Circular Motion

$r = \text{radius}$
$v = \text{velocity const. magnitude}
\alpha = \text{const. magnitude}

circ. of circle = $2\pi r$

$T = \text{time for 1 revolution = period}$

$v = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T} = v$

$F = \frac{m v^2}{r}$

Notes: $\overrightarrow{F}$ and $\overrightarrow{a}$ always point toward the center. $\overrightarrow{a}, \overrightarrow{F}, \overrightarrow{V}$ all have constant magnitude, but change in direction.
Similar Triangles

\[ \frac{\Delta r}{r_0} = \frac{\Delta v}{v_0} \]

but \( \Delta r = v_0 \Delta t \) and \( \Delta v = a \Delta t \)

\[ \therefore \quad \frac{v_0 \Delta t}{r_0} = \frac{a \Delta t}{v_0} \]

\[ \Rightarrow \quad \frac{v_0^2}{r_0} = a \]
Car on Flat Curve (static friction holds car on curve)

Important: want to find maximum $v$ (car) and minimum $R$ (road).

$\Rightarrow$ maximum static frictional force needed (no skidding) $\Rightarrow f_{\text{fric}} = \mu_s N$

$\begin{align*}
\text{Side View} & \quad \vec{a} \quad \vec{N} \\
\text{Top View} & \quad \vec{V} \quad F
\end{align*}$

$\begin{align*}
\begin{cases}
\text{⊥ direction} & \quad \vec{N} - mg = 0 \\
\mu_s = \text{Coef. of friction} \\
\text{∥ direction} & \quad f = \mu_s N = \mu mg = ma \\
\text{but } a = \frac{V^2}{R} \\
\therefore \quad \mu_s mg = m \frac{V^2}{R}
\end{cases}
\end{align*}$

\[
\begin{align*}
\mu_s g &= \frac{V^2}{R} \\
\therefore \quad V &= \sqrt{R \mu_s g} \\
\therefore \quad R &= \frac{V^2}{\mu_s g}
\end{align*}

How to set speed limit
How to build a road
\[ F = m \frac{v^2}{r} \]

Only force is \( T \) so

\[ T = m \frac{v^2}{r} \]

But \( T = Mg \)

so \( Mg = m \frac{v^2}{r} \)

\[ v^2 = \frac{Mgr}{m} \]

\[ v = \sqrt{\frac{Mgr}{m}} \]
Centripetal Force
$\vec{F}_c$ (a real force)
The force that makes motion circular.

“Centrifugal Force”
Nonexistent/Pseudo/Fictitious

Appears in accelerating (circular) reference frame. The apparent force is due to the car’s frame being non-inertial. (It’s accelerating.) The only real force on the car is the inward centripetal force.
Non uniform circular motion – motion in vertical circle

\[ F_r = -m \frac{v^2}{r} = -mg - T \]

\[ F_r = m \frac{v^2}{r} = -mg + T \]
Non uniform circular motion – motion in vertical circle

\[ F_r = m \frac{v^2}{r} = T \]

\[ F_\perp = -mg \]

\[ F_r = -m \frac{v^2}{r} = -T \]

\[ F_\perp = -mg \]
Car on banked Curve \((N_x)\) holds car on curve, no friction needed

- **Top View**
  - \(F_{\text{NET}} = N_x\)
  - \(N_x = N \sin \theta\)
  - \(N_y = N \cos \theta\)
  - \(\theta\) is the degree of banking
  - \(w = -mg\)

- **Side View**
  - \(\sum F_y = 0 \Rightarrow N \cos \theta - mg = 0\)
  - \(\Rightarrow N = \frac{mg}{\cos(\theta)}\)
  - \(F_x = N_x = mV^2/R\)
  - \(N \sin \theta = mV^2/R\)
  - \(\Rightarrow mg \sin \theta / \cos \theta = mV^2/R\)
  - \(\tan \theta = V^2 / g R\)