Circular Motion

Notes: \( \vec{F} \) and \( \vec{a} \) always point toward the center. \( \vec{a} \), \( \vec{F} \), \( \vec{V} \) all have constant magnitude, but change in direction.
Similar Triangles

\[
\frac{\Delta r}{r_0} = \frac{\Delta v}{v_0}
\]

but \( \Delta r = v_0 \Delta t \) and \( \Delta v = a \Delta t \)

\[
\therefore \quad \frac{v_0}{r_0} = \frac{a \Delta t}{v_0}
\]

\[
\Rightarrow \quad \frac{v_0^2}{r_0} = a
\]
Car on Flat Curve (static friction holds car on curve)
Important: want to find maximum v (car) and minimum R (road).
⇒ maximum static frictional force needed (no skidding) ⇒ $f_{\text{fric}} = \mu_s N$

\[ \begin{align*}
\text{\perp direction} & & \text{\perp direction} \\
\vec{N} - mg = 0 & & f = \mu_s N = \mu mg = ma \\
\mu_s = \text{Coef. of friction} & & \therefore \quad \mu_s mg = m \frac{V^2}{R} \\
\end{align*} \]

How to build a road
How to set speed limit
\[ F = m \frac{v^2}{r} \]

Only force is \( T \) so
\[ T = m \frac{v^2}{r} \]

But \( T = Mg \)

so \( Mg = m \frac{v^2}{r} \)

\[ v^2 = \frac{Mgr}{m} \]

\[ v = \sqrt{\frac{Mgr}{m}} \]
Centripetal Force
$\vec{F}_c$ (a real force)
The force that makes motion circular.

“Centrifugal Force”
Nonexistent/Pseudo/Fictitious

Appears in accelerating (circular) reference frame. The apparent force is due to the car’s frame being non-inertial. (It’s accelerating.) The only real force on the car is the inward centripetal force.
Non uniform circular motion – motion in vertical circle

\[ F_r = -m \frac{v^2}{r} = -mg - T \]

\[ F_r = m \frac{v^2}{r} = -mg + T \]
Non uniform circular motion – motion in vertical circle

\[ F_r = m \frac{v^2}{r} = T \]

\[ F_\perp = -mg \]

\[ F_r = -m \frac{v^2}{r} = -T \]

\[ F_\perp = -mg \]
Car on banked Curve \((N_x \text{ holds car on curve, no friction needed})\)

\[
\begin{align*}
\sum F_y &= 0 \quad \Rightarrow \quad N \cos \theta - mg = 0 \\
\Rightarrow N &= \frac{mg}{\cos(\theta)}
\end{align*}
\]

\[
\begin{align*}
F_x &= N_x = \frac{mV^2}{R} \\
\Rightarrow N \sin \theta &= \frac{mV^2}{R}
\end{align*}
\]

\[
mg \sin \theta / \cos \theta = \frac{mV^2}{R}
\]

\[
\tan \theta = \frac{V^2}{gR}
\]