\[ \vec{F} = m\vec{a} \]
\[ F_x = ma_x \]
\[ F_y = ma_y \]
\[ \vec{F}_{total} = \sum \vec{F}_{individual} \]

**Free body diagram**

System choice

Sign convention choice!!

“system’” cuts at point of internal force action

CONSISTANT sign convention choice around corners!!
\[ F_{f_{(\text{max})}} = \mu_s N \]

\[ 0 \leq \mu_s < 1 \]

\[ F_f = \mu_k N \]
Newton’s 1st Law (law of inertia)
“A body at rest, or in uniform motion, tends to stay at rest or in uniform motion unless an unbalanced force acts on it.”

Newton’s 2nd Law
“An object is accelerated when an unbalanced force is applied to it. The rate of acceleration is proportional to the force and inversely proportional to the objects mass”
\[
a = \frac{F}{m} \quad \text{or} \quad F = ma
\]

Newton’s 3rd Law
“For every action (force) there is an equal and opposite reaction (force).”

Newton’s gravitational force law
1) F acts along the line connecting the centers of the objects (for spherical objects)
   “central force”
2) F magnitude
\[
F = \frac{G Mm}{r^2}
\]
\[
G = 6.67 \times 10^{-11} \text{ [m}^3/\text{kg s}^2]\]

A few basic principles allows quantitative (mathematical) understanding of a tremendous body of phenomena.
What physics is all about!!! [all fits on 1 page!!]
Newton’s 1st Law (law of inertia)
“A body at rest, or in uniform motion, tends to stay at rest or in uniform motion unless an unbalanced force acts on it.”

A body’s inertia is proportional to its mass [mass proportional to (but not=) weight]
1 kilogram = mass of 1 liter of water --- 1 gram = mass of 1 milliliter of water

uniform motion = constant velocity motion = constant speed in a straight line motion

Need to measure deviation from constant velocity motion (uniform motion)!

acceleration \[ \vec{a} = \frac{\vec{v}_{final} - \vec{v}_{initial}}{\Delta t} \]

straight line but speed changes

speed constant but direction changes
Newton’s 2\textsuperscript{nd} Law

“An object is accelerated when an unbalanced force is applied to it. The rate of acceleration is proportional to the force and inversely proportional to the objects mass.”

$$\boxed{a = \frac{F}{m}} \quad \text{or} \quad F = ma$$

Could call “mass” objects tendency to resist acceleration “ottra”.

For a given force, the bigger the mass the smaller the acceleration

\[ \begin{align*}
2N & \quad \frac{1}{2} \text{kg} \\
& \quad \Rightarrow \quad a = 4 \text{ m/s}^2 \\
& \quad \frac{\text{kgm}}{\text{s}^2} \\
2N & \quad 1 \text{kg} \\
& \quad \Rightarrow \quad a = 2 \frac{\text{m}}{\text{s}^2}
\end{align*} \]

Force of Gravity at Earth’s Surface toward the Earth’s center

\[ F = mg \quad g = 9.8 \text{ m/s}^2 \]
Object of mass $M$ at the surface of the earth.

Weight of an object, near the surface of the earth, = the gravitational force of the earth on the object

$= mg$

$$a = g = \frac{G M_e}{R_e^2}$$

all objects accelerate near earth’s surface

with $g = 9.8 \text{ m/s}^2$  \((32 \text{ ft/s}^2)\)
Newton’s 3rd Law

“For every action (force) there is an equal and opposite reaction (force).”

2 bodies (A & B) interact

Force of B on A

equal and oppositely directed

Force of A on B

man on refrig.

man on refrig.

man on refrig.

apple on Earth

apple on Earth

Moon

Table on Books

Books on table

Table
Newton's force reaction force (3\textsuperscript{rd}) Law & momentum conservation in rocket demo

Newton’s 2\textsuperscript{nd} Law

\[ F = ma \quad \text{or} \quad F = \frac{\Delta p}{\Delta t} \]

where \( p = mv \)

A+B system: no external forces \( \Rightarrow \) momentum conservation

\[ F_{\text{tot}} = F_A + F_B = 0 \quad \Rightarrow \quad \frac{\Delta p_{\text{tot}}}{\Delta t} = 0 \quad \Rightarrow \quad \Delta p_{\text{tot}} = \Delta p_A + \Delta p_B = 0 \]

\[ \downarrow \]

\[ \Delta p_A = -\Delta p_B \]
Velocity, acceleration, and force in rocket demo

**Observation**

\[ \bar{v} = \frac{\Delta x}{\Delta t} = \frac{10\text{m}}{4\text{s}} = 2.5 \frac{\text{m}}{\text{s}} \]

**One Way**

\[ a = \left(\text{assume const} \ a\right) \]

\[ \Delta x = \frac{1}{2} at^2 \Rightarrow a = \frac{2(\Delta x)}{t^2} \]

\[ a = \frac{2(10\text{m})}{(4\text{s})^2} = 1.25 \frac{\text{m}}{\text{s}^2} \]

\[ v_f = \left(1.25 \frac{\text{m}}{\text{s}^2}\right)(4\text{s}) = 5 \frac{\text{m}}{\text{s}} \]

\[ F = ma \]

\[ F = (113\text{kg})(1.25\text{m/s}^2) = 141 \text{ N (about 32 lb)} \]

**Another Way**

\[ v_f = \left(\text{assume } a = \text{const} \right) \bar{v} = \frac{v_f + v_i}{2} \]

\[ 2.5 \frac{\text{m}}{\text{s}} = \frac{v_f + 0}{2} \Rightarrow v_f = 5 \frac{\text{m}}{\text{s}} \]

\[ a = \frac{v_f - v_i}{\Delta t} \]

\[ a = \frac{(5-0)\text{m}}{4\text{s}} = 1.25 \frac{\text{m}}{\text{s}^2} \]
\[ F_{\text{on CO2}} = - F_{\text{on board}} \]
\[ F_{\text{on Dave}} = - F_{\text{on CO2}} \]
\[ F_{\text{external on system}} = 0 \implies a_{\text{system}} = 0 \]

**Internal forces only**
Consider object of mass \( m \) on a surface

Free body diagram (of all forces on the object)

\[ N - 2^{\text{nd}} \text{ Law} \]

[for total force = sum of individual forces]

\[ \sum F_y = N - mg = ma_y = 0 \]

Object does “hop” or “burrow” so \( a_y = 0 \)

The sum of individual forces in y-direction are “balanced” so that the net force (in y-direction) zero.

There could be “unbalanced” force in x-direction.
Note: balanced forces in y-direction are assumed and ignored.
Now define **system** carefully
to solve specific question asked !!!

Internal forces (e.g. $T$) do not appear- only outside to inside forces.

2 masses

```
<table>
<thead>
<tr>
<th>m_2</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>m_1</td>
<td>P</td>
</tr>
</tbody>
</table>
```

*Ask* $a = ?$

$$a = \frac{P}{(m_1+m_2)}$$

```
<table>
<thead>
<tr>
<th>m_2</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>m_1</td>
<td>P</td>
</tr>
</tbody>
</table>
```

“**system**” cut at point of internal force action

*Ask* $T = ?$

$$T = m_2 \cdot a = m_2 \frac{P}{(m_1+m_2)}$$

4-6a
3 coupled masses

\[ (3) \ T_2 = m_3 \ a \]
\[ (2) \ T_1 - T_2 = m_2 \ a \]
\[ (1) \ P - T_1 = m_1 \ a \]

very hard way

Add (1)+(2)+(3) \[ \Rightarrow \ P = (m_1 + m_2 + m_3) a \]

Ask \( a = ? \)

easy way

\[ P = (m_1 + m_2 + m_3) \ a = Ma \]
\[ a = \frac{P}{M} \]

Carefully define “system” to attack question asked!!
Ask $T_2 =$?

$T_2 = m_3 \cdot a$

$T_2 = m_3 \cdot \frac{P}{M}$

$P = (m_1 + m_2 + m_3) \cdot a = Ma$

$a = \frac{P}{M}$

“system” cut at point of internal force action
Man holding rope attached to box

Note: It is total F that = ma

\[ F = ma \]

\[ T - mg = ma \]

\[ T > mg \quad a > \# \]

\[ T < mg \quad a < \# \]

\[ T = mg \quad \Delta a = 0 \]

\[ \frac{1}{V} = \text{const.} \]

\[
\begin{align*}
\text{Total force} &= ma \\
T &= m(a + g) \quad \text{Big } a \quad \text{Big } T
\end{align*}
\]

\[ \text{Also} \]

\[ T = m(a + g) \]
\[ \sum F = ma \]

\[ P - (m_1 + m_2 + m_3)g = (m_1 + m_2 + m_3) a \]

\[ a = \frac{P - (m_1 + m_2 + m_3)g}{(m_1 + m_2 + m_3)} \]

\[ \text{Example:} \]
\[ m_1 = 12 \text{ kg}, \quad m_2 = 0.8 \text{ kg}, \quad m_3 = 10 \text{ kg}, \quad P = 36 \text{ N} \]

\[ a = \left( \frac{36 - \left[ \frac{12 + 8 + 10}{12 + 8 + 10} \right] \text{ N}}{3.0 \text{ Kg}} \right) = +2 \frac{\text{m}}{\text{s}^2} \]

\[ g = +2 \frac{\text{m}}{\text{s}^2} \]
Q. What is $T$ between $m_2$ and $m_3$?

$$T - m_3 g = m_3 a$$

$$T = m_3 [a + g]$$

recall $a = \frac{p - (m_1 + m_2 + m_3)g}{m_1 + m_2 + m_3}$

$$T = (1.0)(10g) \left[ + \frac{2m_1 s^2}{s^2} + 10m_3 s^2 \right]$$

$$= 12 \times \frac{m_3 s^2}{s^2} = 12 N$$

“system” cut at point of internal force action
Going around corner, carefully choose sign convention consistently !!!
Choosing “system” to answer question!!!
\[ m_3g = (m_1 + m_2 + m_3)a \]

\[
\frac{m_3}{(m_1 + m_2 + m_3)} g = a
\]

\[
\frac{3}{(1+2+3)} 9.8 = a
\]

\[
[\text{m/m}] [\text{m/s}^2] 4.9 = a
\]
Forces from outside box (system) acting to inside = $M_{\text{inside box}} \ a_{\text{box}}$

$$a = \frac{(m_c - m_A)}{(m_A + m_B + m_c)} g$$

$$= \frac{(3-1) \ [\text{kg}]}{(1+2+3) \ [\text{kg}]} \ 9.8 \ [\text{m/s}^2]$$

$$a \approx \frac{10}{3} \ \text{m/s}^2$$
Choosing “systems” to answer question!!!
Inclined plane abc’s

\[ N = \text{Normal force of plane on object} \]

\[ W = \text{Force of gravity on object (Pull of earth on object)} = mg \]
Demo

Forces add like vectors

\[ \mathbf{F}_{\text{tot}} = 0 \iff \mathbf{a} = 0 \]

\[ F_{\text{tot}} = 0 \iff a = 0 \]

\[ P = mg \sin \theta \]

\[ N = mg \cos \theta \]

\[ \begin{align*}
  m &= 1 \text{ kg} \\
  \theta &= 30^\circ \\
  \sin 30^\circ &= \frac{1}{2} \\
  \cos 30^\circ &= \frac{\sqrt{3}}{2} \\
  P &= 1(9.8) \times \frac{\sqrt{3}}{2} \\
  P &= 5N \\
  N &= 1(9.8) \times \frac{1}{2} \\
  N &= 8.7 \text{ N}
\end{align*} \]
\[ F_y = N - mg \cos \theta = 0 = a_y \]  
(no motion in y dir)

\[ F_x = mg \sin \theta = ma_x \]

\[ a_x = g \sin \theta \]
Balanced force components in perp. Dir. \( N - Mg \cos(\theta) = 0 \)

\[-Mg \sin(\theta) + mg = (M+m)a\]

\[
\frac{[-M \sin(\theta) + m] \cdot g}{(M+m)} = a
\]
Recall object on table

\[ N - mg = ma_y = 0 \]

Impose force parallel to surface (no friction)

\[ P = ma_x \]

Now add frictional force parallel to surface

- \( F_f \) frictional force of surface on object
- \( F_f \) opposes motion

Two Cases

- Static Friction (no motion)
- Kinetic Friction (moving)
Stationary Object: Static Friction
(pulled with force P)

\[-F_f + P = ma_x = 0\]

For static friction ("always")

\[F_f = P\]

P increases \(\Rightarrow F_f\) increases

There is a largest value of \(F_f\) for which the object remains at rest.

\[F_f(\text{max}) = \mu_s N\]

\[0 \leq \mu_s < 1\]

\[\mu_s = \text{coefficient of static friction}\]

In this simple case \(N = mg\)
Moving Object: Kinetic Friction (sliding) (projected at velocity $v_i$)

\[-F_f = m a_x\]

For kinetic friction (“always”)

\[F_f = \mu_k N\]

\[N - mg = ma_y = 0\]

\[N = mg\]

\[m a_x = -\mu_k N\]

\[m a_x = -\mu_k mg\]

\[a_x = -\mu_k g\]

\[v = v_i -\mu_k g t\]

Friction slows object down

Note $v_i$ shown in figure IS NOT A FORCE !!!!
Moving Object: Kinetic Friction (sliding)
(pulled with force $P$)

$-F_f + P = ma_x$

For kinetic friction ("always")

$F_f = \mu_k N$

$N = mg$

$N - mg = ma_y = 0$

$ma_x = P - \mu_k N$

Note: There are two possibilities in this kinetic case.

a) non zero acceleration

\[
a_x = \frac{P - \mu_k N}{m}
\]

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b) zero acceleration

\[
P = \mu_k N
\]

$\quad a_x = 0$

object moving

$v = \text{constant}$
Note: normal force is not always \( mg \)

For kinetic friction ("always")

\[
F_f = \mu_k N
\]

\[
ma_x = P - \mu_k N
\]

\[
ma_x = P - \mu_k (mg + R)
\]

Vertical force contributes to retarding horizontal force
Pulling a Tablecloth with Dishes on It

First consider dishes + tablecloth as "system":

\[ F = (m_1 + m_2) a \]
\[ \Rightarrow a = \frac{F}{m_1 + m_2} \quad (1) \]

Now consider dishes as system:

\[ f_s = m_1 a = \frac{m_1}{m_1 + m_2} F \text{ using (1).} \]

But we also know that

\[ N - W = 0 \Rightarrow N = W = m_1 g. \]

And \( f_s \leq \mu_s N = \mu_s m_1 g \).

So \( \frac{m_1}{m_1 + m_2} F \leq \mu_s m_1 g \Rightarrow F \leq \mu_s g (m_1 + m_2) \)

**If \( F \) becomes greater than this value, tablecloth starts sliding under dishes!**

\[ \bullet \text{if a smaller than this value the tablecloth + dishes move together} \]

\[ \bullet \text{if a is larger than this value the tablecloth starts sliding under the dishes} !!!! \]

so pull it with big \( a \) !!!

\[ a \leq \mu_s g \quad !!!!! \]

\[ a (m_1 + m_2) \leq \mu_s g (m_1 + m_2) \]

M. Kalekar

M. Croft

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If $P$ is greater than this dishes break free from sheet

\[ f = m_d \frac{P}{(m_d + m_s)} \leq \mu_s m_d g \]

\[ P \leq \mu_s g(m_d + m_s) \]

Dishes moving with sheet case

\[ P = (m_d + m_s) a \]

\[ a = \frac{P}{(m_d + m_s)} \]

\[ N = m_d g \]

\[ f \leq \mu_s N \]

\[ f \leq \mu_s m_d g \]
dishes sliding on sheet case

\[ P - f = m_s a_s \]

big \( P \gg f \)
big \( a_s \gg g \)

\[ f = \mu (m_d g) = m_d a_d \]

\[ \mu g = a_d \]

small \( \mu \)
small \( a_d \ll g \)

\[ a_d \ll \ll a_s \]

sheet accelerates rapidly out from under dishes!!
Suppose there is a frictional force on \( m_c \).

\[
\text{Tot. external force} = (\text{tot. mass}) \cdot a
\]

\[
-a_A g + m_c g - \mu m_A g = (m_A + m_B + m_c) \cdot a
\]

\[
a = \frac{g}{m_A + m_B + m_c} \left[ m_c - m_A - \mu m_B \right]
\]

Let \( \mu = 0.5 \)

\[
a = \frac{10 \left[ 3 - 1 - (0.5)^2 \right]}{1 + 2 + 3} = \frac{10}{6} (1) = 1.66 m/s^2
\]

\[
a = \left[ \frac{3.33}{\text{no friction}} - \frac{1.66}{\text{due to friction}} \right] = 1.66 m/s^2
\]
\[ \mu_S \text{ static friction coefficient determination} \]

\[ \begin{align*}
    y, \perp \text{ to plane: } & \quad N - mg \cos(\theta) = ma_y = 0 \\
    x, \parallel \text{ to plane: } & \quad -F_f + mg \sin(\theta) = ma_x
\end{align*} \]

\[ F_f = mg \sin(\theta) \]

experiment: increase \( \theta \) until \( F_f \) reaches maximum

then \( F_f = F_S = \mu_S N = \mu_S mg \cos(\theta) \)

\[ \mu_S \frac{mg \cos(\theta)}{\cos(\theta)} = mg \sin(\theta) \]

\[ \mu_S = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta) \]

observe \( \theta = 21^\circ \Rightarrow \tan(\theta) = 0.38 = \mu_S \]

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kinetic friction coefficient determination

\[ N - mg \cos(\theta) = ma_y = 0 \]

\[ -F_f + mg \sin(\theta) = ma_x \]

for kinetic case

then \( F_f = F_K = \mu_K N = \mu_K \cdot mg \cos(\theta) \)

\[ -\mu_K \cdot mg \cos(\theta) + mg \sin(\theta) = ma_x \]

\[ a_x = g \left[ \sin(\theta) - \mu_K \cos(\theta) \right] \]

experiment: observe \( a_x \) at given \( \theta \)
Air/Fluid Resistance to Moving Object
Case object moving fast (golf ball...)

\[ v_{\text{rel}} = \text{speed relative to air} \quad (\text{air at rest } v_{\text{rel}} = v) \]
\[ v_t = \text{terminal velocity} \quad (\text{air moving at } v_a / v_{\text{rel}} = v + v_a) \]

for falling object of mass \( m \)
\[ A = \text{cross-sectional area of object} \]
\[ \rho = \text{density of gas(fluid)} \]
\[ C = \text{drag coeff.} \]

\[ F_{\text{res}} = [C \rho A/2] \ v_{\text{rel}}^2 \]

\[ ma = mg - \frac{C \rho A}{2} \ v^2 \]

if \( a=0 \quad v=v_t \)

\[ 0 = g - \frac{C \rho A}{2m} \ v_t^2 \]

\[ v_t = \sqrt{\frac{2mg}{CA \rho}} = \sqrt{gL} \]

\[ L = \frac{2m}{C \rho A} \]

sky diver example
\( m = 70 \text{ Kg}; \quad A = 0.7 \text{ m}^2 \)
\( C = 0.4 \)
\( \rho(\text{air}) = 1.2 \text{ Kg/m}^3 \)
\( v_t = 64 \text{ m/s (140 mph)} \quad L = 417 \text{ m} \)