<u>Kinetic Energy</u> (energy of motion)   E or KE

$$K = \frac{1}{2} m |v|^2 = \frac{1}{2} m (v_x^2 + v_y^2 + v_z^2)$$

Units

$$[kg \frac{m^2}{s^2}] = J$$

(Joule)

---

example   baseball   m=0.15 kg

pitched at   v = 69 mph = 36.5 m/s

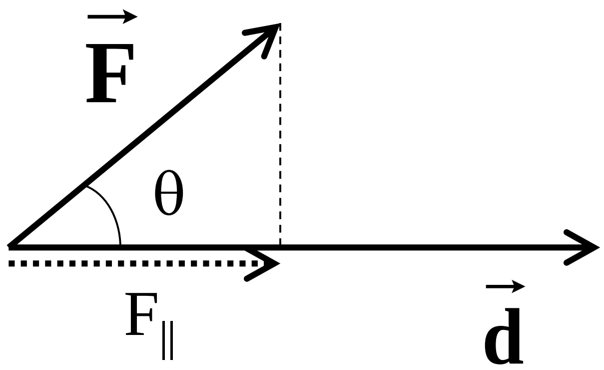
$$K = \frac{1}{2} m v^2 = \frac{1}{2} (0.15)(36.5)^2 \ [\mathrm{kg}\ (\mathrm{m/s})^2]$$

v = 69 mph $K = 100 \ J$

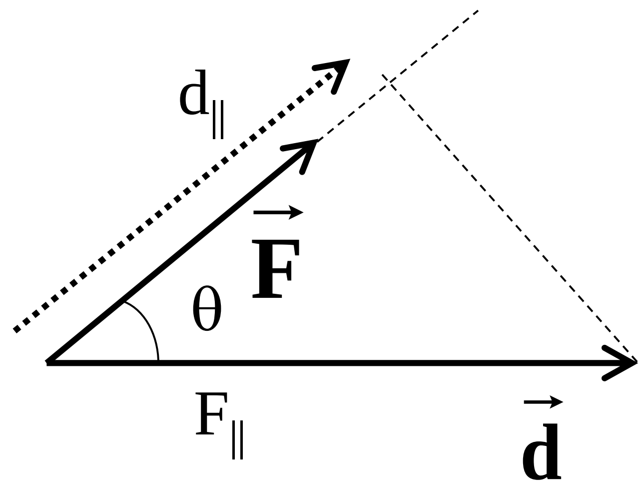
v = 100 mph $K = 210 \ J$

!!! Lethal energies

Work done by force F acting over displacement d



OR



$W = F_{\parallel}\, d$

component (projection of)
F along d

$W = F\, d_{\parallel}$

component (projection of)
d along F

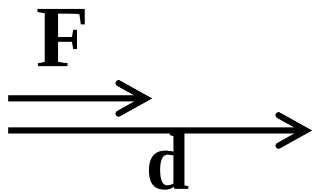
$$W = F\, d \cos(\theta)$$

Units

$$(\mathbf{N})\,\mathbf{m} = (\mathbf{Kg}\,\frac{\mathbf{m}}{\mathbf{s}^2})\,\mathbf{m} = \mathbf{Kg}\,\frac{\mathbf{m}^2}{\mathbf{s}^2} = \mathbf{J} \quad !!$$

Collinear F and d

$W = -F\,d$

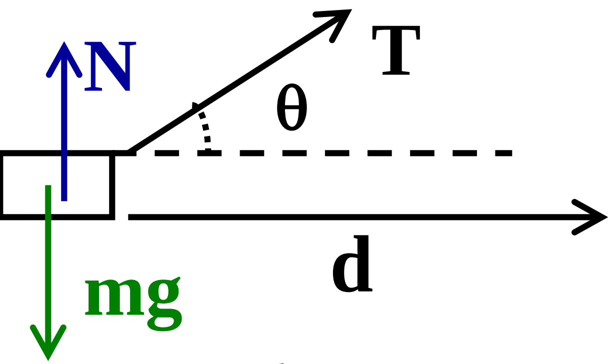


$W = +F\,d$

Note kinetic friction force always opposes displacement $\therefore$ always does - work

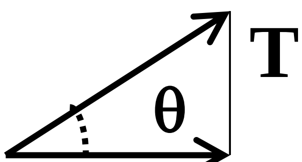
---

example **box on frictionless plane** **Constant force not // d**



$\vec{N} \perp \vec{d}$ **note**

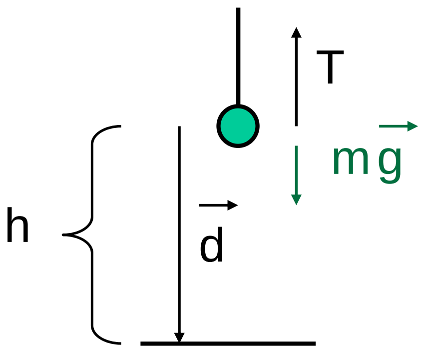
$mg \perp \vec{d}$ $\therefore$ do no work



$T_{||} = T\cos(\theta)$

$W_{tot} = T_{||}\,d = T\cos(\theta)\,d$

Ex: object lowered on a string



$$\vec{F} = T - m\vec{g}$$

$$|\vec{d}| = h$$

Work done by gravity

$W_g = mgh$ (positive work)

Work done by tension

$W_T = -Th$ (negative work)

Total Work Done on Object

$$\left.\begin{aligned} W &= (T - mg)\bullet h \\ W &= W_g - W_T \end{aligned}\right\} \text{Net Work}$$

If object lowered with constant velocity, then $mg - T = 0$ and $W = 0$

If object accelerates down, then $0 < W < mgd$

Work-Energy Theorem

Object: m; constant force F; constant accelertation a=F/m

$$v = v_0 + at \Rightarrow a = \frac{v - v_0}{t} \qquad x = [\frac{v + v_0}{2}]t$$

$$W = F\,x = ma\,x$$

$$W = m\,(\frac{v - v_0}{t})\{[\frac{v + v_0}{2}]t\}$$

true for non-const. a also !!!

$$W = \frac{m}{2}(v^2 - v_0^2) \Rightarrow \boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2}$$

$$\frac{1}{2}mv^2 = K = \text{Kinetic Energy}$$

$$\boxed{W = K - K_0 \quad \text{or} \quad W = \Delta K}$$

Total work done on object= change in kinetic energy

(work done by total force)

$$[K] = kg\,(\frac{m^2}{s^2}) = \text{Joule} = Nm \qquad \cancel{[K] = slug\,(\frac{ft^2}{s^2}) = ft\,lb}$$

Power

A measure of the rate at which work is done.

Average Power $$\bar{P} = \frac{\Delta W}{\Delta t}$$

Instantaneous Power $$P = \frac{dW}{dt}$$

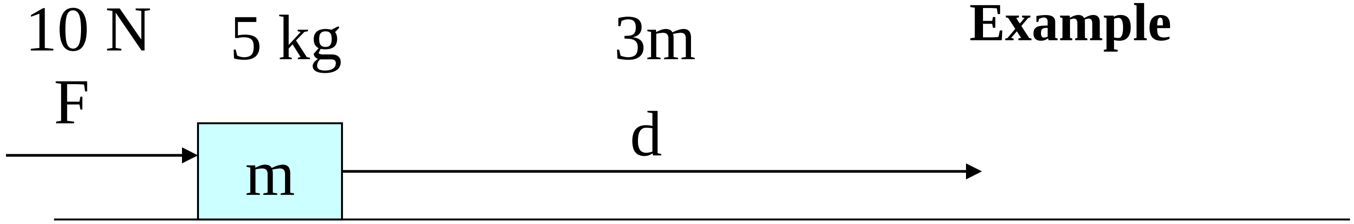
Power= work per unit time [J/s= Watt]

$$P = \frac{W}{t}$$

SI unit: J/s = watt, W

1 horsepower = 1 hp = 746 W

Example



$$W = F_{\parallel}\, d = 10\ (3)\ \text{Nm} = 30\ \text{J}$$

Assume: no other forces (no friction) & starts from rest ($v_i = 0$)

$$W = \Delta K$$

What is v_f ?

$$W = F\,d = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$

$$30\ \text{J} = \tfrac{1}{2}\,(5\ \text{kg})\ v_f^2$$

$$v_f = 3.5\ \text{m/s}$$

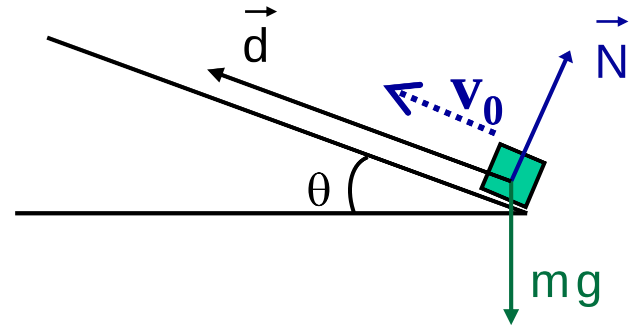
If this took 3 sec what was the average power input by F ?

$$P = W/\,\Delta t = 30\ \text{J}/3\text{s} = 10\ \text{watts}$$

Object projected up inclined plane with speed v_0

How far up ?

Work Energy Theorem



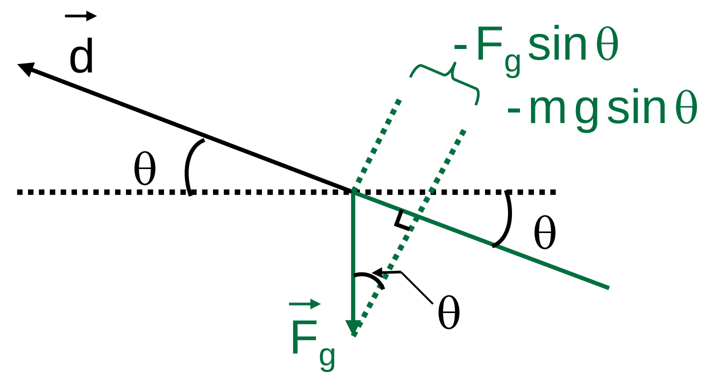
Normal force does no work

$N_{//d} = 0$: $N_{\perp}$ to d

$W_N = N_{\|} d = 0$

$$W_g = \underbrace{F_{g\|}}_{} d$$

$$\boxed{W_g = (-mg\sin\theta)d}$$



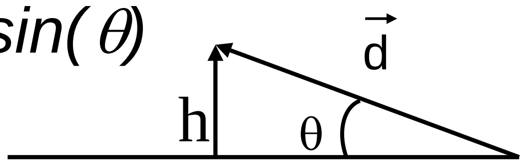
$$\mathbf{W = W_N + W_g = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2}$$

$\therefore\ -\frac{1}{2}mv_0^2 = -mgd\,\underline{\sin(\theta)}$

$$\boxed{\therefore\quad d = \frac{V_0^2}{2g\sin\theta}}$$

Note: h=height up plane

$h = d\sin(\theta)$



$-\frac{1}{2}mv_0^2 = -mgh$

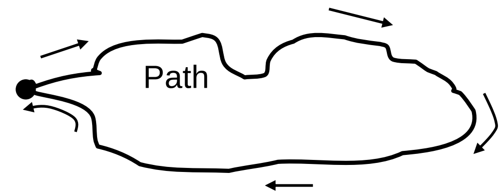
Conservative Force in general

3 ways to define a *conservative force*

F = conservative force if:

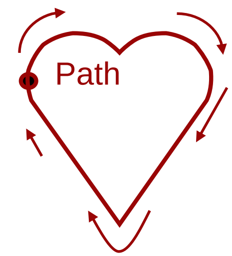
1. the work done by F in any round trip motion of an object is zero

$\Delta W_F = 0$

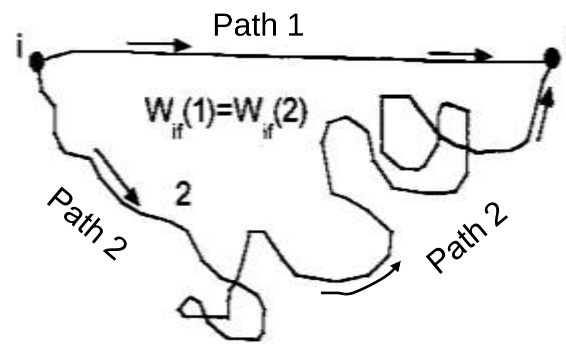


2. the change in kinetic energy, K, caused by F in any round trip motion of an object is zero.

$\Delta K_F = 0$



3. the work done by F when an object moves from an initial point to a final point depends only on these two points and not on the path taken between them.



conservative force examples:

- Gravity - Electric fields - The force from a spring

Potential Energy & Conservation of Energy

Work Energy Theorem

$$\Delta K = W_{if}$$

Now define change in Potential Energy

$$\Delta U = -W_{if} \quad *$$

for conservative forces

$$\Delta K - W_{if} = 0$$

$$\boxed{\Delta K + \Delta U = 0} \quad *$$

kinetic energy change

potential energy change

or $\boxed{K + U = E}$

kinetic energy

potential energy

total mechanical energy

$E_{\text{conserved}}^{*}$

Energy conservation

$$E_i = E_f$$

initial & final positions/times

$$E = K + U \qquad E_i = E_f$$

Energy conservation

In the case of gravitational force/potential (near the earth's surface)

$$U = mg\,(y-y_o)$$

$$E = \frac{1}{2}mv^2 + mg\,(y - y_o)$$

kinetic energy change

potential energy change

y_o is some arbitrary position where U=0

Can choose zero of potential anywhere one wants
- once choice made keep same

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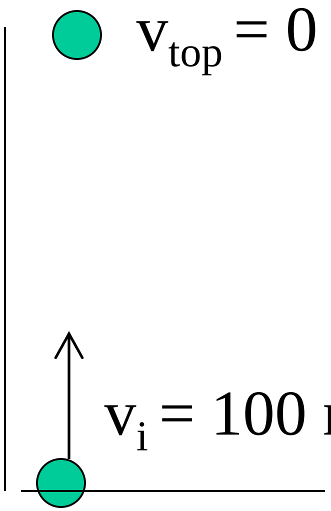
Example

Throw an object up – how high does it go?

$$E_i = E_f \Longrightarrow \frac{1}{2}mv^2 = mgh \Longrightarrow \boxed{h = \frac{v_i^{\ 2}}{2g}}$$

$$h = \frac{100^2}{2(9.8)} \quad \frac{\left(\frac{m}{s}\right)^2}{\left(\frac{m}{s^2}\right)}$$



y= h

$v_{top} = 0$: know

$$E_f = 0 + mgh$$

$$h = 510 \text{ m}$$

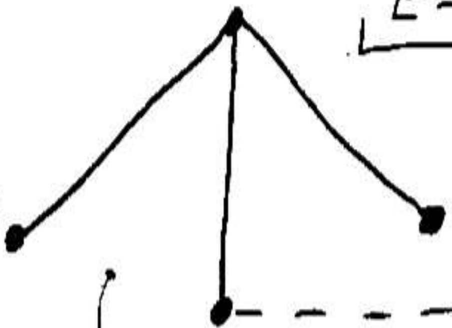
$v_i = 100$ m/s

$$E_i = \frac{1}{2}mv_i^{\ 2} + 0$$

U = 0

y = 0

Pendulum



$$\underbrace{E = \frac{1}{2} m v^2 + m g h}$$

$h=0,\ U=0$

top of swing

$V=0$

$KE=0$

$\underline{E = U}$

all PE

at bottom

$h=0$, $U=0$

PE

$$\underbrace{E = \frac{1}{2} m v_b^2 = KE}$$

any point $E = \frac{1}{2} m v^2 + m g h$

$E = KE + PE$

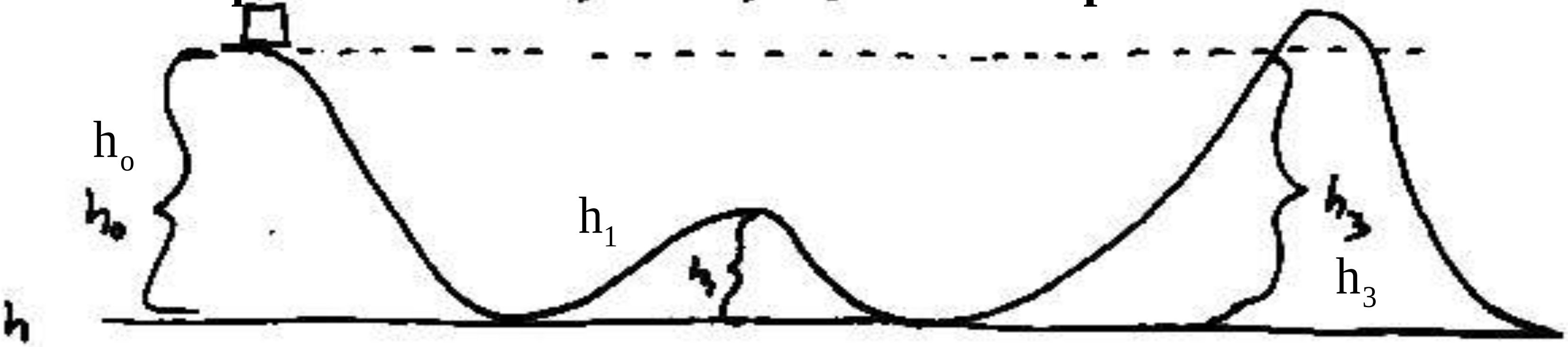
Energy transferred back & forth but tot Energy constant

$$E_{bottom} = \frac{1}{2} m v_b^2 = E_{top} = m g h_{top}$$

$$\Longrightarrow \quad \underbrace{\frac{1}{2} m v_b^2 = m g h_{top}}$$

$$\boxed{V_b = \sqrt{2 g h_{top}}}$$

Example roller coaster – starts from rest at top of 1st hill



$$E_i = \underbrace{\frac{1}{2}mv_o^2}_{\mathbf{0}} + mg\,h_o$$

$$\Downarrow$$

$$E_i = mg\,h_o$$

$$E_1 = \frac{1}{2}mv_1^2 + mg\,h_1$$

$$E_3 = \frac{1}{2}mv_3^2 + mg\,h_3$$

but $h_3 = h_0$

$$\Downarrow$$

$$E_3 = \frac{1}{2}mv_3^2 + mg\,h_0$$

$$E_i = E_1$$

$$\Downarrow$$

$$mg\,h_o = \frac{1}{2}mv_1^2 + mg\,h_1$$

$$\Downarrow$$

$$\frac{1}{2}mv_1^2 = mg\,(h_1 - h_o)$$

$$\Downarrow$$

$$v_1 = \sqrt{2g\,(h_1 - h_o)}$$

$$E_1 = E_3$$

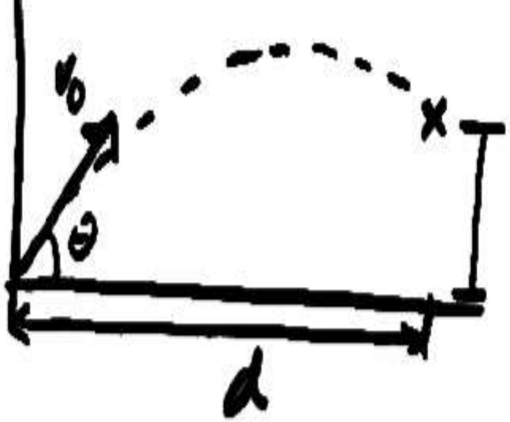
$$\Downarrow$$

$$mg\,h_0 = \frac{1}{2}mv_3^2 + mg\,h_0$$

$$\Downarrow$$

$$0 = \frac{1}{2}mv_3^2 \quad \Rightarrow \quad 0 = v_3$$

Object comes to rest before top of 3rd hill (when h=h_0)



$h = ?$ ← ①

V at h is ? ②

$y = v_0 \sin\theta\, t - \frac{g}{2} t^2$ (a) $\quad v_y = v_0 \sin\theta - g t$ (c)

$x = v_0 \cos\theta\, t$ (b) $\quad v_x = v_0 \cos\theta$ (d)

$$d = v_0 \cos\theta\, t \Rightarrow t = \frac{d}{v_0 \cos\theta}$$

$$\therefore\ y = v_0 \sin\theta \frac{d}{v_0 \cos\theta} - \frac{g}{2}\left(\frac{d}{v_0 \cos\theta}\right)^2$$

$$h = y = d\frac{\sin\theta}{\cos\theta} - \frac{g}{2 v_0^2}\frac{d^2}{\cos^2\theta}$$

can find $\vec{v}$ with (c) and (d)

or by energy

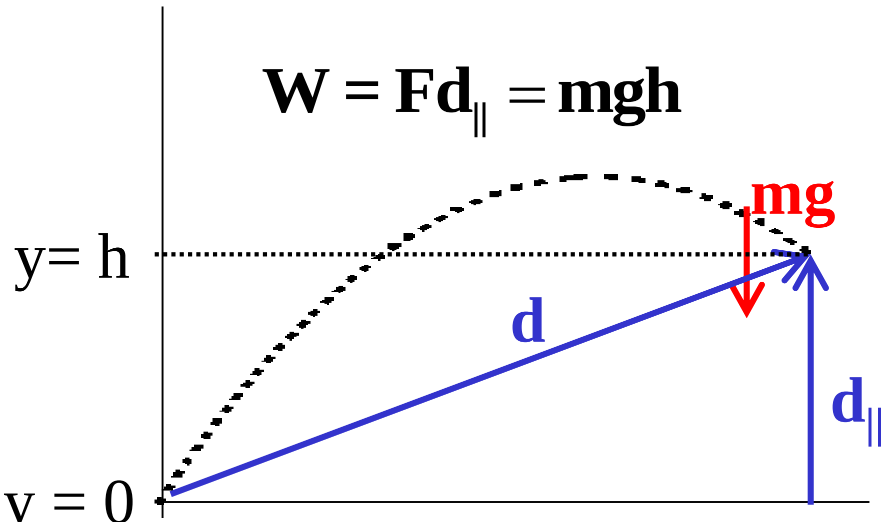
$$E_i = E_f$$

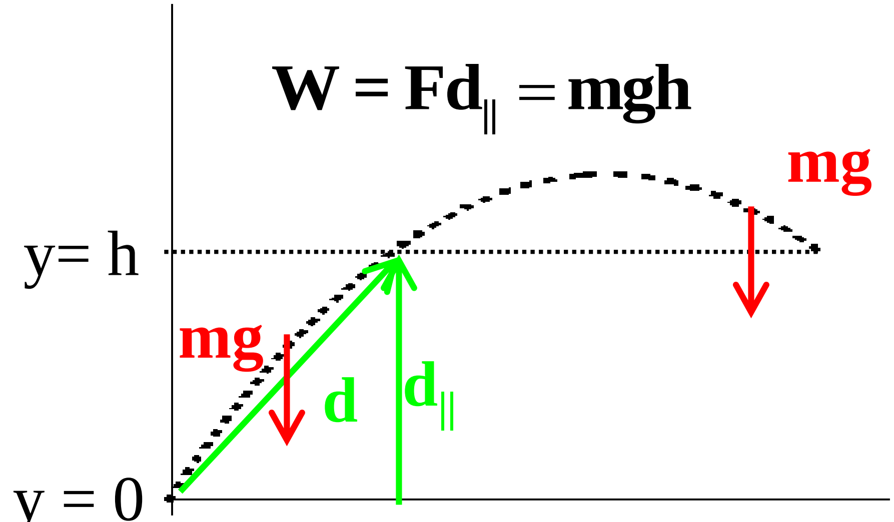
$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + m g h$$

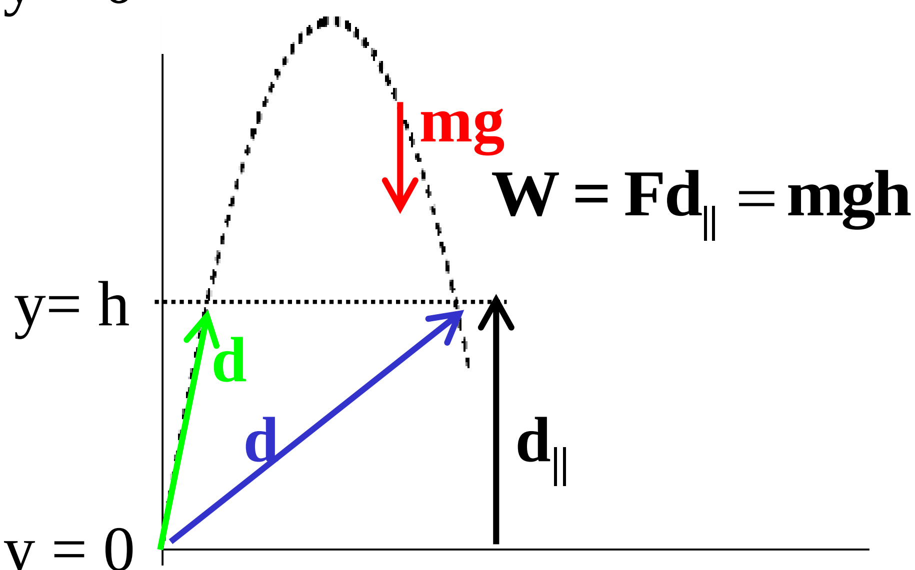
$$v = \sqrt{v_0^2 - 2 g h}$$

Energy conservation fast way to solve for h vs v

Ball thrown upward near surface of the earth







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All these cases have the same the $d_{\parallel}$=h and the work done by gravity is the same = mgh !!!!

In all these cases (by the W-E theorem) the change in kinetic energy

ΔK=mgh !!!!

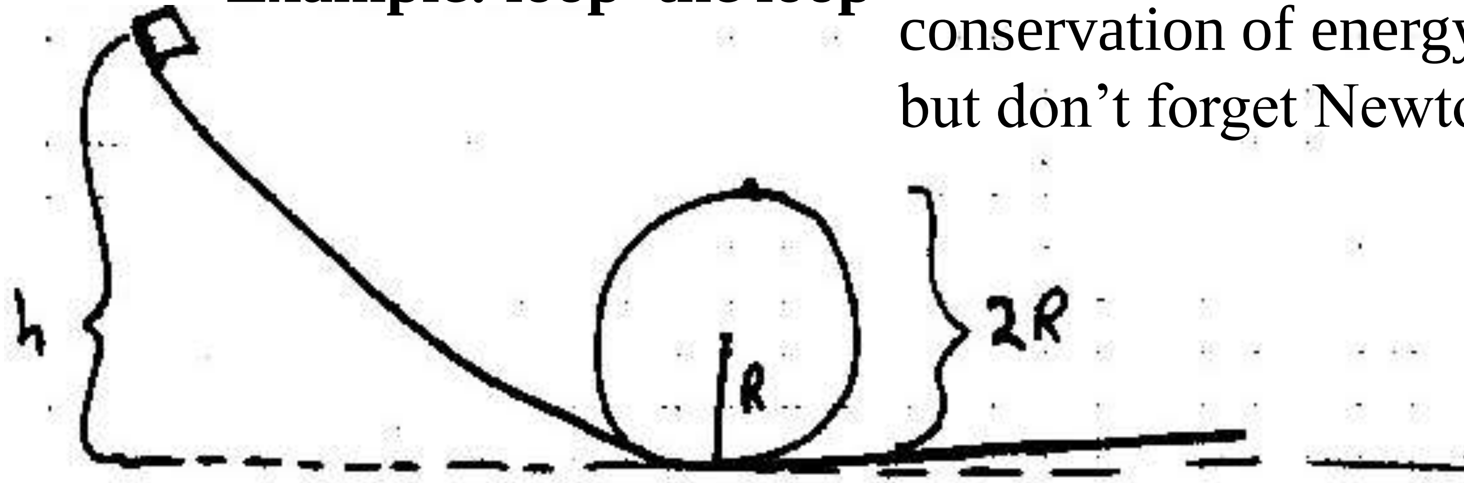
Indeed, in every case you invent the work done by gravity will depend only on the vertical change in height (h) and **ΔK=mgh** !!!!

Gravity is a very dependable

conservative force

Example: loop the loop

conservation of energy is great
but don't forget Newton's Law's !!!



$$E_i = mgh \qquad E_f = \tfrac{1}{2}mv^2 + mg(2R)$$

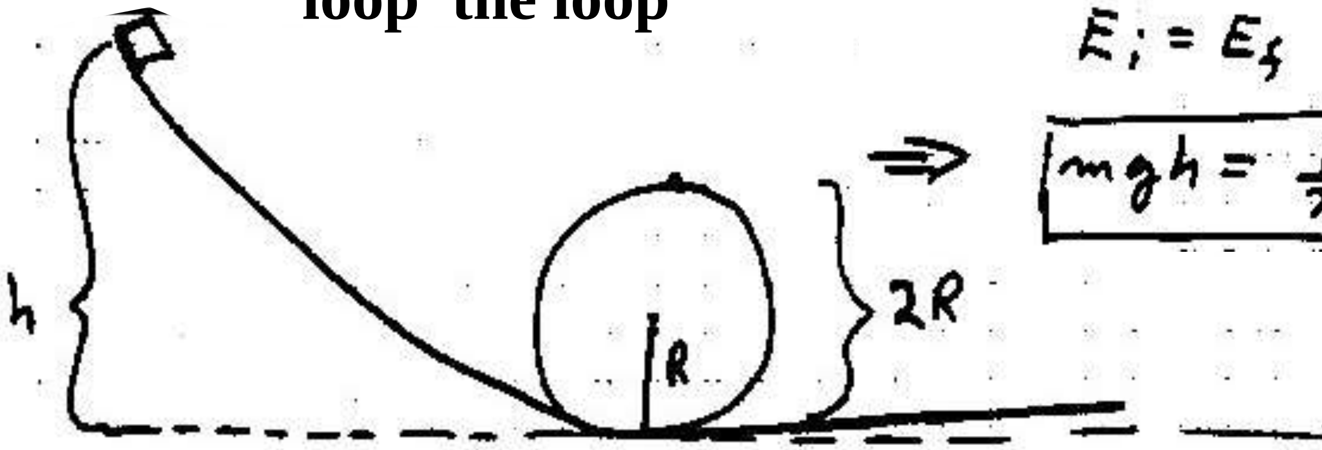
$$E_i = E_f$$

$$\Rightarrow \boxed{mgh = \tfrac{1}{2}mv^2 + mg\,2R}$$

Q. How high does h have to be to have v go to 0 At top of loop? **(wrong question if you want to survive !!)**

A. h=2R **But you fall off before you get there !!**

loop the loop



$$E_i = E_f$$

$$\Rightarrow \boxed{mgh = \frac{1}{2} m v^2 + mg\,2R}$$

$$E_i = mgh \qquad E_f = \frac{1}{2} m v^2 + mg(2R)$$

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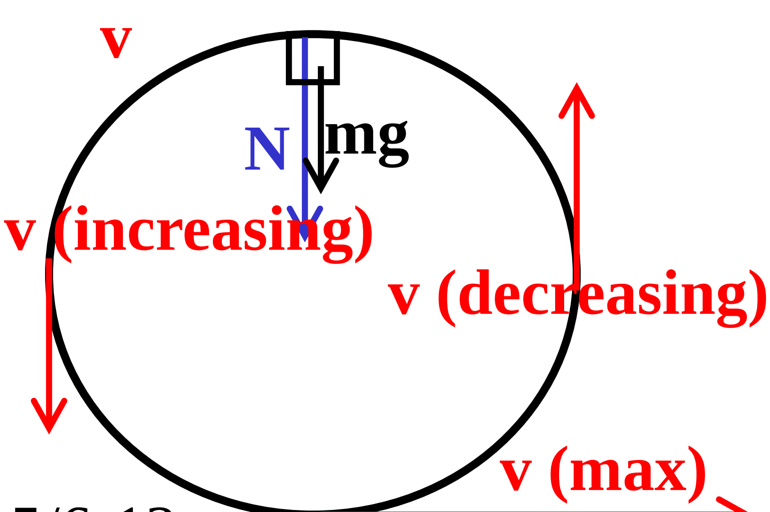
v is min at top of loop



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at top of loop

$$-N - mg = -m\frac{v^2}{R}$$

If N just goes to zero

then $mg = \frac{mv^2}{R} \Rightarrow \boxed{v^2 = gR}$

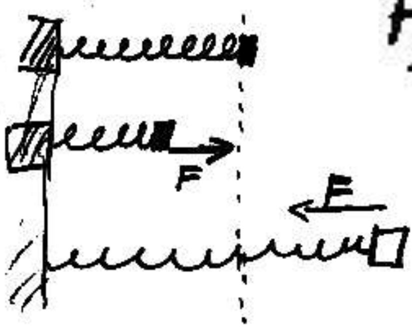
$$mgh = \frac{1}{2} m\, gR + mg\,2R$$

$$\Rightarrow \boxed{h = \frac{1}{2}R + 2R = \frac{5}{2}R}$$

Another Conservative Force: A spring

Spring: Energy Considerations

$x = 0 \leftarrow$ the equilibrium (no strech, no compression)



$F_{spring} = -kx$ Hooks Law

compressed

extended

Always acts to restore position to $x=0$

- **Force restores to equilibrium (x=0)**

- **the larger the displacement the larger the restoring force**

$$\mathbf{F = -kx}$$

$$\mathbf{W = -\frac{k}{2}x^2 + \frac{k}{2}x_i^{\ 2}}$$

$$\mathbf{U = \frac{1}{2}kx^2}$$

U=0 at x=0, i.e. $x_i = 0$

$$\mathbf{E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2}$$

$$\mathbf{W = \int_{x_i}^{x} F\,dx = \int_{x_i}^{x} -kx\,dx = -\frac{k}{2}x^2\Big|_{x_i}^{x} = -\frac{k}{2}x^2 + \frac{k}{2}x_i^{\ 2}}$$

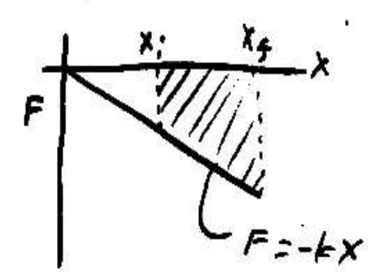
$$\mathbf{\Delta U = -W = \frac{k}{2}x^2 - \frac{k}{2}x_i^{\ 2}}$$

Recall last we we proved i.e.

$F = -kx$

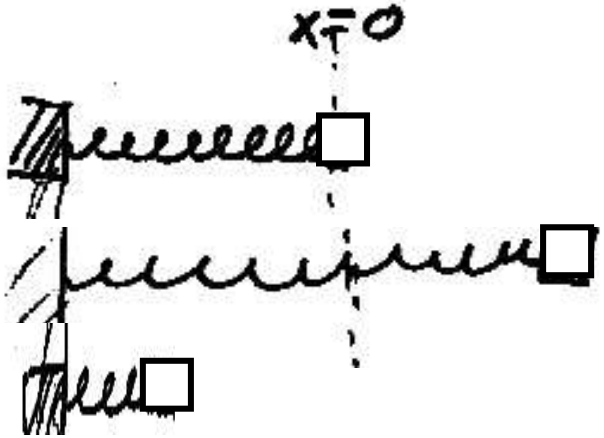
$\Downarrow$

$W_{if} = -\frac{1}{2}kx_f^2 + \frac{k}{2}x_i^2$



cyclic transfer :: kinetic energy ⇔ potential energy

$$\mathbf{E} = \frac{1}{2}\mathbf{m}\mathbf{v}^2 + \frac{1}{2}\mathbf{k}\mathbf{x}^2$$



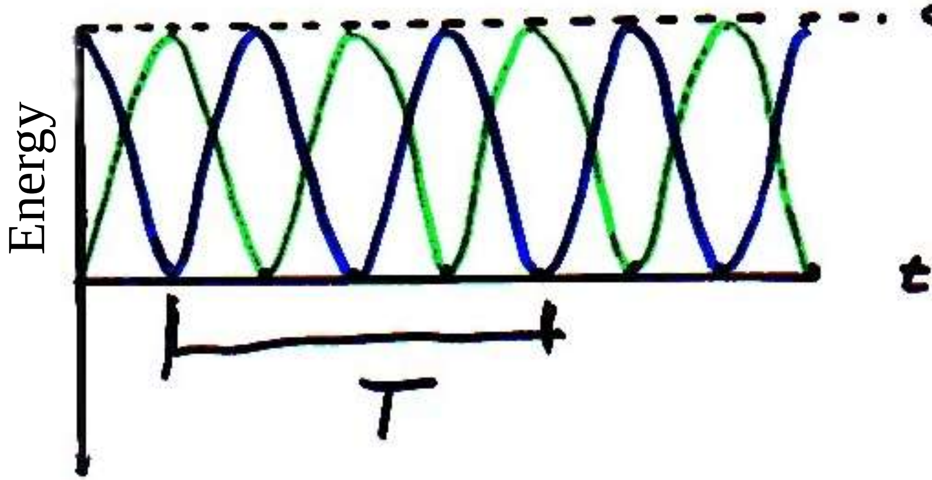
x = 0 $\mathbf{E} = \frac{1}{2}\mathbf{m}\mathbf{v}^2_{\mathbf{max}}$

x= A $\mathbf{E} = \frac{1}{2}\mathbf{k}\mathbf{A}^2$

x= -A $\mathbf{E} = \frac{1}{2}\mathbf{k}\mathbf{A}^2$

$$\frac{1}{2}\mathbf{k}\mathbf{A}^2 = \frac{1}{2}\mathbf{m}\mathbf{v}^2_{\mathbf{max}}$$

$$\mathbf{v}_{\mathbf{max}} = \mathbf{A}\sqrt{\frac{\mathbf{k}}{\mathbf{m}}}$$



$\leftarrow E = \frac{1}{2} k A^2$

$= \frac{1}{2} m \omega^2 A^2$

$$E = \underbrace{\frac{1}{2} m v^2}_{\text{K.E.}} + \underbrace{\frac{1}{2} k x^2}_{\text{Pot. Energy}}$$

Now include non-conservative forces in Work Energy Theorem

Work Energy Theorem

$$\underbrace{W_1 + W_2 + \ldots + W_n}_{\text{conservative forces}} + \underbrace{W_{nc}}_{\text{Work done by non-conservative force}} = \Delta K \quad \text{change in KE}$$

$$\underbrace{- \Delta U_1 - \Delta U_2 - \ldots - \Delta U_n}_{\text{changes in potential energies associated with conservative forces}} + W_{nc} = \Delta K$$

non-conservative forces

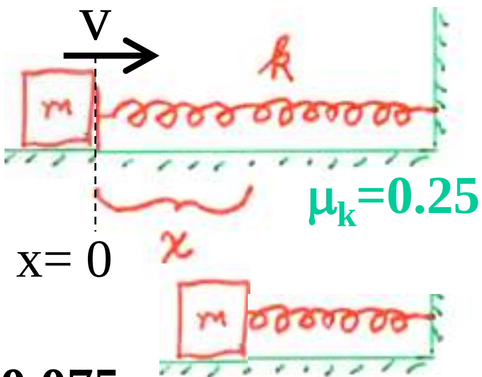
- kinetic friction (-work)
- rocket, explosion, kick (+work)

$$W_{nc} = \Delta K + \Delta U_1 + \Delta U_2 + \ldots + \Delta U_n$$

Work done by non-conservative forces $\boxed{W_{nc} = \Delta E}$ change in total mechanical energy

non-conservative force example

$m = 2.5$ kG $k = 320$ N/m



$x_f = 0.075$ m

v= ?

$$E_i = U_i + K_i = 0 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2$$

$$E_f = U_f + K_f = \frac{1}{2}kx^2 + 0 = \frac{1}{2}kx_f^2$$

$$W_f = E_f - E_i$$

NC friction,
- work

(displacement)

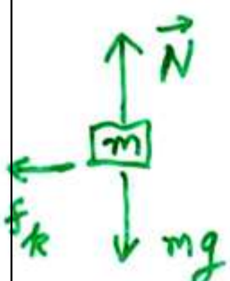
[force]

$$[-\mu_k mg](x_f) = \frac{1}{2}kx_f^2 - \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx_f^2 + \mu_k mg\, x$$

$$v = \sqrt{\frac{k}{m}x_f^2 + 2\mu_k g\, x_f} = \sqrt{\frac{320}{2.5}(0.075)^2 + 2(.25)9.8(0.075)}\ \frac{m}{s}$$

$$v = 1.04\ \frac{m}{s}$$



$N - mg = 0 \Rightarrow N = mg.$

SO FRICTION $f_k = \mu_k N = \mu_k mg.$

$$W_f = -\mu_k mg\, x$$

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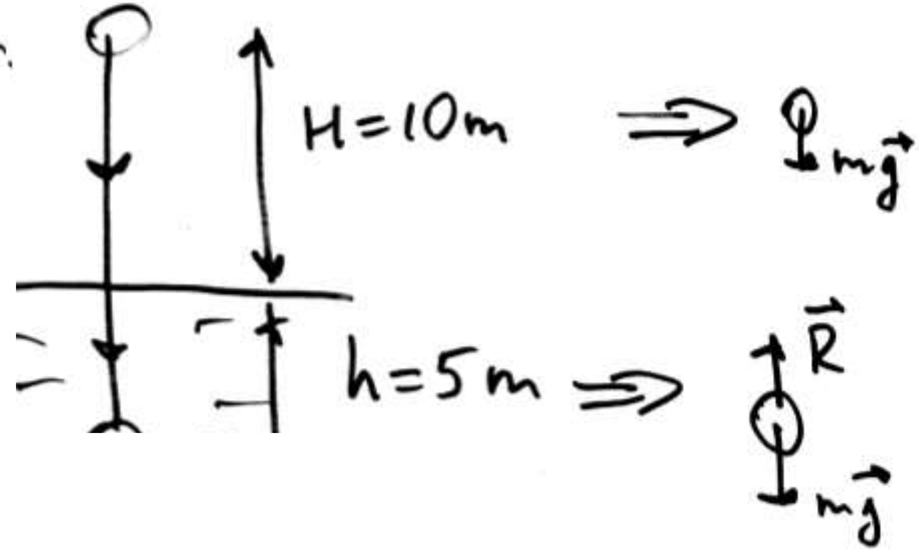
non-conservative force example

Example. A 70 kg diver steps off a 10 m tower and drops straight down into the water. If he comes to rest 5.00 m beneath the surface of the water, determine the average resistance force that the water exerts on the diver.



$$\sum W_{non} = E_f - E_i$$

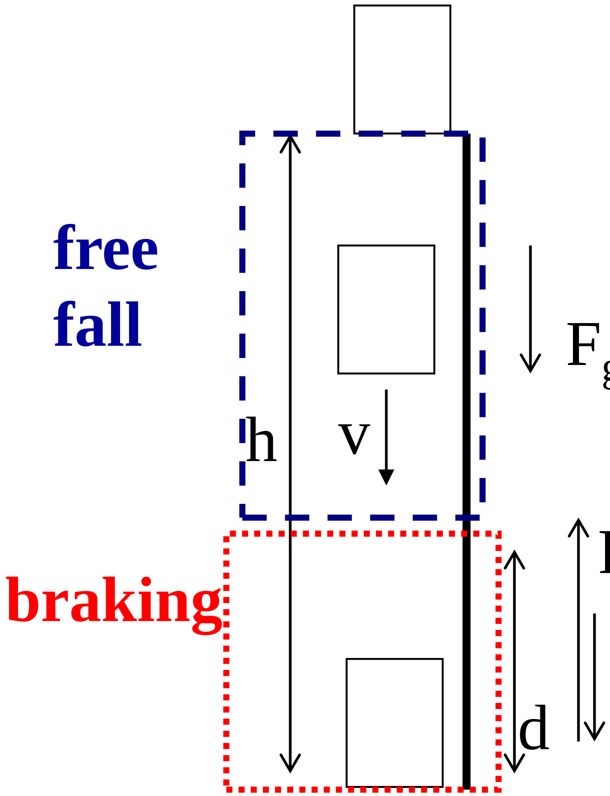
$$\left.\begin{array}{l}\Delta U = mg\,\Delta y = -mg\,(H+h) \\ \Delta K = 0\end{array}\right\} \rightarrow \Delta E = -mg(h+H)$$

$$W_{non} = -Rh \qquad \text{(force * displacement * cos } \theta\text{)}$$

So, $\quad Rh = mg(h+H)$

$$R = \frac{mg(H+h)}{h} = 70 \cdot 9.8 \cdot \frac{10+5}{5}\ N = 2.1\ kN$$

Consider an elevator that falls from a height h, but which is brought to rest by the brakes just as it reaches the ground



$W_f = \Delta E = mgh$

$F_g = mg$

Velocity before brake
$\frac{1}{2}mv^2 = mg(h-d)$

<u>if F_f only acts over d then</u>

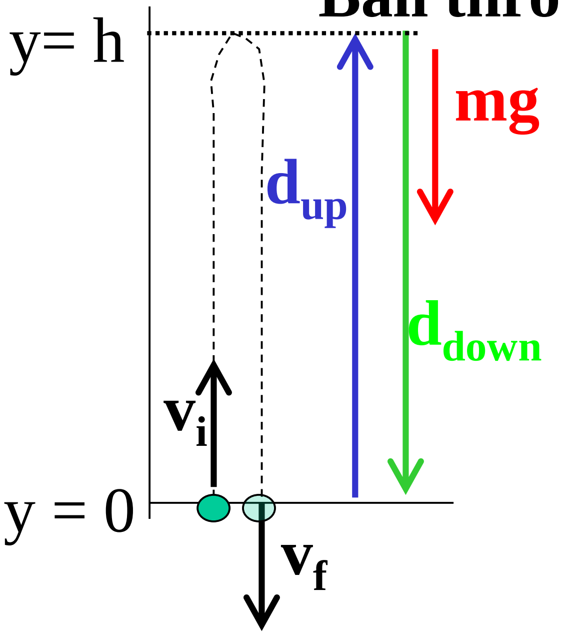
F_f = frictional braking force

$F_g = mg$

$W_f = \Delta E = mgh = F_f\, d$

$F_f = mgh/d$

Ball thrown upward with v_0



Recall

$$\boxed{W = F_{\parallel} d = \frac{1}{2} m v_f^{\,2} - \frac{1}{2} m v_i^{\,2}}$$

$$\boxed{W = \Delta K}$$

$$W_{up} = -mg\,h = -\frac{1}{2} m v_0^{\,2}$$

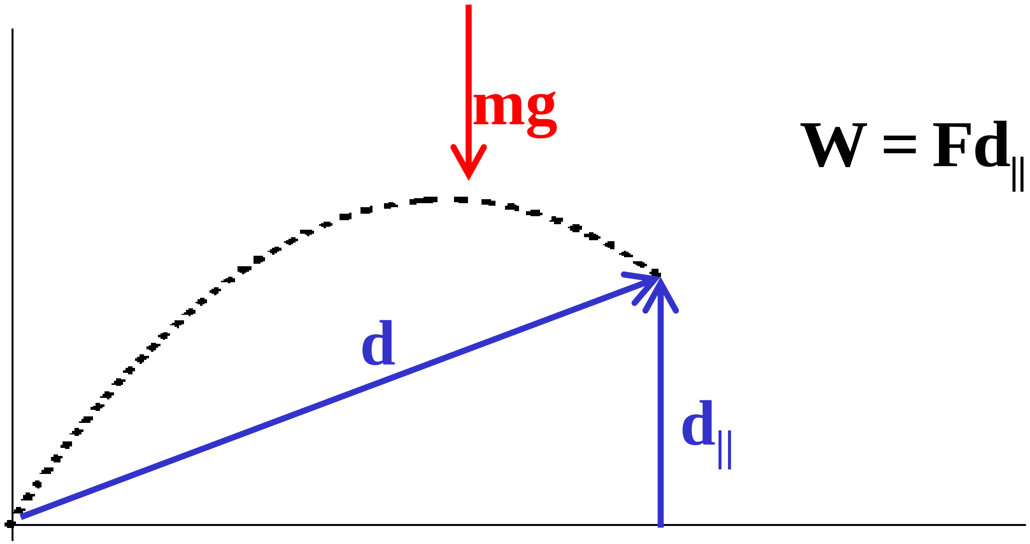
$$W_{down} = +mg\,h = \frac{1}{2} m v_f^{\,2}$$

$$W_{round\ trip} = W_{up} + W_{down} = 0$$

$$\Rightarrow \Delta K_{\ round\ trip} = 0$$

Also follows from energy conservation v_0

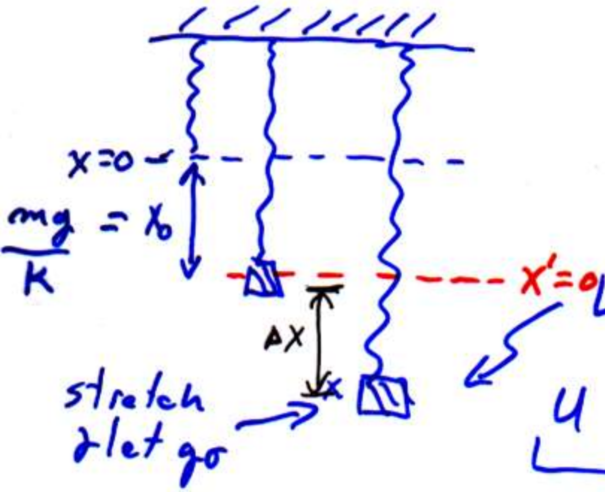
$$\Rightarrow \quad v_f = v_0$$



$$W = F d_{\parallel}$$

A note on a why a weight hanging vertically from a spring can be treated like one on a frictionless surface.

motion about new equlibrium position



start $x=0$ $\quad U=0$

$$U = U_{grav} + U_{spring}$$

$$U = \frac{1}{2}kx^2 + mgx$$

$$U = \frac{1}{2}k\left[x^2 + \underbrace{\frac{2mg}{k}}_{2x_0\ !!}x\right]$$

$$\therefore \quad U = \frac{k}{2}\left[x^2 + 2x_0x \underbrace{+x_0^2 - x_0^2}_{0}\right] \quad \text{complete square} = \frac{k}{2}\left[(x+x_0)^2 - x_0^2\right]$$

Appendix

$$U = \frac{1}{2}k(x+x_0)^2 \underbrace{- \frac{1}{2}kx_0^2}_{\text{constant}}$$

$$U' = U + \frac{1}{2}kx_0^2$$

shift Pot. Energy by constant

$$\boxed{U' = \frac{1}{2}kx'^2} \quad \boxed{x' = x + x_0}$$

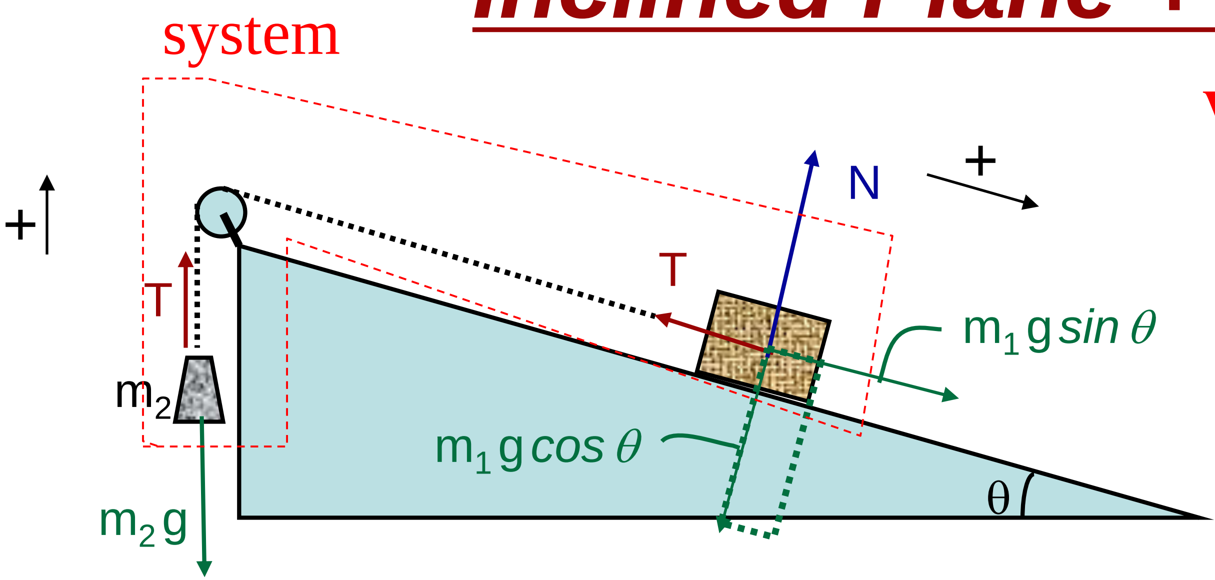
$U'=0$ at $x' = x + x_0 = 0$

$\Delta x = \Delta x'$ $\quad$ gravity included!

Include gravity $\Leftrightarrow$ just shift O

Inclined Plane + a Pulley

What is "a" of objects?



T is an internal force

⊥ to plane

$N - m_1 g \cos\theta = 0$

system F = ma (along the direction of potential motion)

$$-m_2 g + m_1 g \sin\theta = (m_2 + m_1)a$$

Tot external force on system

Tot mass of system

$$\therefore \quad a = g\frac{m_1 \sin\theta - m_2}{m_1 + m_2}$$

Review peoblem