newton's 3^{rd} Law

$$F_A(B) = - F_B(A)$$

- $F_A(B)$: force on A, due to B
- $F_B(A)$: force on B, due to A

(simple but powerful !!)

Newton's 2^{rd} Law $\quad F = ma \quad or \quad F = \dfrac{\Delta p}{\Delta t} \quad where \quad p = mv$

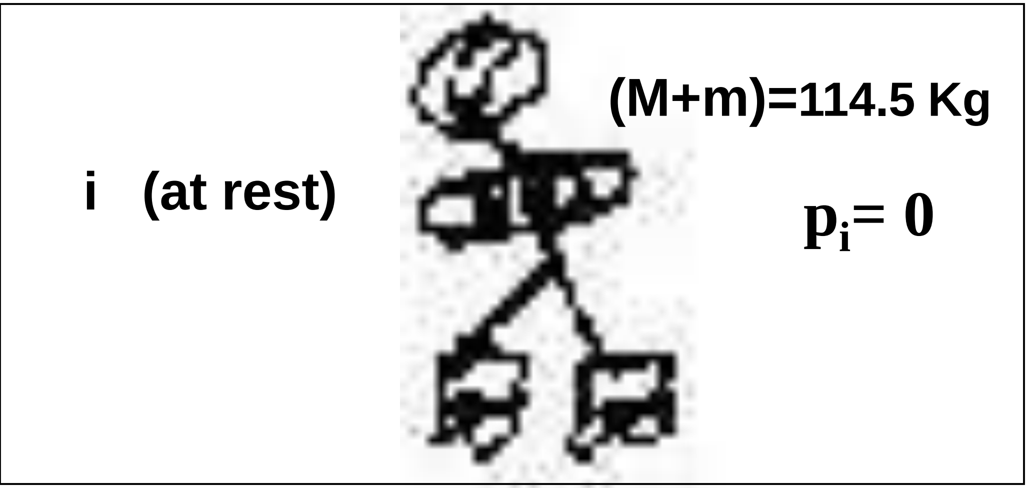
A+B system: no external forces $\Rightarrow$ momentum conservation

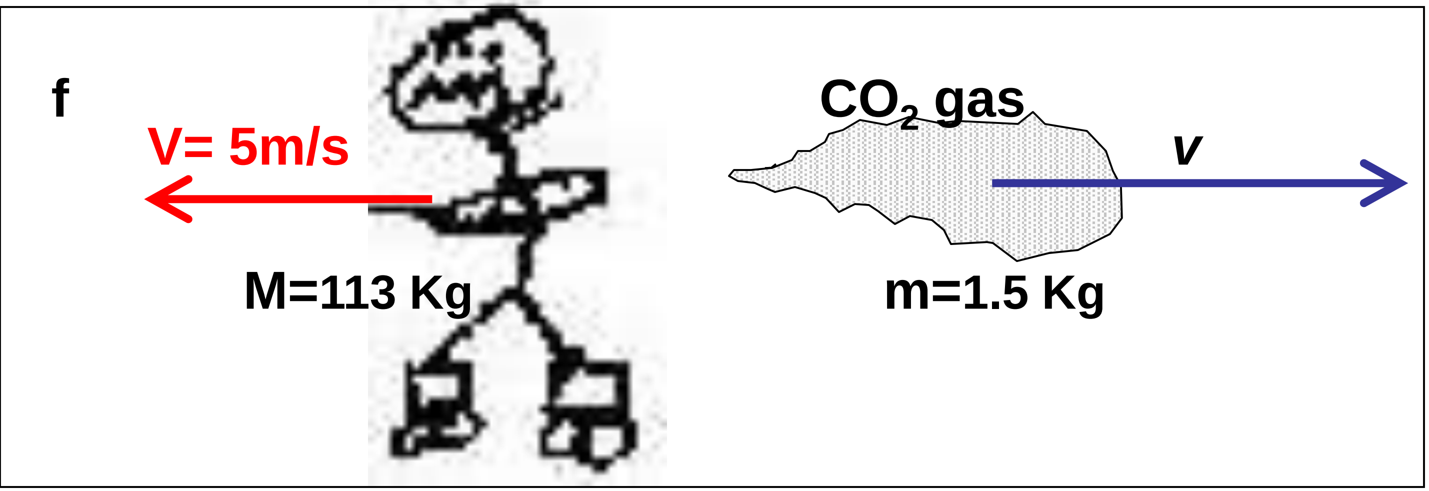
$$F_{tot} = F_A + F_B = 0 \quad \Rightarrow \quad \frac{\Delta p_{tot}}{\Delta t} = 0 \quad \Rightarrow \quad \Delta p_{tot} = \Delta p_A + \Delta p_B = 0$$

$$\Downarrow$$

$$\boxed{\Delta p_A = -\Delta p_B}$$

Conservation of momentum in rocket demo





m=1.5 Kg= mass tank after-before=(16.5-15) Kg

initial momentum → $p_i = p_f$ ← final momentum

$$0 = +MV - m\,v$$

$$m\,v = MV$$

$$v = \left[\frac{M}{m}\right]V$$

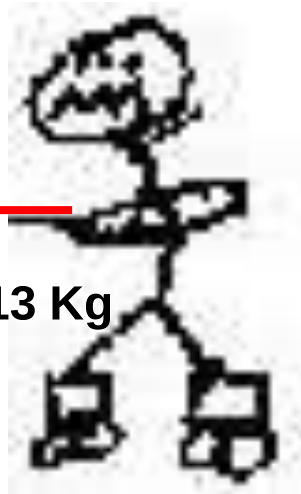
$$v = [75.3]\, 5\frac{m}{s}$$

$$v = 377\frac{m}{s}$$

for CO_2 gas

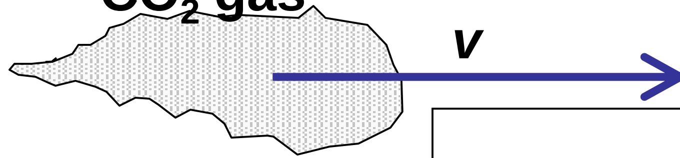
Division of energy & momentum in rocket demo

V= 5m/s



M=113 Kg

CO_2 gas



m=1.5 Kg

momentum conservation

MV= m *v* $\Rightarrow \frac{v}{V} = \frac{M}{m}$

$$\frac{KE_{gas}}{KE_{prof}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}MV^2} = \frac{m}{M}\left(\frac{v}{V}\right)^2 = \frac{m}{M}\left(\frac{M}{m}\right)^2 = \frac{M}{m}$$

$$\frac{KE_{gas}}{KE_{prof}} = \frac{M}{m} = \frac{113}{1.5} = 75!!$$

Energy to gas!!

$$\frac{p_m}{p_M} = \frac{mv}{MV} = 1$$

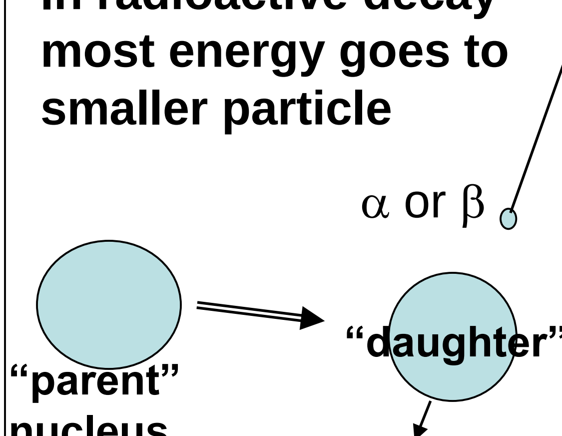
in general

momentum equally to both !!

$$\frac{KE_m}{KE_M} = \frac{M}{m}$$

Energy goes to the little guy!!

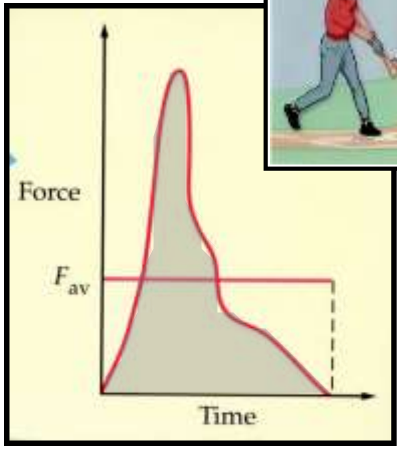
In radioactive decay most energy goes to smaller particle



Impulse (force causing a sudden, or cumulative, change in momentum)

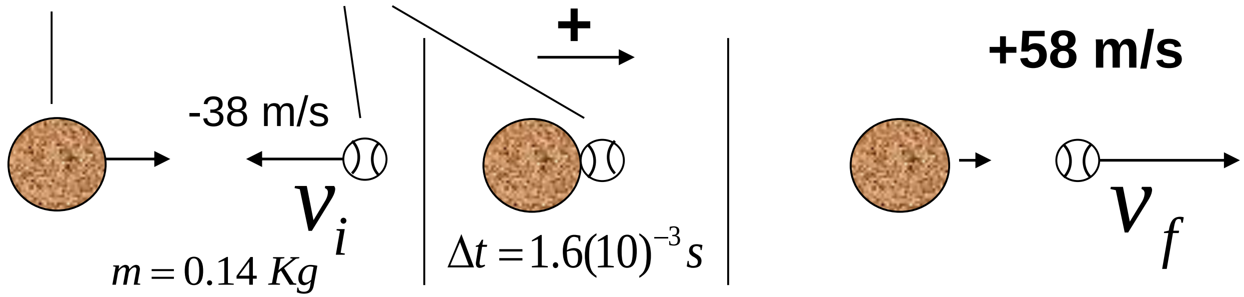
$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t}$$

Impulse: $\vec{F}\Delta t = \Delta \vec{P}$



Area under curve is the impulse.

bat & baseball example



$$\Delta P_{baseball} = mv_f - mv_i = m[v_f - v_i]$$

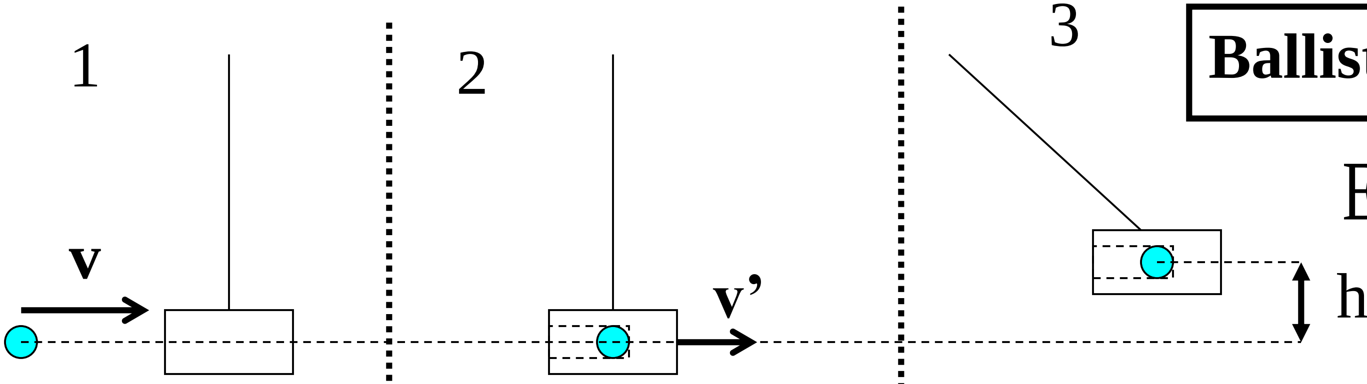
$$\Delta P_{baseball} = 0.14[(+58) - (-38)]Kg\frac{m}{s} = 13.4 Kg\frac{m}{s}$$

$$F_{ave}\Delta t = \Delta P$$

$$F_{ave} = \frac{\Delta P}{\Delta t} = \frac{13.4}{1.6(10)^{-3}} Kg\frac{m}{s^2} = 8400 N$$

1900 lb.

Ballistic Pendulum



$$E_3 = (M + m)\, g\, h$$

$\mathbf{p_1 = mv}$

$\mathbf{p_2 = (m+M)\, v'}$

$$E_2 = \frac{1}{2}(M+m)v'^2$$

no external forces

Momentum conservation

$\mathbf{p_1 = p_2}$

$\mathbf{mv = (m+M)v'}$

$$\frac{m}{(m+M)}\, v = v'$$

Energy conservation

$\mathbf{E_2 = E_3}$

$$\frac{1}{2}(M+m)v'^2 = (M+m)gh$$

$$\frac{1}{2}(M+m)\left[\frac{m}{(m+M)}\, v\right]^2 = (M+m)gh$$

$$v^2 = \left[\frac{(M+m)}{m}\right]^2 2gh$$

$$v = \left[\frac{(M+m)}{m}\right]\sqrt{2gh}$$

1-D Elastic Collision

Kinetic Energy is conserved

Later

Energy is conserved

$$\frac{1}{2}\mathbf{m}_1\mathbf{v}_1{}^2 = \frac{1}{2}\mathbf{m}_1(\mathbf{v}_1')^2 + \frac{1}{2}\mathbf{m}_2(\mathbf{v}_2')^2$$

Momentum is conserved:

$$\mathbf{m_1 v_1 = m_1 v_1' + m_2 v_2'}$$

SOME ALGEBRA (see next page)

$$\mathbf{v}_1' = -\mathbf{v}_1 \frac{(\mathbf{m}_2 - \mathbf{m}_1)}{(\mathbf{m}_2 + \mathbf{m}_1)}$$

$$\mathbf{v}_2' = \frac{2\mathbf{m}_1}{(\mathbf{m}_2 + \mathbf{m}_1)}\mathbf{v}_1$$

SOME ALGEBRA

Energy cons.

$$\frac{1}{2}\mathbf{m}_1\mathbf{v}_1^{\,2} = \frac{1}{2}\mathbf{m}_1(\mathbf{v}_1^{'})^2 + \frac{1}{2}\mathbf{m}_2(\mathbf{v}_2^{'})^2$$

$$\mathbf{m}_1[\mathbf{v}_1^{\,2} - \mathbf{v}_1^{'\,2}] = \mathbf{m}_2\mathbf{v}_2^{'\,2}$$

$$\mathbf{m}_1(\mathbf{v}_1 - \mathbf{v}_1^{'})(\mathbf{v}_1 + \mathbf{v}_1^{'}) = \mathbf{m}_2\mathbf{v}_2^{'\,2}$$

$$\mathbf{m}_2\mathbf{v}_2^{'}(\mathbf{v}_1 + \mathbf{v}_1^{'}) = \mathbf{m}_2\mathbf{v}_2^{'\,2}$$

$$(\mathbf{v}_1 + \mathbf{v}_1^{'}) = \mathbf{v}_2^{'}$$

$$(\mathbf{v}_1 + \mathbf{v}_1^{'}) = \frac{\mathbf{m}_1}{\mathbf{m}_2}(\mathbf{v}_1 - \mathbf{v}_1^{'})$$

$$\mathbf{v}_1(1 - \frac{\mathbf{m}_1}{\mathbf{m}_2}) = -\mathbf{v}_1^{'}(\frac{\mathbf{m}_1}{\mathbf{m}_2} + 1)$$

$$\mathbf{v}_1^{'} = -\mathbf{v}_1\frac{(\mathbf{m}_2 - \mathbf{m}_1)}{(\mathbf{m}_2 + \mathbf{m}_1)}$$

Mom. cons.

$$\mathbf{m}_1\mathbf{v}_1 = \mathbf{m}_1\mathbf{v}_1^{'} + \mathbf{m}_2\mathbf{v}_2^{'}$$

$$\mathbf{m}_1(\mathbf{v}_1 - \mathbf{v}_1^{'}) = \mathbf{m}_2\mathbf{v}_2^{'}$$

$$\mathbf{v}_2^{'} = \frac{\mathbf{m}_1}{\mathbf{m}_2}(\mathbf{v}_1 - \mathbf{v}_1^{'})$$

$$\mathbf{v}_2^{'} = \frac{2\mathbf{m}_1}{(\mathbf{m}_2 + \mathbf{m}_1)}\mathbf{v}_1$$

Recall 1-D Elastic Collision

Kinetic Energy is conserved

Energy conserved

$$v_1' = -v_1 \frac{(m_2 - m_1)}{(m_2 + m_1)}$$

Momentum conserved:

$$v_2' = \frac{2m_1}{(m_2 + m_1)} v_1$$

Special Case 1

Let $m_1 = m_2 = m$

$$v_1' = 0 \qquad v_2' = v_1$$

Later

https://www.msu.edu/~brechtjo/physics/airTrack/airTrack.html

http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/AirTrack/AirTrack.html

http://www.walter-fendt.de/ph14e/ncradle.htm

http://www.sciencejoywagon.com/explrsci/media/airtrack.htm

Recall 1-D Elastic Collision

Kinetic Energy is conserved

Energy conserved

$$v_1' = -v_1 \frac{(m_2 - m_1)}{(m_2 + m_1)}$$

Momentum conserved:

$$v_2' = \frac{2m_1}{(m_2 + m_1)} v_1$$

Special Case 2

$m_1 << m_2$

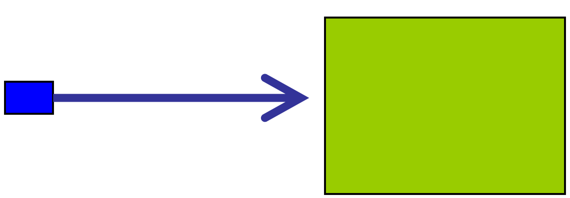
Say $m_2 = 9\ m_1$

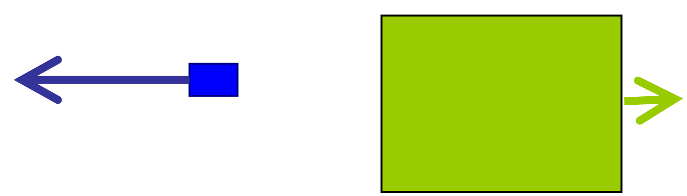
$$v_1' = -v_1 \frac{(9-1)m_1}{(10)m_1} = -v_1 \frac{8}{10}$$

m_1 bounces back with 80% speed

$$v_2' = \frac{2m_1}{(9+1)m_1} v_1 = \frac{2}{10} v_1$$

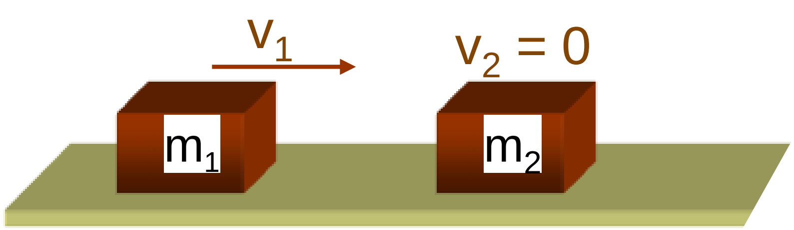
m_2 lumbers off at 20% init. speed of 1

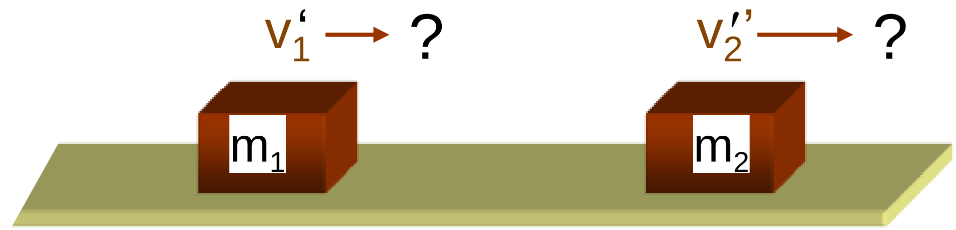




Recall 1-D Elastic Collision

Kinetic Energy is conserved





Energy conserved

$$v_1' = -v_1 \frac{(m_2 - m_1)}{(m_2 + m_1)}$$

Momentum conserved:

$$v_2' = \frac{2m_1}{(m_2 + m_1)} v_1$$

Special Case 3

$m_1 << m_s$
Say $m_1 = 9\ m_2$

$$v_1' = -v_1 \frac{(1-9)m_2}{(10)m_2} = v_1 \frac{8}{10}$$

m_1 looses little speed – plows through

$$v_2' = \frac{2\ (9m_2)}{(9+1)m_2} v_1 = 1.8 v_1$$

m_2 blasts off at ~ 2 v_1