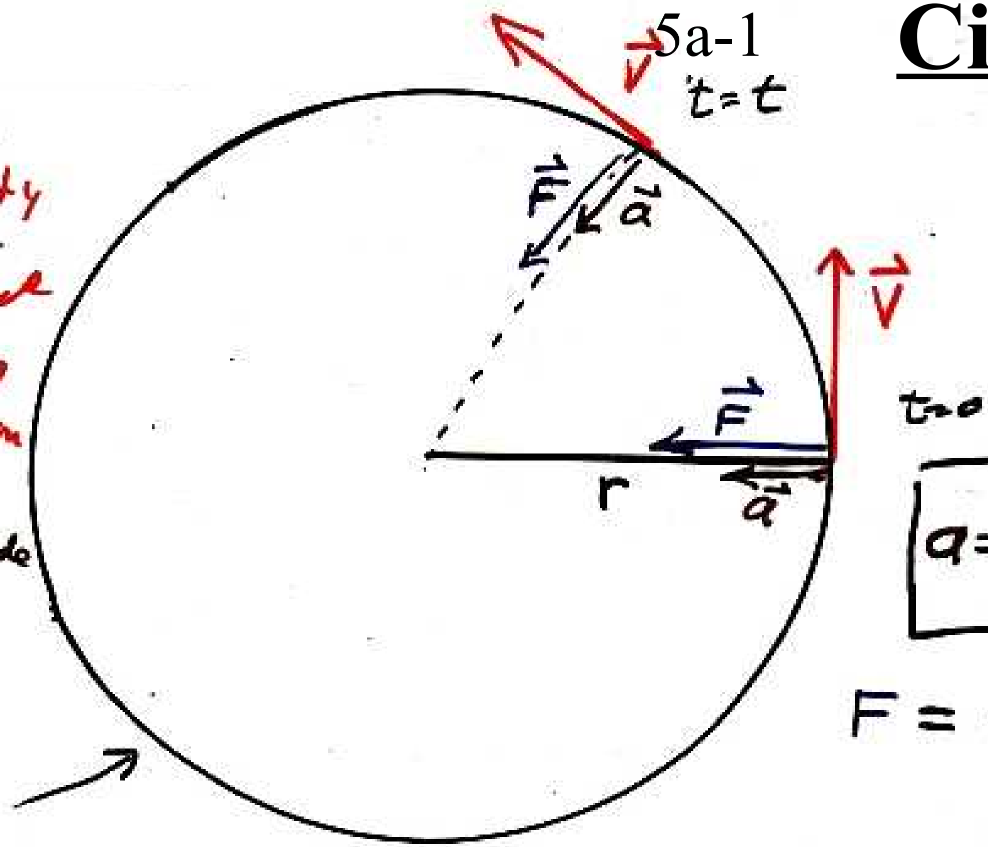
Circular Motion

r = radius

v = velocity const. magnitude changing direction

a = const. magnitude



$$a = \frac{v^2}{r}$$

$$F = \frac{m v^2}{r}$$

circ. of circle = $2\pi r$

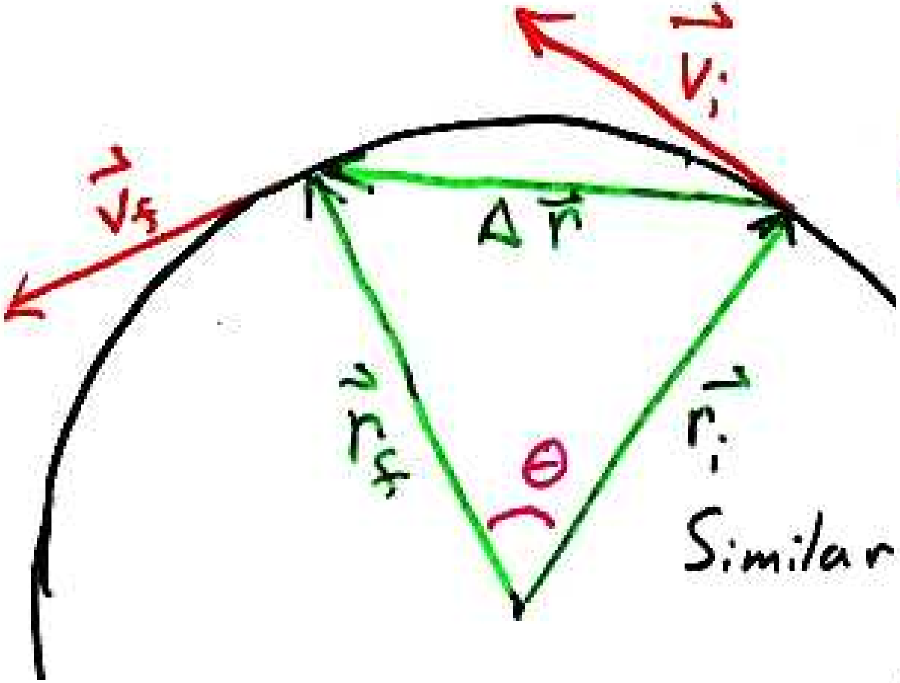
T = time for 1 revolution = period

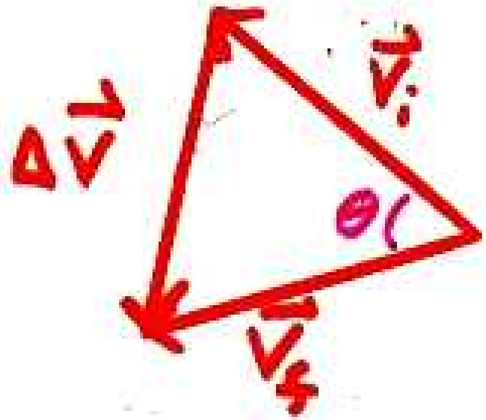
$$V = \frac{\text{distance}}{\text{time}} = \frac{2\pi r}{T} = V$$

$$= \frac{\text{circ.}}{\text{period}}$$

Notes: $\vec{F}$ and $\vec{a}$ always point toward the center.

$\vec{a}$ $\vec{F}$ $\vec{V}$ all have constant magnitude, but change in direction.





$|\vec{v}_i| = v_0$

Similar Triangles

$$\frac{\Delta r}{r_0} = \frac{\Delta v}{v_0}$$

but $\Delta r = v_0 \Delta t$ and $\Delta v = a \Delta t$

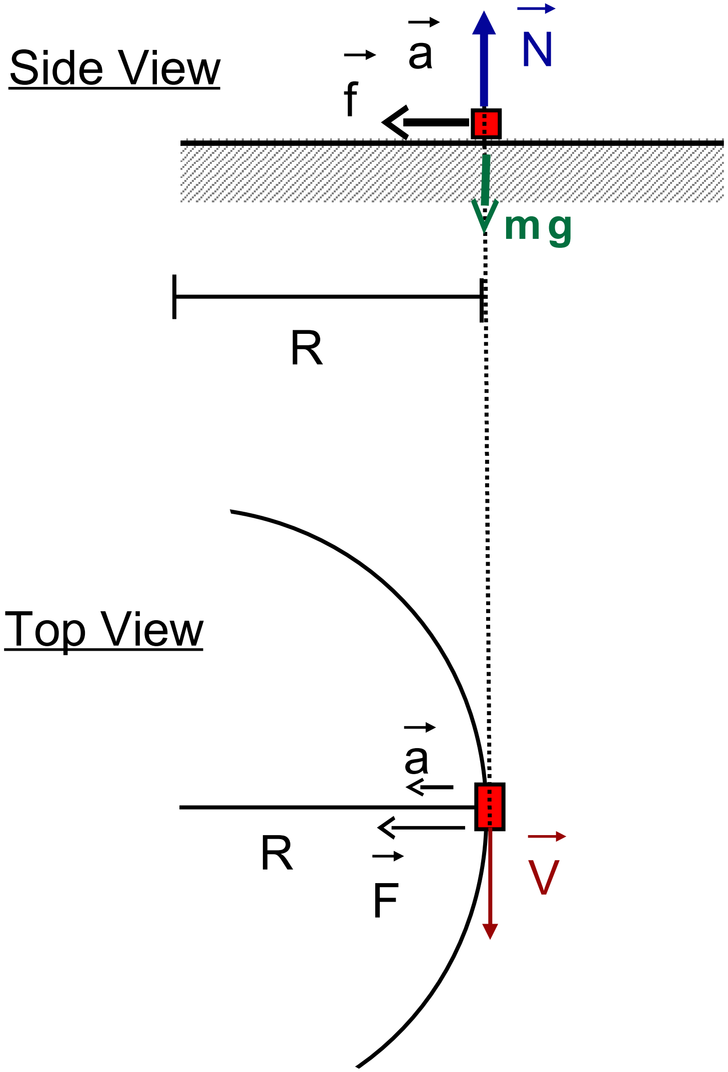
$$\therefore \quad \frac{v_0 \cancel{\Delta t}}{r_0} = \frac{a \cancel{\Delta t}}{v_0}$$

$$\Rightarrow \quad \boxed{\frac{v_0^2}{r_0} = a}$$

Car on Flat Curve (static friction holds car on curve)

Important: want to find maximum v (car) and minimum R (road).

$\Rightarrow$ maximum static frictional force needed (no skidding) $\Rightarrow f_{fric} = \mu_s N$



$\perp$ direction

$\vec{N} - mg = 0$

μ_s=Coef. of friction

|| direction

$f = \mu_s N = \mu mg = ma$

$$\text{but } a = \frac{V^2}{R}$$

$$\therefore \mu_s mg = m\frac{V^2}{R}$$

$$\mu_s g = \frac{V^2}{R}$$

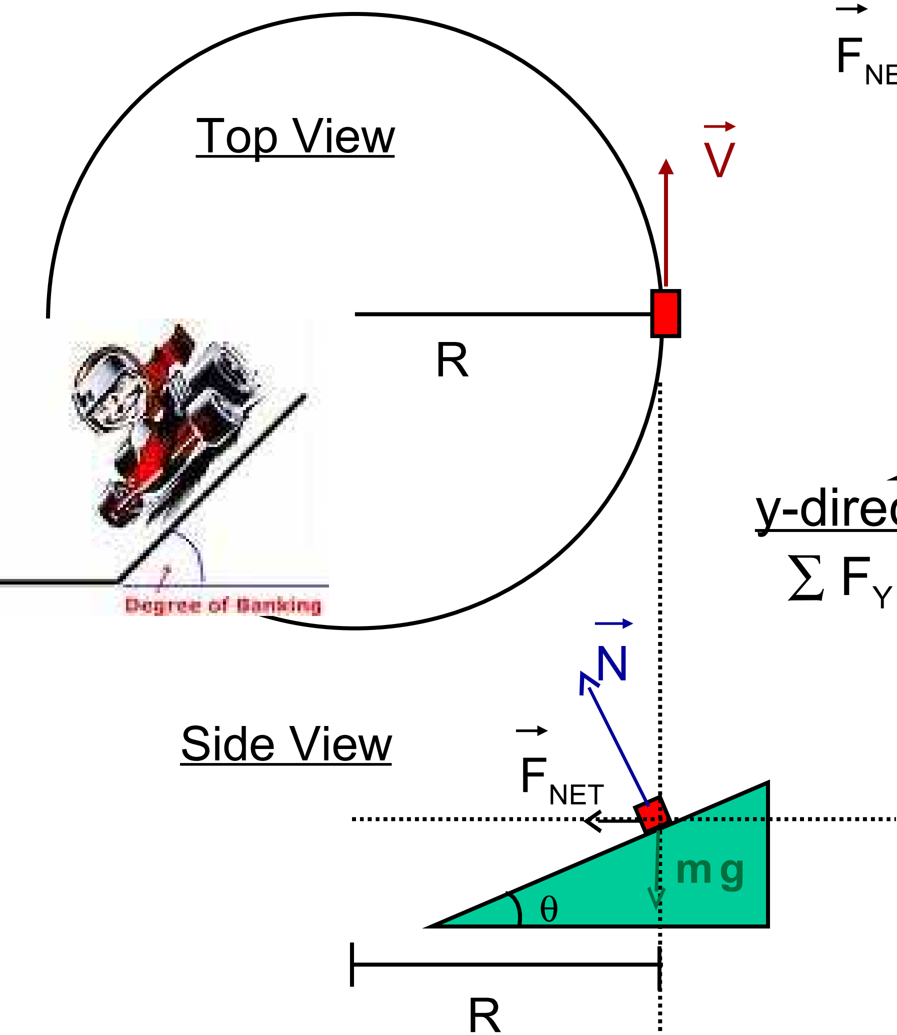
$$V = \sqrt{R\mu_s g}$$

How to set speed limit

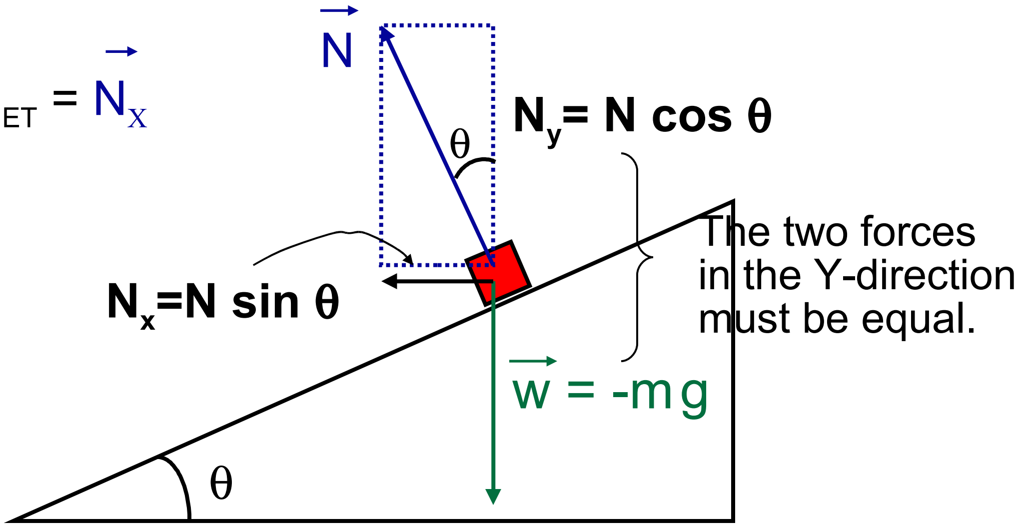
$$R = \frac{V^2}{\mu_s g}$$

How to build a road

Car on banked Curve (N_x holds car on curve, no friction needed)



$\vec{F}_{NET} = \vec{N}_X$



y-direction

$$\sum F_Y = 0 \Rightarrow N\cos\theta - mg = 0$$

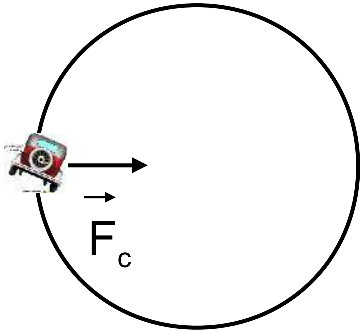
$$\Rightarrow N = mg / \cos(\theta)$$

x-direction:

$$\vec{F}_X = \vec{N}_X = mV^2/R \qquad \Rightarrow N \sin\theta = mV^2/R$$

$$mg \sin\theta / \cos\theta = mV^2/R$$

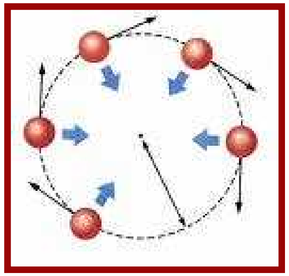
$$\tan\theta = V^2 / gR$$



Centripetal Force

$\vec{F}_c$ (a real force)

The force that makes motion circular.



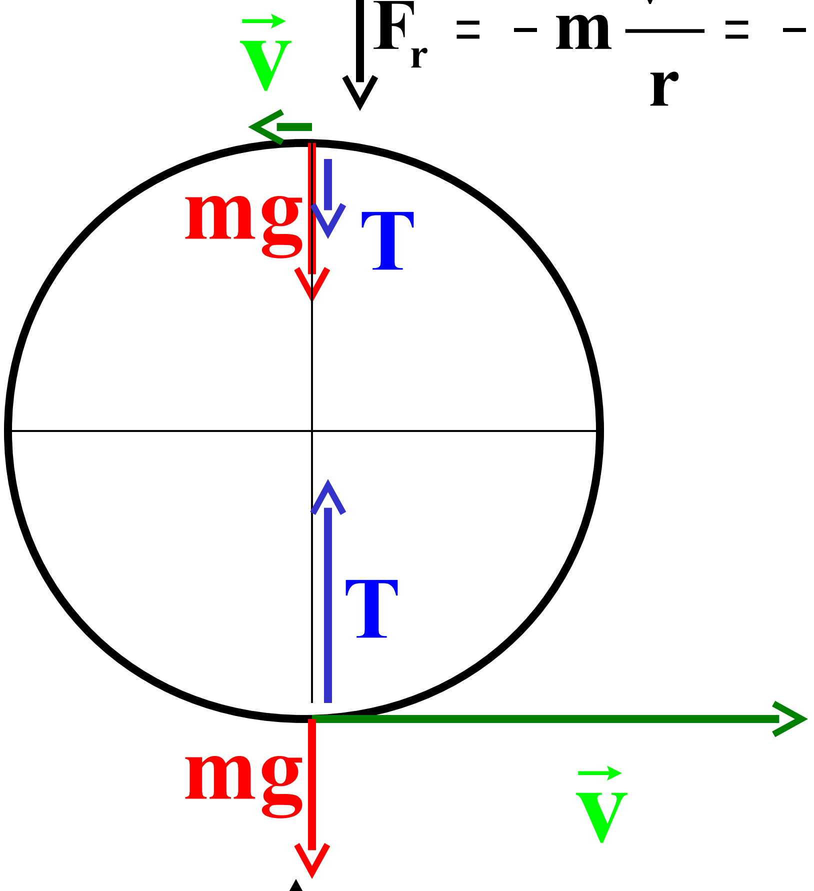
"Centrifugal Force"

Nonexistent/Pseudo/Fictitious

Appears in accelerating (circular) reference frame. The apparent force is due to the car's frame being non-inertial. (It's accelerating.) The only real force on the car is the inward centripetal force.

Non uniform circular motion – motion in vertical circle

$$F_r = -m\frac{v^2}{r} = -mg - T$$

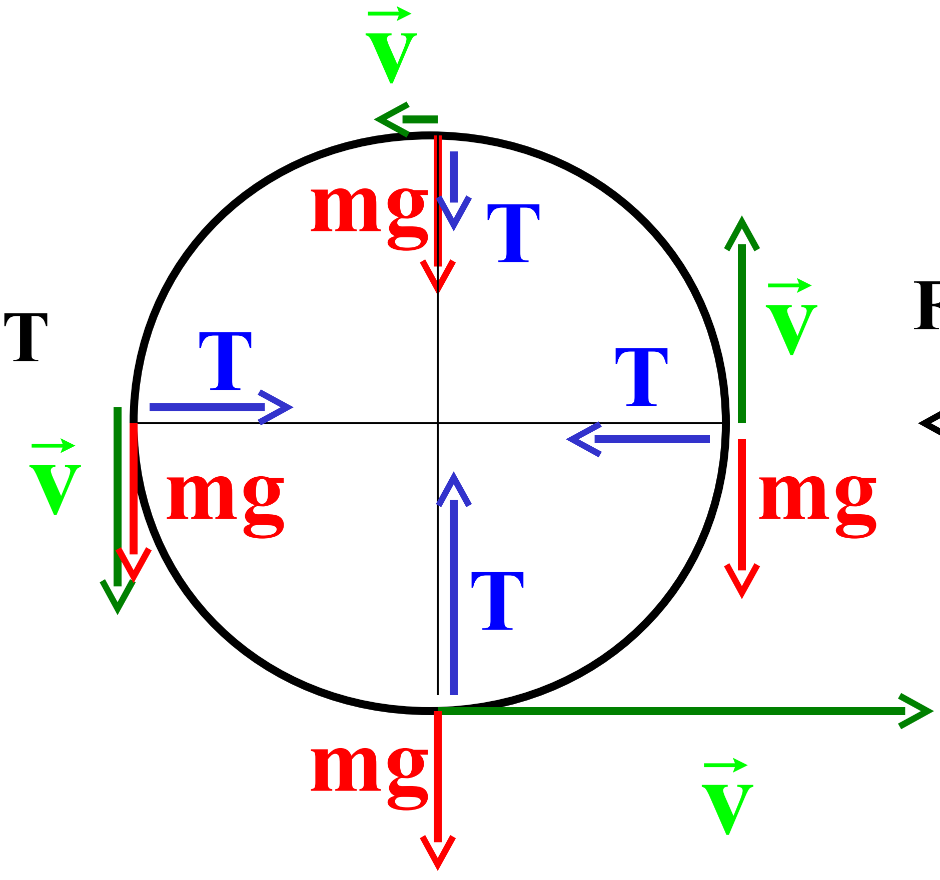


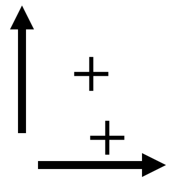
$$F_r = m\frac{v^2}{r} = -mg + T$$

Non uniform circular motion – motion in vertical circle

$$F_r = m\frac{v^2}{r} = T$$

$$F_\perp = -mg$$





$$F_r = -m\frac{v^2}{r} = -T$$

$$F_\perp = -mg$$

$$F = m\frac{v^2}{r}$$

Only force is T so

$$T = m\frac{v^2}{r}$$

But $T = Mg$

so $Mg = m\dfrac{v^2}{r}$

$$v^2 = \frac{Mgr}{m}$$

$$v = \sqrt{\frac{Mgr}{m}}$$