Recall Newton’s Law:
\[ \mathbf{F} = \frac{\Delta (m \mathbf{v})}{\Delta t} \]
\[ \mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} \]

Define momentum:
\[ \mathbf{p} = m \mathbf{v} \]

\[ \therefore \mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t} = \text{Newton's Law written differently} \]

Note: if \( F_{\text{External}} = 0 \) then
\[ \frac{\Delta \mathbf{p}}{\Delta t} = 0 \quad \text{OR} \quad \Delta \mathbf{p} = 0 \]

No change in \( \mathbf{p} \)

\[ \therefore \mathbf{p}_i = \mathbf{p}_f \quad \text{if} \quad \mathbf{F} = 0 \]

Conservation of Mass

\[ M_i = M_f \]

Mass Conservation

Initial
\[ M_i = \sum m_k \]
\[ = m_1 + m_2 + m_3 + \ldots \]

Final
\[ M_f = \sum m_n \]

Conservation of Mass

Will see next semester

Einstein: \( E = mc^2 \) \quad \text{OR} \quad m = \frac{E}{c^2} \]

Conservation of (Mass + Energy)
Generalize 2-Particle System... N-Particle System

\[ \vec{F}_{\text{ext}} = \frac{\Delta(m_1\vec{v}_1)}{\Delta t} + \frac{\Delta(m_2\vec{v}_2)}{\Delta t} \]

\[ = \frac{\Delta[m_1\vec{v}_1 + m_2\vec{v}_2]}{\Delta t} \]

\[ \vec{F}_{\text{ext}} = \frac{\Delta(p_1 + p_2)}{\Delta t} \]

\[ \vec{P} = \vec{p}_1 + \vec{p}_2 \]

(You get the idea.)

If \( \vec{F}_{\text{ext}} = 0 \),

Then \( \vec{P} = \text{constant} \)

\( \vec{P} \) conserved

Corollary: If there are outside forces and/or particles enter or leave the system, then total momentum is not conserved.
\[
\vec{F}_{\text{external}} = 0 \implies \Delta \vec{P} = 0 \quad \text{Isolated system}
\]

### General

\[
\begin{align*}
\vec{P}_{\text{init}} &= \vec{P}_{\text{final}} \\
\left( \sum_{j}^{\text{all}} m_j \vec{v}_j \right)_{\text{init}} &= \left( \sum_{j}^{\text{all}} m_j \vec{v}_j \right)_{\text{final}}
\end{align*}
\]

### 1-D, N bodies

\[
\begin{align*}
\vec{P}_{\text{init}} &= \vec{P}_{\text{final}} \\
\left( \sum_{j}^{\text{all}} m_j \vec{v}_j \right)_{\text{init}} &= \left( \sum_{j}^{\text{all}} m_j \vec{v}_j \right)_{\text{final}}
\end{align*}
\]

### Momentum Conservation

### 2-D or N bodies

\[
\begin{align*}
(P_x)_{\text{init}} &= (P_x)_{\text{final}} \\
\left( \sum_{j}^{\text{all}} m_j (v_x)_j \right)_{\text{init}} &= \left( \sum_{j}^{\text{all}} m_j (v_x)_j \right)_{\text{final}}
\end{align*}
\]

\[
\begin{align*}
(P_y)_{\text{init}} &= (P_y)_{\text{final}} \\
\left( \sum_{j}^{\text{all}} m_j (v_y)_j \right)_{\text{init}} &= \left( \sum_{j}^{\text{all}} m_j (v_y)_j \right)_{\text{final}}
\end{align*}
\]

7-2a
m_1 (with v) hits m_2 (rest) and sticks with new velocity, v'

\[
m_1 \text{ (at rest)} \rightarrow \begin{cases} \text{m}_1 + \text{m}_2 \end{cases} \rightarrow v' \\
p_i = p_f \\
m_1 \cdot v + 0 = (m_1 + m_2) \cdot v' \\
v' = \frac{m_1 \cdot v}{[m_1 + m_2]}
\]

7-3
1-D. P conservation example (explosion)

\( \text{i : Two masses (rest )with compressed (mass less) spring between} \)

\( m_1 \)

\( m_2 \)

\( \text{f : masses moving spring drops out} \)

\( \text{Later - reverse the objects' v's} \)

\( p_i = p_f \)

\( 0 = m_1 v'_1 + m_2 v'_2 \)

\( -m_1 v'_1 = m_2 v'_2 \)

\( v'_1 = -\frac{m_2}{m_1} v'_2 \)

Speed ratio = inverse mass ratio

\( p_i = 0 \)

\( p_f = 0 \)
Simple 1D collision

\[ P_i = m_1 v_i + 0 \quad \Rightarrow \quad P_f = m_1 v_i' + m_2 v_2' \]
\[ P_i = P_f \]

\[ m_1 v_i = m_1 v_i' + m_2 v_2' \]

Example 2

| \hline
| \text{Example} | \text{Example} | \text{Example} |
| \hline
| m_1 = 2 \text{ kg} | m_2 = 20 \text{ kg} |
| v_i = 5 \text{ m/s} | v_2 = 0.7 \text{ m/s} |
| v_i' = ? |

\[ v_i' = \frac{m_1 v_i - m_2 v_2'}{m_1} = \frac{2(5) - 20(0.7)}{2} = -2 \text{ m/s} \]
\[ m_1 \text{ bounces backward} \]

7-5
Consider Energy in Collisions

- p always conserved !!
- Kinetic Energy not always conserved!!
  (some E can be carried away as heat)

“Elastic collisions” ⇒ E is conserved
“Inelastic collisions” ⇒ some E is converted to heat (or material deformation)

In fact in most of the real world, E is at best approximately conserved

Examples

Elastic

Inelastic

Atoms in a box

Highly Elastic

7-6
Newton’s force reaction force (3rd) Law & momentum conservation in rocket demo

Newton’s 3rd Law

\[ F_A(B) = - F_B(A) \]

due to B
due to A

force on A
force on B

(simple but powerful !!)

Newton’s 2nd Law

\[ F = ma \quad or \quad F = \frac{\Delta p}{\Delta t} \]

where \( p = mv \)

A+B system: no external forces \( \Rightarrow \) momentum conservation

\[ F_{tot} = F_A + F_B = 0 \quad \Rightarrow \quad \frac{\Delta p_{tot}}{\Delta t} = 0 \quad \Rightarrow \quad \Delta p_{tot} = \Delta p_A + \Delta p_B = 0 \]

\[ \Delta p_A = -\Delta p_B \]
Conservation of momentum in rocket demo

\[ (M+m) = 114.5 \text{ Kg} \]

\[ \text{i (at rest) } \quad p_i = 0 \]

\[ \text{f } \quad V = 5 \text{m/s} \]

\[ M = 113 \text{ Kg} \]

\[ m = 1.5 \text{ Kg} \]

\[ \text{CO}_2 \text{ gas} \]

\[ \text{for } \text{CO}_2 \text{ gas} \]

\[ \text{m} = 1.5 \text{ Kg} = \text{mass tank after-before} = (16.5 - 15) \text{ Kg} \]

\[ v = 377 \frac{\text{m}}{\text{s}} \]
Division of energy & momentum in rocket demo

\[ V = 5 \text{m/s} \]

\[ \text{M} = 113 \text{ Kg} \]

\[ \text{m} = 1.5 \text{ Kg} \]

\[ \frac{\text{KE}_{\text{gas}}}{\text{KE}_{\text{prof}}} = \frac{\frac{1}{2} \text{mv}^2}{\frac{1}{2} \text{MV}^2} = \frac{\text{m} \left( \frac{\text{v}}{\text{V}} \right)^2}{\text{M} \left( \frac{\text{M}}{\text{m}} \right)^2} = \frac{\text{M}}{\text{m}} \]

\[ \frac{\text{KE}_{\text{gas}}}{\text{KE}_{\text{prof}}} = \frac{\text{M}}{\text{m}} = \frac{113}{1.5} = 75!! \]

Energy to gas!!

\[ \frac{\text{p}_\text{m}}{\text{p}_\text{M}} = \frac{\text{mv}}{\text{MV}} = 1 \]

in general

momentum equally to both !!

\[ \frac{\text{KE}_\text{m}}{\text{KE}_\text{M}} = \frac{\text{M}}{\text{m}} \]

Energy goes to the little guy!!

In radioactive decay
most energy goes to smaller particle

\[ \alpha \text{ or } \beta \]

“parent” nucleus

“daughter”

7-7b
**Impulse** (force causing a sudden, or cumulative, change in momentum)

\[
F = \frac{\Delta \vec{P}}{\Delta t}
\]

**Impulse:** \( \vec{F} \Delta t = \Delta \vec{P} \)

**bat & baseball example**

\[
\Delta P_{\text{baseball}} = mv_f - mv_i = m[v_f - v_i]
\]

\[
\Delta P_{\text{baseball}} = 0.14[(+58) - (-38)]Kg \frac{m}{s} = 13.4Kg \frac{m}{s}
\]

\[
F_{\text{ave}} \Delta t = \Delta P
\]

\[
F_{\text{ave}} = \frac{\Delta P}{\Delta t} = \frac{13.4}{1.6(10)^{-3}} Kg \frac{m}{s^2} = 8400N
\]

1900 lb.
Ballistic Pendulum

Energy conservation

$$E_2 = E_3$$

$$\frac{1}{2} (M+m) v'^2 = (M+m)gh$$

$$\frac{1}{2} (M+m) \left[ \frac{m}{(m+M)} v \right]^2 = (M+m)gh$$

$$v'^2 = \left[ \frac{(M+m)}{m} \right]^2 2gh$$

$$v = \left[ \frac{(M+m)}{m} \right] \sqrt{2gh}$$

Momentum conservation

$$p_1 = mv$$

$$p_2 = (m+M) v'$$

$$E_2 = \frac{1}{2} (M+m)v'^2$$

no external forces
Kinetic Energy is conserved

Energy is conserved

\[ \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 (v_1')^2 + \frac{1}{2} m_2 (v_2')^2 \]

Momentum is conserved:

\[ m_1 v_1 = m_1 v_1' + m_2 v_2' \]

SOME ALGEBRA (see next page)

\[ v_1' = -v_1 \frac{(m_2 - m_1)}{(m_2 + m_1)} \]

\[ v_2' = \frac{2m_1}{(m_2 + m_1)} v_1 \]
\( \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 (v'_1)^2 + \frac{1}{2} m_2 (v'_2)^2 \)

\( m_1 [v_1^2 - v'_1^2] = m_2 v'_2^2 \)

\( m_1 (v_1 - v'_1) (v_1 + v'_1) = m_2 v'_2^2 \)

\( m_2 v'_2 (v_1 + v'_1) = m_2 v'_2^2 \)

\( (v_1 + v'_1) = v'_2 \)

\( (v_1 + v'_1) = \frac{m_1}{m_2} (v_1 - v'_1) \)

\( v_1 (1 - \frac{m_1}{m_2}) = -v'_1 (\frac{m_1}{m_2} + 1) \)

\( v'_1 = -v_1 \frac{(m_2 - m_1)}{(m_2 + m_1)} \)

\( v'_2 = \frac{2m_1}{(m_2 + m_1)} v_1 \)
Recall 1-D Elastic Collision

Kinetic Energy is conserved

Energy conserved

\[ v_1' = -v_1 \left( \frac{m_2 - m_1}{m_2 + m_1} \right) \]

Momentum conserved:

\[ v_2' = \frac{2m_1}{m_2 + m_1} v_1 \]

Special Case 1

Let \( m_1 = m_2 = m \)

\[ v_1' = 0 \quad v_2' = v_1 \]

Later

https://www.msu.edu/~brechtjo/physics/airTrack/airTrack.html
http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/AirTrack/AirTrack.html
http://www.walter-fendt.de/ph14e/ncradle.htm
http://www.sciencejoywagon.com/explrscti/media/airtrack.htm
Recall 1-D Elastic Collision  

Kinetic Energy is conserved

Energy conserved
\[ v_1' = -v_1 \frac{(m_2 - m_1)}{(m_2 + m_1)} \]

Momentum conserved:
\[ v_2' = \frac{2m_1}{(m_2 + m_1)} v_1 \]

Special Case 2

Say \( m_2 = 9 m_1 \)

\( m_1 \ll m_2 \)

\[ v_1' = -v_1 \frac{(9 - 1)m_1}{(10)m_1} = -v_1 \frac{8}{10} \]

\( m_1 \) bounces back with 80% speed

\[ v_2' = \frac{2m_1}{(9 + 1)m_1} v_1 = \frac{2}{10} v_1 \]

\( m_2 \) lumbers off at 20% init. speed of 1

7-10c
Recall 1-D Elastic Collision

Energy conserved
\[ v_1' = -v_1 \frac{(m_2 - m_1)}{(m_2 + m_1)} \]

Momentum conserved:
\[ v_2' = \frac{2m_1}{(m_2 + m_1)} v_1 \]

Special Case 3

\[ m_1 << m_s \]
Say \( m_1 = 9m_2 \)

\[ v_1' = -v_1 \frac{(1-9)m_2}{(10)m_2} = v_1 \frac{8}{10} \]
\[ v_2' = \frac{2(9m_2)}{(9+1)m_2} v_1 = 1.8v_1 \]

\( m_1 \) looses little speed – plows through
\( m_2 \) blasts off at ~ 2 \( v_1 \)